ZBIGNIEW ZARZYCKI*, KAMIL URBANOWICZ*

The influence of hydraulic pressure conduits’ parameters on the course of unsteady flow with cavitation

Keywords
Cavitation, unsteady flow, waterhammer, hydraulic systems, method of characteristics.

Summary
The paper attempts to assess the influence of closed conduits’ parameters on maximum pressure values recorded as a result of waterhammer and on the time of keeping unsteady flow with cavitation. A number of numerical tests, necessary for carrying out this research program, were performed using the authors’ own programs written in Matlab and based on effective cavitation transient models, which take into account unsteady hydraulic friction. In particular, the results of two key models were investigated; i.e. bubbly cavitation model (BCM) and column separation model (CSM).

* Faculty of Mechanical Engineering, Szczecin University of Technology, Department of Mechanics and Machine Elements, Piastów 19, 70-310 Szczecin, Poland. Phone: +48 091 449 42 57, fax: +48 091 449 45 64, e-mail: zbigniew.zarzycki@ps.pl, kurbanowicz@ps.pl.
1. Introduction

Unsteady liquid flow in hydraulic systems takes place as a result of violent changes of flow velocity. Usually, these changes are caused by a sudden opening and closing of a valve as well as by a sudden turning on or turning off of the pump operating in a system. Such flows are also referred to as waterhammer effect. Waterhammer is accompanied by a sudden pressure pulsation, which is very destructive and can lead to serious accidents.

When pressure drops to the level of saturated vapour pressure, cavitation areas are formed (i.e. areas filled with vapour), which are responsible for cavitational erosion of hydraulic elements. Therefore, the key issue is to assess both maximum values of pressure taking place at those moments (i.e. values which far exceed nominal values) and the duration of time in which unsteady liquid flow with cavitation actually takes place.

In earlier studies [1, 2, 3], mathematical models of unsteady liquid flow with cavitation taking into consideration unsteady hydraulic resistance (i.e. bubbly cavitation model BCM and column separation model CSM) were developed. These models quite accurately reflect the physics of the problem of unsteady liquid flow with cavitation. They also make it possible to analyse how some specific parameters of a long hydraulic line (initial pressure, initial velocity, length, the pipe’s inside diameter, the liquid’s viscosity) affect the course of unsteady liquid flow with cavitation.

Simulation studies were conducted for experimental data obtained from two systems known in the literature on hydraulic installations (Bergant–Simpson and Sanada–Kitagawa–Takenaki’ systems – detailed information about these systems can be found in attachment A) [4, 5, 6, 7]. This allows the authentication of tendencies observed in this present study. The results of analysis were presented in the form of diagrams showing tendencies of pressure changes and time duration changes of unsteady liquid flow with cavitation.

The knowledge of such tendencies for all parameters describing a given hydraulic system can be used while both analysing and designing new hydraulic systems and modernising already existing systems.

2. Mathematical models of cavitational flow in pressure pipes

In order to simulate cavitational liquid flow two commonly applied models – CSM (column separation model) and BCM (bubbly cavitation model) - were used. CSM is a discrete model. It assumes that cavitation takes place at a given place of liquid flow and that it leads to a disturbance of this flow’s continuity. In this case, liquid flow in all the flow areas is given by equations [4, 7]:

...
The influence of hydraulic pressure conduits’ parameters on the course …

- momentum equation:

$$\frac{dp}{dx} + \rho_1 \cdot \frac{dv}{dt} + \rho_1 \cdot g \cdot \sin \gamma + \frac{2}{R} \cdot \tau_w = 0$$  \hspace{1cm} (1)

- continuity equation:

$$\frac{dp}{dt} + \rho_1 \cdot c^2 \frac{dv}{dx} = 0$$  \hspace{1cm} (2)

where: \( c \) – is the velocity of propagation of pressure waves [m/s], \( g \) – acceleration of gravity [m/s²], \( p \) – pressure [Pa], \( t \) – time [s], \( v \) – mean flow velocity [m/s], \( x \) – location of cross-section [m], \( R \) – radius of pipeline [m], \( \gamma \) - pipeline’s angle of inclination [°], \( \rho_l \) – liquid density [kg/m³], \( \tau_w \) – shearing stress at pipeline’s wall [N/m²].

BCM is a continuous, homogeneous cavitation model, where cavitation is assumed to take place along the axis of a pipeline. The fundamental flow equations are given below [7]:

- momentum equation:

$$\rho_m \cdot \frac{d}{dt} \left( \frac{v}{\alpha} \right) + \frac{dp}{dx} + \frac{2}{R} \cdot \tau_w + \rho_m \cdot g \cdot \sin \gamma = 0$$  \hspace{1cm} (3)

- continuity equation:

$$\frac{1}{c^2} \cdot \frac{d}{dt} \left( \rho_1 - \rho_v \right) \cdot \frac{d\alpha}{dt} + \rho_m \cdot \frac{d}{dx} \left( \frac{v}{\alpha} \right) = 0$$  \hspace{1cm} (4)

where: \( \rho_m \) – is the density of mixture [kg/m³], \( \alpha \) - coefficient of liquid phase concentration, \( \rho_v \) – vapour density [kg/m³].

Equations (1), (2), (3) and (4) were solved using the characteristics’ method. Calculation procedures helpful in simulating flow parameters can be seen in earlier studies [1, 2, 3].

3. Simulation results

Below you can see the results of computer simulations, in which the influence of hydraulic parameters on unsteady liquid flow with cavitation was investigated. Maximum pressure values were analysed for the first “\( p_1 \)” and the second “\( p_2 \)” pressure amplitude (Fig. 1). It is commonly known that, sometimes
as a result of superposition of pressure waves caused by a sudden turning off of a valve and then a shut-off of cavitation area, the maximum pressure can take place not at the first, but at the second pressure amplitude [8, 9,10]. The time of unsteady flow with cavitation “tₑ” was assumed as the time of the last cavitational area (Fig. 1).

The influence of the following parameters on the course of pressure changes during waterhammer with transient cavitation was analysed in detail; initial pressure “pₑ”, initial velocity “vₒ”, pipeline’s length “L”, liquid’s viscosity “ν”, pipeline’s inside diameter “D”. These parameters are graphically presented in Figures 2–6.

The presented diagrams shown in Figures 2–6 clearly present certain tendencies of maximum pressure values at the cross-section at the shut-off valve and the time values of unsteady liquid flow with cavitation. Particular attention should be paid to both the diagram of maximum pressures (Fig. 2) for Bergant – Simpson’s system for a given changeable initial pressure “pₑ” and the diagrams showing the tendency of “D” pipeline’s inside diameter’s influence (Fig. 6).

While analysing the influence of the initial pressure “pₑ”, you can observe a stepwise decrease of maximum pressure at the valve for initial pressure values of approximately 1.6 MPa. This decrease can be due to the fact that pressure waves do not superimpose, because the time of discontinuity area between the first and the second pressure amplitude is too short (Fig. 2 – Bergant – Simpson’s system; te-pₑ diagram). In many other simulations, not presented in this paper, it was also observed that the ratio between p₁/p₂ higher than unity (maximum pressure of waterhammer effect occurs at the first amplitude of pressure) always takes place when there is a very short liquid column separation, i.e. when a single and small cavitation area is created.
a) The influence of initial pressure $p_p$ ($p_p=\text{var}; (v_o, L, c, \nu, \rho, D)=\text{const}$)

Bergant-Simpson’s system

Sanada-Kitagawa-Takenaki’s system

Fig. 2. The influence of initial pressure $p_p$
Rys. 2. Wpływ ciśnienia początkowego $p_p$
b) The influence of initial velocity $v_o$ ($v_o$=var; ($p_p$,$L$,$c$,$\nu$,$\rho$,$D$)=const)

b) Wpływ prędkości początkowej $v_o$ ($v_o$=var; ($p_p$,$L$,$c$,$\nu$,$\rho$,$D$)=const)

Bergant-Simpson’s system

Sanada-Kitagawa-Takenaki’s system

Fig 3. The influence of initial velocity $v_o$

Rys. 3. Wpływ prędkości początkowej $v_o$
c) The influence of the pipeline’s length $L$ ($L=\text{var}$; $(p, \nu, c, v, \rho, D)=\text{const}$)

c) Wpływ długości nacięgu $L$ ($L=\text{var}$; $(p, \nu, c, v, \rho, D)=\text{const}$)

Fig. 4. The influence of pipeline’s length $L$
Rys. 4. Wpływ długości nacięgu $L$. 
d) The influence of liquid’s kinematic viscosity $\nu$ ($\nu$=var; $(p,p_x,L,c,p,D)$=const)

d) Wpływ lepkości kinematycznej $\nu$ ($\nu$=var; $(p,p_x,L,c,p,D)$=const)

Fig 5. The influence of kinematic viscosity of liquid $\nu$

Rys. 5. Wpływ lepkości kinematycznej $\nu$
c) The influence of the pipeline’s inside diameter D (D=var and c(D)=var; (p, v, L, ρ)=const)

On the other hand, the analysis of the influence of the inside diameter of the pipeline “D” (Fig. 6) shows that one deals here with a very interesting case. It is hard to explain why when pressure “p1” decreases at the amplitude, the pressure at the second amplitude “p2” initially increases (from a value at which the ratio between p1/p2 is definitely higher than unity) and then starts to fall. A similar
situation is also observed in diagrams showing the times of unsteady liquid flow with cavitation “$t_e$”. This particular course of “$p_2$” can be explained by a complicated influence of the inside diameter on unsteady liquid flow with cavitation. This diameter affects, through the coefficient of hydraulic resistance and Reynolds number, hydraulic resistance and the velocity of pressure waves’ propagation. When the diameter increases, both hydraulic losses and the velocity of pressure waves’ propagation decrease.

4. Conclusions

The paper presents the results of numerical simulations in which effective models of transient cavitation were used. As it can be clearly seen from the presented comparison of two different hydraulic systems (i.e. Bergant – Simpson and Sanda-Kitagawa-Takenaka’s systems), the tendencies of both pressure changes and the time of unsteady liquid flow with cavitation are very similar. Owing to that, it can well be assumed that for other hydraulic systems similar tendencies will be also kept.

The analysis of each parameter separately made it possible to determine the influence of this parameter on the maximum pressure and the length of time of unsteady liquid flow with cavitation. Summing up the results of the analysis of the diagrams it can be said that when

\[ p_t \uparrow \text{ Then } p_1 \uparrow \text{ and } p_2 \uparrow \text{ and } t_e \downarrow \]
\[ v_s \uparrow \text{ Then } p_1 \uparrow \text{ and } p_2 \uparrow \text{ and } t_e \uparrow \]
\[ L \uparrow \text{ Then } p_1 \uparrow \rightarrow \text{ and } p_2 \downarrow \text{ and } t_e \uparrow \]
\[ v \uparrow \text{ Then } p_1 \rightarrow \uparrow \text{ and } p_2 \downarrow \text{ and } t_e \downarrow \]
\[ D \uparrow \text{ Then } p_1 \downarrow \text{ and } p_2 \downarrow \downarrow \text{ and } t_e \uparrow \downarrow \]

The above analysis makes it possible to optimise parameters of a long hydraulic line in case of flow discontinuity.
Attachment A

Bergant – Simpson’s experimental system I:

The liquid’s kinematic viscosity: $1 \times 10^{-6}$ [m$^2$/s]
The pipe’s length: $L=37.2$ [m]
The pipeline’s inside diameter: $D=0.0221$ [m]
The liquid’s initial velocity (in a steady motion): $v_o=1.50$ [m/s]
The liquid’s density: $\rho_l=1000$ [kg/m$^3$]
The vapour’s density: $\rho_v=0.8$ [kg/m$^3$]
The velocity of pressure waves’ propagation: $c=1319$ [m/s]

Sanada – Kitagawa – Takenaki’s experimental system II:

The liquid’s kinematic viscosity: $1 \times 10^{-6}$ [m$^2$/s]
The pipe’s length: $L=200$ [m]
The pipeline’s inside diameter: $D=0.0152$ [m]
The liquid’s initial velocity (in a steady motion): $v_o=1.45$ [m/s]
The liquid’s density: $\rho_l=1000$ [kg/m$^3$]
The vapour’s density: $\rho_v=0.8$ [kg/m$^3$]
The velocity of pressure waves’ propagation: $c=820$ [m/s]
References


Manuscript received by Editorial Board, October 10, 2008

Wpływ parametrów hydraulicznych przewodów ciśnieniowych na przebieg niestacjonarnego przepływu z kawitacją

Streszczenie

W pracy podjęto próbę oszacowania wpływu parametrów przewodów zamkniętych na wartości maksymalnych ciśnień występujących w wyniku uderzenia hydraulicznego, jak i na czas utrzymywania się przepływu nieustalonego z kawitacją. Niezbędne do realizacji tej pracy liczne badania numeryczne, wykonane zostały z wykorzystaniem własnych programów napisanych w Matlabie bazujących na efektywnych modelach kawitacji przejściowej, w których uwzględniono niestacjonarne opory hydrauliczne. W szczególności rozpatrywano wyniki otrzymane z dwóch kluczowych modeli: kawitacji pęcherzykowej (bubbly cavitation model – BCM) oraz rozerwania słupe cieczy (column separation model – CSM).