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Factorisation algorithm-based method used for the calculation of network system's reliability

K e y w o r d s

Rreliability, factorisation algorithm, network system.

S ł o w a k l u c z o w e

Niezawodność, algorytm faktoryzacji, układy sieciowe.

S u m m a r y

This study presents a factorisation algorithm to be considered as a method for reliability assessment of complex technical systems, in particular, network systems. The well-known, classical methods for the calculation of structural reliability are ineffective, or inapplicable in the case of actually exploited complex network systems. Difficulties connected with the calculation of their reliability structure are considered the main problems. A factorisation algorithm based on graph theory allows the calculation the reliability of the network system without the determination of its reliability structure, and it can also be used in the case of systems of known structures, giving results that are compatible with classical methods.

This paper also presents the assumptions, basic principles, and advantages of the factorisation algorithm used for network systems. Examples of analyses using the presented method for simple network systems and their practical use in the assessment of the reliability of a fragment of a real gas network are included. The obtained results confirm the usability of the method for structural reliability assessment of network systems, as well as the facility of

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comparing structural reliability of designed and modernised network systems. It was also observed that the method in question can be improved in the future by taking under consideration in the calculation such parameters as the number, reliability and localisation of supply sources in the network structure.

Introduction

Reliable operation of electrical power distribution, water distribution, or computer networks is very important, and the widespread application and complexity of network systems results in an increased number of system failures. Beside customer inconveniences, it also results in financial losses for suppliers and users. In some cases, network failures may result in threats to life and health, including ecological hazards.

The reliability of network systems and the optimisation of their structures aimed at the assurance of required reliability levels and the safety of their operations have been a target of numerous studies for many years [1, 2, 3, 5, 6, 9, 10]. The difficulty of calculation reliability structures of such network systems is a main problem in calculation reliability structure of these systems, which is required in majority of known calculation methods [6, 8, 10].

The most complex (with respect to the structure complexity) network systems occur in urban areas, where they must be consequently developed and modernised. Calculating the reliability of these networks in design phase or before planned modernisation provides the possibility of comparing various variants for the selection of the best option. That is why there is a need for calculation methods allowing simple and fast analysis of the reliability of even complex network systems.

This paper presents the assumptions and basic principles of the method of network system structural reliability assessment based on factorisation algorithms with use of graph theory and their reduction. The presented method allows the calculation of the network reliability using various assumptions related to the system aptitude. There are also possible method modifications aimed at better modelling of the real network. Computerisation of the presented method together with its modification will allow fast computation for various structures and initial assumptions.

Methods of structural reliability assessment of network systems

Methods used in the calculation of technical system and network structural reliability can be divided into the following [6]:

– Analytical methods based on the analysis of random events and processes,
– Simulation methods based on the simulation of random events and processes, and
– Mixed methods combining analytical and simulation methods.

The method of "complete survey" (survey of all states) is one of the best-known analytical methods [4, 6, 8]. This method can be independently used for any complex system on a number of individual elements. The rapidly growing number of the system states, including an increase in the number of elements is considered the main disadvantage of this method, and this method is ineffective for complex networks systems.

The method of "minimal operating paths and minimal cuts" is a method limiting the number of the tested system states being minimal operating paths or minimal cuts that are taken into consideration [4, 5, 8].

Damages tree method is also considered as a method of structural analysis, which uses Boole's algebra. Logic functors realizing logical product or logical sum are used for the damages tree construction.

In order to make calculations, the damages tree is transformed into a form in which each entrance to logical functors corresponds to failure of another element. In the case of complex systems, these transformations are difficult and ineffective.

Methods of simple and complex decomposition are used for testing the reliability structure of the system with respect to a single element (or group of elements k) called a "decomposition element" [8]. The efficiency of this method depends mainly on the selection of the decomposition element. However, procedures of this element selection are not determined, and the application of this method in the case of complex systems is also very difficult.

Simulation methods are considered a separate group of calculation methods. Application of these methods depends on the system structure, mathematical relationships characterising the tested system and relationships between elements, including required reliability characteristics.

Methods developed specially for concrete complex technical systems can also be distinguished among the methods aimed at the assessment of technical systems. They are often developed as combinations of the methods mentioned above. They additionally allow, independently on structural reliability, calculation probability of continuous amount of the factor of needed quality. This group comprises highly specialised methods and computer programs used in reliability studies of only concrete network types (for example, water distribution or electrical power networks), usually on a fixed level (for example distribution networks) [6].

The selection of the method used in calculations depends mainly on possibility of using the method in a concrete situation. Usually, it depends on the following:

– The type and complexity of the system reliability network,
– The initial simplified assumptions considered in a given method,
– The type of information needed for the system reliability calculation,
– The labour consumption and time needed for making the calculations, and
– The possibility of obtaining the required reliability factors, including the accuracy of the results.

Assumptions and the basis of the structural reliability of network systems based on a factorisation algorithm

The mentioned network systems, such as natural gas and water distribution and electrical power grids, or computer networks, have physical structures, which are similar to graphs. That is why these networks are often modelled based on graphs. Obviously, suitable simplifying assumptions should be made. Despite the models of methods selected, calculation methods posses some of same generalised features, and they can be used for the calculation of the structural reliability of various network systems.

The factorisation method can be classified as such a method. In its standard form, the factorisation algorithm allows the calculation of the probability of the existence of connections between a distinguished assemblage of vertexes in a coherent and undirected graph. Assumptions related with the method, the procedure for modelling network structures using the graph and the possibilities of the reliability calculation using the factorisation algorithm, including its modifications, are presented below.

One of basic graph definitions assumes that the graph is defined as an assemblage of vertexes, which can be connected with their edges in such manner that each edge begins and ends in one of the vertexes. It can be described as follows [7]:

$$G = (V, E) \tag{1}$$

where: V – assemblage of the graph vertexes, $V = (v_1, v_2, ..., v_{n-})$,
E – assemblage of the graph edges, $E = (e_1, e_2, ..., e_{m-})$.

The graph is undirected if its edges have no defined directions (Fig. 1). Thus, we can describe that assemblage of edges E is a family of bi-elementary sub-assemblages of the assemblage of vertexes V:

$$E \subseteq \{\{u,v\} : u,v \in V, u \neq v\} \tag{2}$$

However, in case of the directed graph, each edge has a defined direction (Fig. 2).

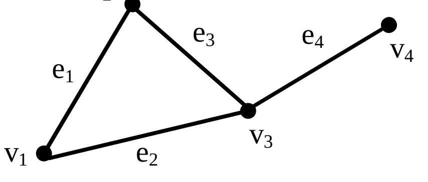

Fig. 1. Not-directed graph
Rys. 1. Graf nieskierowany

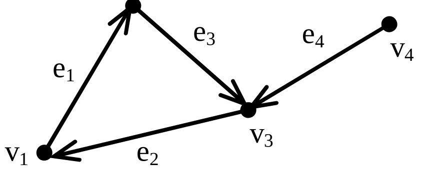

Fig. 2. Directed graph
Rys. 2. Graf skierowany

In order to derive a loop (Fig. 3), i.e. edge, which both ends form the same vertex, assemblages {v} or a single-element assemblage or a bi-element multi-assemblages {v,v} should be accepted in the graph definition.

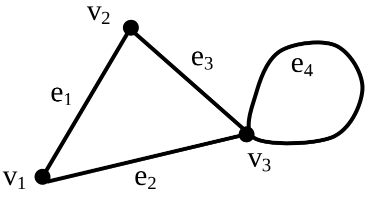

Fig. 3. Graph with loop
Rys. 3. Graf z pętlą

It is assumed that graph is coherent if, for each of its vertexes, there is a track to another vertex. Whereas, the track is defined as determined by the route of the edges, allowing relocation from a chosen initial vertex to a chosen final vertex [7].

If it is assumed that the tested network, for example a water or gas distribution system, is represented by undirected graph, the following must be true:

– Each edge (e_i) of the graph represents a single connection in a real network. For example, this would be a section of the pipe-line with mounted fittings connecting two furcation points (or one furcation point and final point, for example, a customer connection point) in a water distribution system.
– Graph vertexes (v_i) represent elements of real networks at points where furcation takes place (for example, three-way or four-way junction) or a final point (for example, a valve in the customer connection point).

Edge weights (connection weights) are also used in modelling using graphs. It is a numerical value characterising a given connection, which, depending on particular needs and network type, may determine the connection length, flow capacity, or pipe-line diameter. In the case of the analysis presented in this study, the connection weight is defined as the value of the reliability function at a chosen moment t. This value is marked as R_{ei}. The value of this function results from the value of the pipe-line reliability function and all fitting elements mounted on the pipeline. They will be considered as series connection system. Thus, we can write the following for a given moment t:

$$R_{ei} = Rr_{ei} \cdot \prod_{i=1}^{n} Ra_i \qquad (3)$$

where: Rr_{ei} – the value of the pipe-line reliability function at a distance of single connection,

Ra_i – the value of the fitting element reliability function on this connection.

Based on the above conclusions, we assume that a coherent undirected graph will represent the network system in which probabilities of correct operations of the connections are known. Occurring damages are independent. Such a graph is named a “stochastic” or “probabilistic” network.

The factorisation algorithm can be applied for the calculation of the reliability function value of the real network for which the graph is considered a model. This algorithm allows calculating the function value with no need of reliability structure determination. Additionally, the method allows defining various system states and the calculation of the reliability considered as the probability of the occurrence of the states in question.

The network reliability measure is defined as the probability that the network can realise its function, e.g. assure gas, water or current flow between terminals. Usually, it is a flow from the source to customers connection points. In the case of complex networks having closed connection rings, the flow can be realised in various (alternate) ways, which improves reliability but complicates the process of the calculation.

The factorisation algorithm allows the calculation of the probability of the connection between chosen assemblage (K) of the graph vertexes [1, 2]. It is a “K-terminal network” reliability problem. This means that terminals, which are in the assemblage K, are mutually connected via damaged connections. In a real system, it means that there is the possibility of flow from one of chosen terminals to all rather terminals of this assemblage. In this case, the probability of such event is a measure of the network reliability.

The assemblage of terminals K in each network can be determined in the following manner:

$$2 \le |K| \le |V|$$

Thus, with respect to a number of terminals in the assemblage K, an assumption of various states of aptitude is possible. In practice, two boundary conditions are considered:

– K=2 (two terminal network reliability problem): In a real network, the probability of a connection between chosen two graph vertexes is defined as the probability that from one terminal (for example source) there is a connection, and flow is possible to the second of chosen terminals (for example to a chosen customer).
– K=V (all terminal network reliability problem): The probability of the existence of connections between all graph vertexes. In a real network, it is probability that there is a connection from each terminal, and flow is possible to all other terminals, e.g. to all customers.

In case of networks in which the transmission of the medium from the source to customers takes place, the condition for K=2 can be determined as a reliability measure from the point of view of a single customer. However, if in the assemblage K terminals representing all customers are found, the sources can be determined as the network reliability from the point of view of a supplier.

We can assume the following:

– Graph vertexes (network nodes) are absolutely reliable, $Rv_i=1$.
– There is the probability of the edge (network connection) damage, $Fe_i=1-Re_i$.
– Failures of connections are independent.

States of the graph representing the network can be divided into two assemblages with respect to two possible edge states. Thus, the reliability of K-terminals of the graph can be expressed in the form of a conditional reliability formula [3]:

$$\begin{aligned} R(G_K) &= R_{ei} \cdot R(G_K \,|e_i \textit{ is in an operational state}) + \\ &+ (1-R_{ei}) \cdot (G_K \,|e_i \textit{ is in failure state}) \end{aligned} \tag{4}$$

Topological interpretation of this formula for undirected graphs is expressed by the general factorisation theory:

$$R(G_K) = R_{ei} \cdot R(G_{K'} * e_i) + (1-R_{ei}) \cdot R(G_K - e_i) \tag{5}$$

where: $R(G_K)$ – probability that all terminals from the assemblage K are connected,

Re_i – probability that i-edge is in an operational state,

e_i – arbitrary edge of graph G_K,

$G_{K'} * e_i = (V-u-v+w, E-e_i),\ w = u \cup v$ – graph reduction if connection e_i is in an operational state – from vertexes (V) are detached vertexes on ends of the connection $e_{I,}$ and they are replaced by vertex resulting from their connection, whereas connection e_i are removed from connections assemblage (E),

$$K' = \begin{cases} K & \textit{if } u, v \notin K \\ K-u-v+w & \textit{if } u \notin K \text{ or } v \in K, \end{cases}$$

$G_K - e_i = (V, E-e_i)$ – graph reduction when connection e_i is in failure state – assemblage of vertexes (V) is not changed and connection e_i is removed from the assemblage,

$R(G_{K'}*e_i)$ – probability that vertexes from assemblage K are mutually connected if edge e_i is in an operational state – this edge is removed and vertexes incident with the edge are transformed in single vertex,

$R(G_K - e_i)$ – probability that the vertexes from assemblage K are mutually connected when edge e_i is in failure state – this edge is removed.

The applied formulas (5) via reductive calling for graph representing the network allows its gradual reduction. During the reduction, only the cases that assure coherence of vertexes in the assemblage K are taken into consideration. The reduction allows the calculation of the probability that all vertexes from the assemblage K are mutually connected. Graphs developed during the reduction are simplified, so they posses only one edge and only one vertex less in case of graph $G_{K'}*e_i$ and one edge less than in case of graph $G_K - e_i$.

According to presented model, the calculation of the reliability measure of the network is represented by a coherent undirected graph in which vertexes (i.e. nodes in the network) are absolutely reliable. However, other assumptions can be made if, for example, a road network in an urban area is considered. In such a situation (crossings which are considered the network nodes) can be in failure state with determined probability, and connections between terminals (roads between crossings) are absolutely reliable. The procedure of the calculation of the probability of a connection between a chosen assemblage of road is presented in [9].

Example of the network reliability calculation

The procedure of the system reduction according to (5) is presented below. Relationships, whic h for a given value of the connections reliability function, allow the calculation of the network reliability function if two conditions, K = V (Fig. 4) and K = 2 (Fig. 5), have been determined. Re_i = Ri and Fe_i = Fi were introduced in order to simplify the calculations.

In the third example, values of the reliability function of a fragment of a real natural gas network (Fig. 6) have been calculated, which is supplied by a reduction station in point 1 (WE). Point 10 (WY) is a supplying point for the next network connected to the tested fragment. The values of the network reliability were determined using the factorisation method, Fig. 7.

The calculated probability of the existence of a connection between all network terminals (K = V = 17) means that gas can be sent to all network terminals. This probability determines the network reliability function in chosen time t. The calculated probability that gas can be sent from reduction station WE to point WY (K = 2 = {1, 10}) determines the probability of supplying a network connected to the tested fragment. Calculations were executed for assumed values of the reliability function of connections in network (R_{ei}), and the results are shown in Fig. 7.

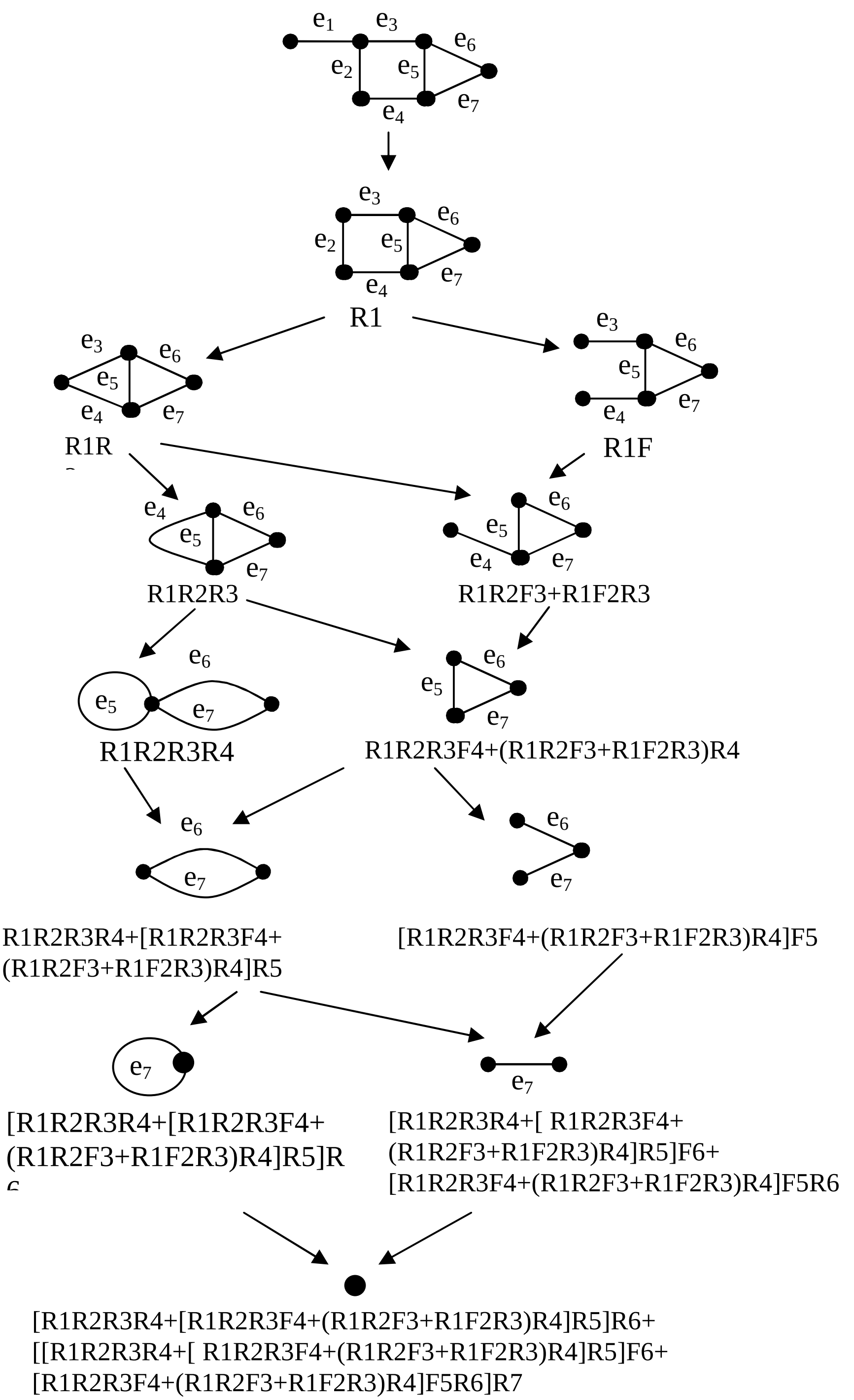

Fig. 4. Network reduction for condition K = V
Rys. 4. Redukcja sieci dla przypadku K = V

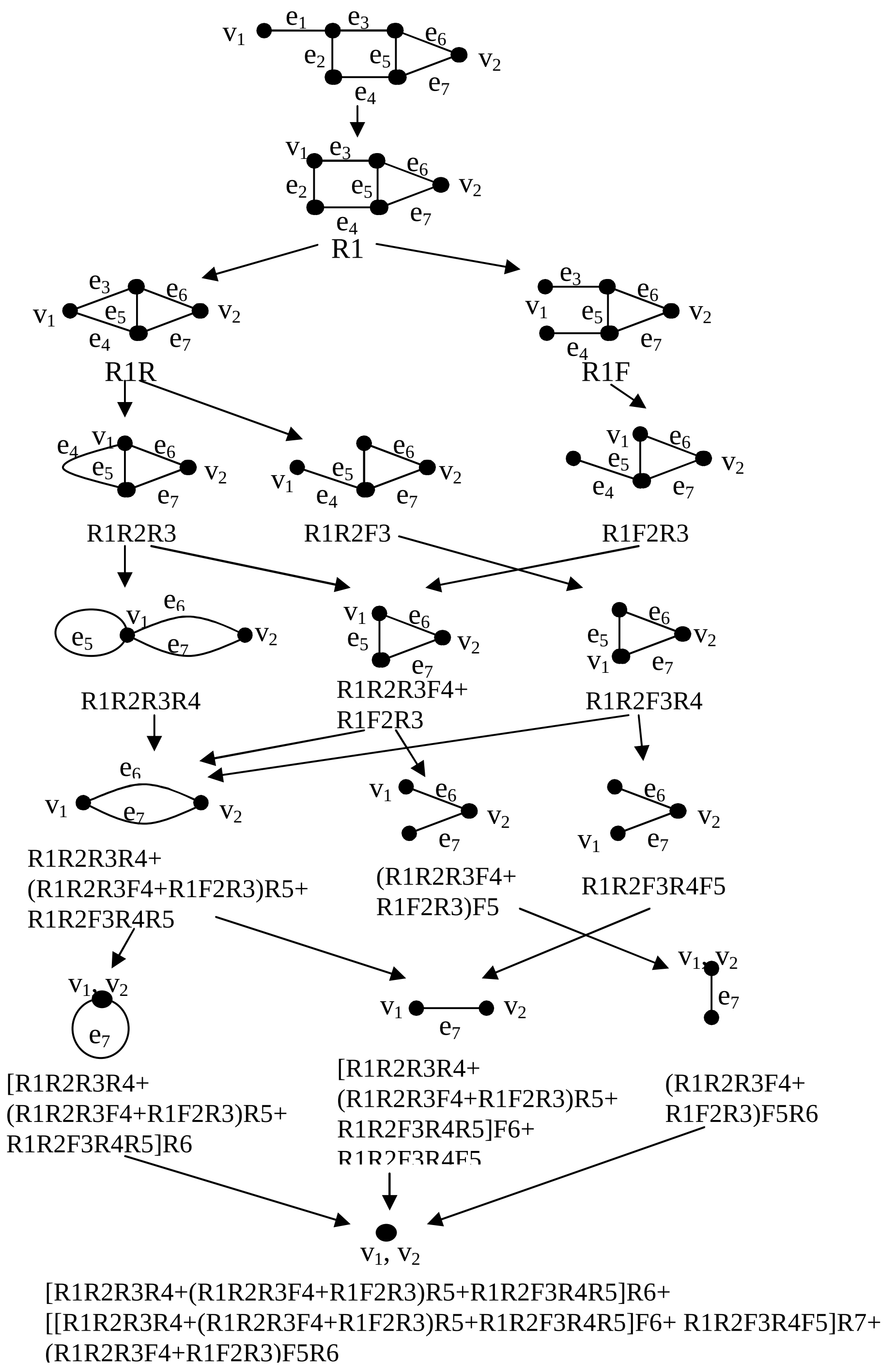

Fig. 5. Network reduction for condition K = 2 = {v_1, v_2}
Rys. 5. Redukcja sieci dla przypadku K = 2 = {v_1, v_2}

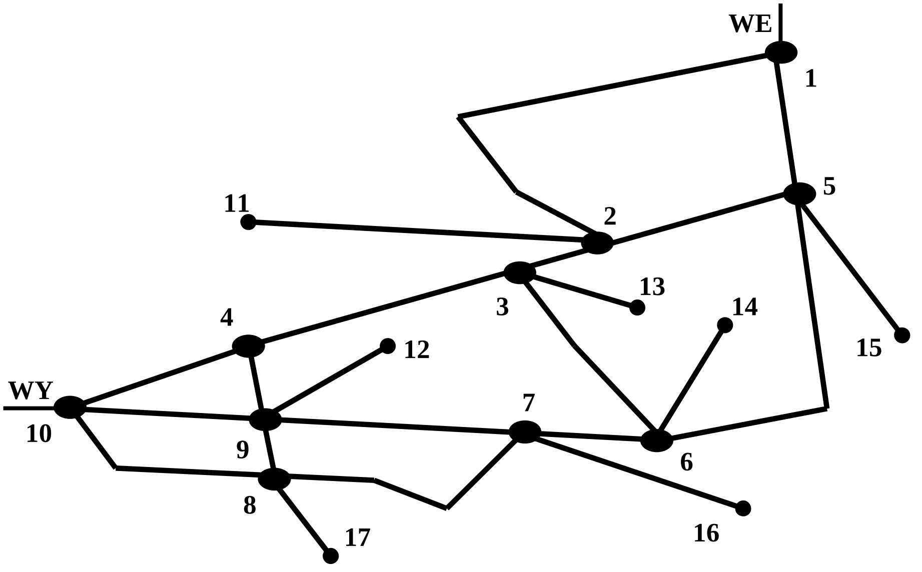

Fig. 6. The connection structure of the chosen fragment of a tested gas pipeline network
Rys. 6. Struktura połączeń fragmentu analizowanej sieci gazowej

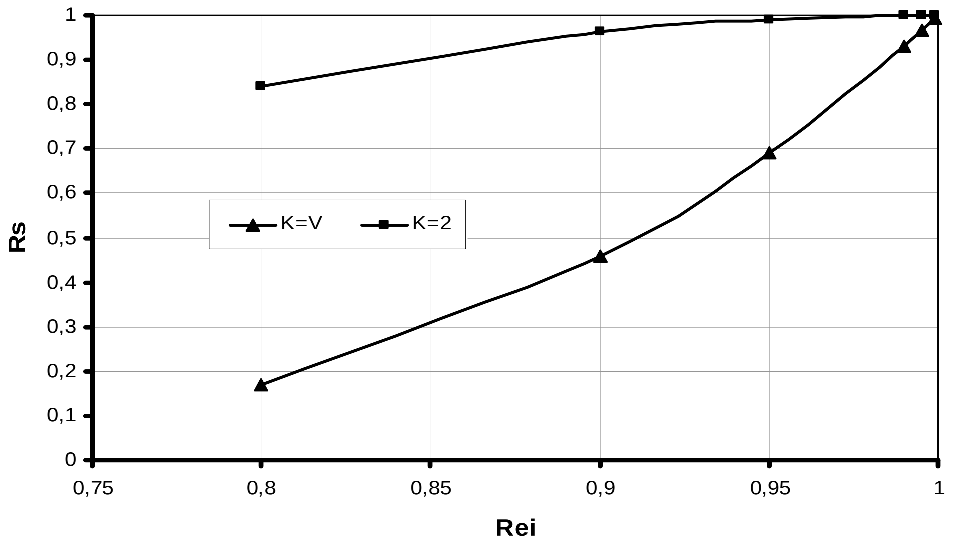

Fig. 7. Reliability function value (R_S) for chosen time t of the gas pipeline network fragment from Fig. 6 in relation to the reliability function value of the network connections (R_{ei})
Rys. 7. Wartości funkcji niezawodności (R_S) dla wybranej chwili t fragmentu sieci gazowej z Rys. 6, w zależności od wartości funkcji niezawodności jej połączeń (R_{ei})

Summary

The presented reduction procedure can be used for the development of a reliability function value of a complex network system for chosen time t. Cited examples prove that, the more complex system, the more difficult and time consuming is its reduction. Thus, computer algorithms allowing the reduction and calculation of the reliability function according to this method are extremely useful. Deriving these algorithms allowed fast and multiple reliability function calculations of the tested network connections.

The obtained results prove the usability of this method. This method can be used for testing reliability structures of designed and modernised gas or water distribution networks. The method in question also allows checking if the required probability level of delivery of given medium to chosen points of the network is assured.

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Metodyka wyznaczania niezawodności układów sieciowych w oparciu o algorytm faktoryzacji

S t r e s z c z e n i e

W artykule zaprezentowano algorytm faktoryzacji jako metodę umożliwiającą szacowanie niezawodności złożonych układów technicznych, a w szczególności układów sieciowych. Znane klasyczne metody wyznaczania niezawodności strukturalnej są mało efektywne lub wręcz niemożliwe do zastosowania w przypadku eksploatowanych współcześnie rozbudowanych układów sieciowych. Podstawowym problemem jest trudność w określeniu dla nich struktury niezawodnościowej. Oparty na teorii grafów algorytm faktoryzacji umożliwia wyznaczenie

niezawodności układu sieciowego bez określania jego struktury niezawodnościowej i może być również wykorzystany w przypadku układów o znanych strukturach, dając wyniki zgodne z metodami klasycznymi.

W opracowaniu przedstawiono założenia i podstawy algorytmu faktoryzacji w zastosowaniu do układów sieciowych oraz jego zalety na tle innych metod. Zamieszczono również przykłady analizy wg prezentowanej metody dla prostych struktur sieciowych i praktyczne wykorzystanie do szacowania niezawodności dla fragmentu rzeczywistej rozdzielczej sieci gazowej. Uzyskane wyniki potwierdziły użyteczność metody w szacowaniu niezawodności strukturalnej układów sieciowych oraz łatwość porównywania niezawodności strukturalnej sieci projektowanych lub modernizowanych. Zauważono również, że metoda ma możliwości dalszego doskonalenia poprzez uwzględnienie w obliczeniach liczby, niezawodności i lokalizacji w strukturze sieciowej jej źródeł zasilania.