SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
4 (168) 2011

BOGDAN LIGAJ[*]

# An analysis of the influence of cycle counting methods on fatigue life calculations of steel

Key words

Service load, cycle-counting methods, two-parameter characteristics of fatigue, fatigue life.

Słowa kluczowe

Obciążenie eksploatacyjne, metody zliczania cykli, dwuparametryczna charakterystyka zmęczeniowa, trwałość zmęczeniowa.

## Summary

In this paper, using the course of stress derived from load measurements of steering knuckle pin of a passenger car, the calculation of fatigue life under service loads is investigated. Adopted courses have been processed to designate a set of sinusoidal cycles by the following methods: peak counting method, simple-range counting method, full cycle counting method, range-pair counting method, and rainflow counting method. Based on set of cycles, with variable parameters $S_{m\,I}$ and $S_{ai}$ block load spectrums have been developed for substitute amplitude $S_{az}$, designated using the author's method, which uses two-parametric fatigue characteristics. The result of this paper is a comparison of fatigue life for load spectrums determined from the established methods of calculating random loads cycles of a broad spectrum, which are characterised by variable values of the coefficient I.

---

[*] University of Technology and Life Sciences in Bydgoszcz, Faculty of Mechanical Engineering, Department of Machine Design, ul. Prof. S. Kaliskiego 7, 85-789 Bydgoszcz, Poland, phone: +48 52 340 82 53, fax: +48 52 340 82 71; e-mail: bogdan.ligaj@utp.edu.pl.

## Nomenclature

$C_{(-1)}$ – constant in the formula describing Wöhler fatigue diagram for oscillating load ($R = -1$),
$I$ – coefficient characterising the width of the random loading spectrum,
$N$ – cycle number – general notation (fatigue life),
$N_0$ – base number of cycles corresponding to fatigue life ($N_0 = 10^6$),
$R$ – cycle asymmetry ratio ($R = S_{min}/S_{max}$),
$R_e$ – material yield point in MPa,
$R_m$ – material tensile strength in MPa,
$S_a$ – sinusoidal cycle stress amplitude in MPa,
$S_{az}$ – substitute stress amplitude for a sinusoidal cycle with parameters $S_m$ and $S_a$ in MPa,
$S_{a(R)}^{(T)}$ – sinusoidal cycle stress amplitude with a specific coefficient value R determining the constant fatigue life amplitude (N = const.) in MPa,
$S_{f\,(-1)}$ – fatigue limit under oscillating load ($R = -1$) for N0 cycle number in MPa,
$S_i$ – local stress values on the “I” level of loading in MPa,
$S_m$ – mean sinusoidal cycle stress in MPa,
$S_{m(R)}^{(T)}$ – average value of a sinusoidal stress cycle with a specific coefficient value R determining the constant fatigue life amplitude (N = const.) in MPa,
$S_{max}$ – maximum sinusoidal cycle stress in MPa,
$i$ – general notation for the loading level ($i = 1,2, ...., p$),
$k$ – exponent in equation $\psi_N = N^{-k}$ ($\log k = 1.973 - \log R_m$),
$m_{(-1)}$ – exponent in the formula describing Wöhler fatigue diagram for oscillating load ($R = -1$),
$\delta_{(N)}$ – the relative difference of fatigue life,
$\psi_N$ – factor of material sensitivity to cycle asymmetry, for $N \neq N_0$,
$\zeta$ – spectrum filling factor,
$\lambda$ – number of repetitions of a program to fatigue fracture.

## Abbreviations:

FCM – full cycle counting method,
PCM – peak counting method,
RCM – simple-range counting method,
RFM – rainflow counting method,
RPM – range-pair counting method,
TFC – two-parametric fatigue characteristics.

## Introduction

Some of the criteria for the selection of the design features of machine elements are calculations in the field of fatigue of materials in the area of fatigue life estimation under random loads. According to the theory of stochastic processes, operating loads can be classified into groups of narrow and broad spectrums. Significant differences are due to fatigue life between designated loads apply to the cycle portion of a particular set of cycle asymmetry coefficient R values and the presence of high amplitude cycles (where the maximum amplitude of $S_a \approx S_{max}$). Fatigue life calculations for loads with a narrow spectrum are implemented by using a block load spectrum (1D spectrum) developed on the basis of a set of cycles determined by one of the counting cycle methods: peak counting method, simple-range counting method, full cycle counting method, range-pair counting method and rainflow counting method. In paper [2], it was found that the choice of the method of counting cycles for specific loads does not significantly affect the results of the fatigue life calculations performed using the fatigue characteristics of $N_{(Sa)}$ (1D characteristics).

For loads with a broad spectrum, the realisation of calculations is based on the fatigue load spectrum describing the variability of the cycle parameters $S_{m\,i}$ and $S_{a\,i}$, which is, for example, the correlation array of in the system $S_a$-$S_m$ or $S_{min}$-$S_{smax}$ (2D spectrum). The need to use this type of spectrum is due to the presence of cycles with cycle asymmetry coefficient loads of the range $-\infty < R < 1$, and their impact on the fatigue life [4]. An important element in the development of the 2D spectrum is the selection of counting cycle methods. Among the known methods [2, 3], when developing broad spectrum loads, it is not advisable to use the peak counting method due to the formation of most severe load conditions that can underrate the estimated durability compared to the experimental results. Other cycle counting methods listed above are identified as recommended for the development of these types of loads. In the calculations of durability two-parametric characteristics of fatigue N are used ($S_a$, $S_m$) allowing to take into consideration influence of cycles with a particular value of R on fatigue life.

The purpose of this study was to compare the results of the fatigue life calculations for load spectrums determined based on selected methods of counting cycles. The calculations are carried out using alternative load spectrums in the form of a block spectrum, developed based on the two-parameter fatigue characteristic, which is a generalisation of the Goodman diagram.

The scope of work includes the characterisation of service loads that allow for the evaluation of load spectrum width and carry the calculations of cycles by the selected counting method. The developed 2D-load spectrum will be transformed into a block 1D-load spectrum based on which fatigue life is calculated. The results of the calculations are subject to a comparative analysis to formulate conclusions.

## Service load

Figure 1 shows the load course of a steering knuckle pin of a passenger car registered in certain driving situations associated with driving on different types of surfaces and at different speeds. Due to their use in the calculation of the fatigue life, they are shown as relative $S_i/S_{max}$, where $S_{max}$ is the maximum load occurring in the course.

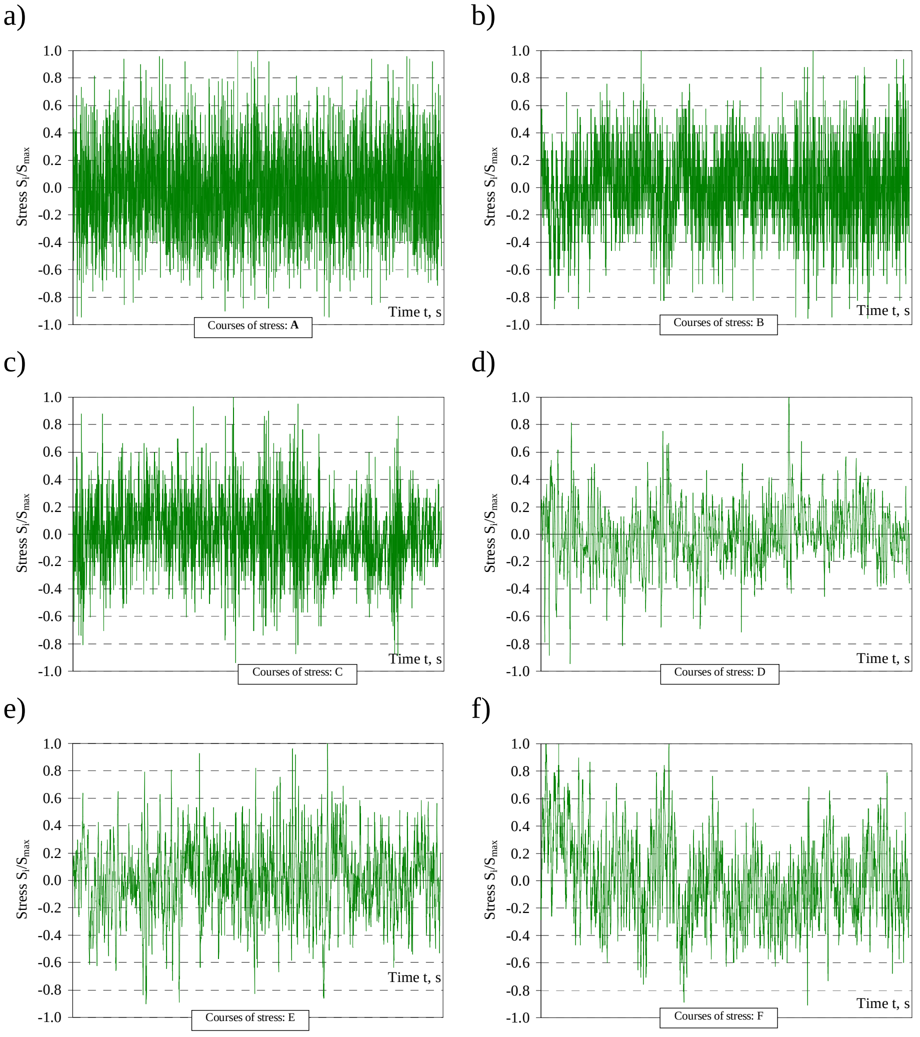


Fig. 1. Course portions of the stress changes in the form of relative values $S_i/S_{max}$

Rys. 1. Fragmenty przebiegów zmian naprężeń w postaci wartości względnych $S_i/S_{max}$

A brief analysis of taken courses shows significant differences in the frequency of load changes and the presence of cycles with high amplitudes. Load analysis in the range of the evaluation of the static and dynamic component of taken courses was carried out based on the mean value and variance. The values of these statistical parameters are shown in Table 1. For all courses, the mean value is close to zero (Table 1, column 2) and variance values show the differences between the courses in the range of the changes in stress. Similar values were obtained for the variance regarding courses marked B and F, C, and E.

In Table 1, the values of skew distribution and kurtosis are shown. Skew is characterised by the degree of asymmetry distribution around the mean value, and kurtosis determines the relative peak or flatness of distribution comparing with the normal distribution [1]. Therefore, on the basis of the results (Table 1, column 4 and 5), and analysis of the value distribution of the load courses, which, due to the volume of work are not presented, it can be assumed that the distribution in the courses is similar to a normal distribution.

Table 1. Statement of chosen statistical load course parameters $S_i/S_{max}$
Tabela 1. Zestawienie wybranych parametrów statystycznych przebiegów obciążeń $S_i/S_{max}$

| Load course | Statistical parameter | | | |
|---|---|---|---|---|
| | Mean value | Variance | Skew | Kurtosis |
| 1 | 2 | 3 | 4 | 5 |
| A | -0.0026 | 0.1400 | 0.0600 | -0.7406 |
| B | -0.0003 | 0.1063 | -0.0212 | -0.2990 |
| C | 0.0001 | 0.0821 | 0.1204 | 0.1837 |
| D | -0.0094 | 0.0556 | 0.0384 | 0.9891 |
| E | 0.0160 | 0.0780 | -0.0748 | 0.2775 |
| F | 0.0217 | 0.1020 | 0.3391 | -0.0292 |

The complex frequency structures of the analysed load courses are illustrated by spectral density function charts shown in Figure 2. The domain of studies are significant changes in the frequency for the courses determined as the A, B, and C for $0 < f < 50$ Hz, and D, E and F for $0 < f < 12$ Hz. Analysis of statistical functions show that the highest value of variation is related to course A in which load cycles occur in the frequency range of 50 Hz. The lowest value of the variance is characterised by course D, which consists of cycles at frequencies up to 12 Hz. The differences between the frequency structure and variance value (which is a measure of the amplitude of the stress changes) load courses are associated with force factors that emerge during certain driving situations. All the analysed load courses can be classified into a group of a broad load spectrum, which is indicated by the characteristic form of the graphs of the spectral density function.

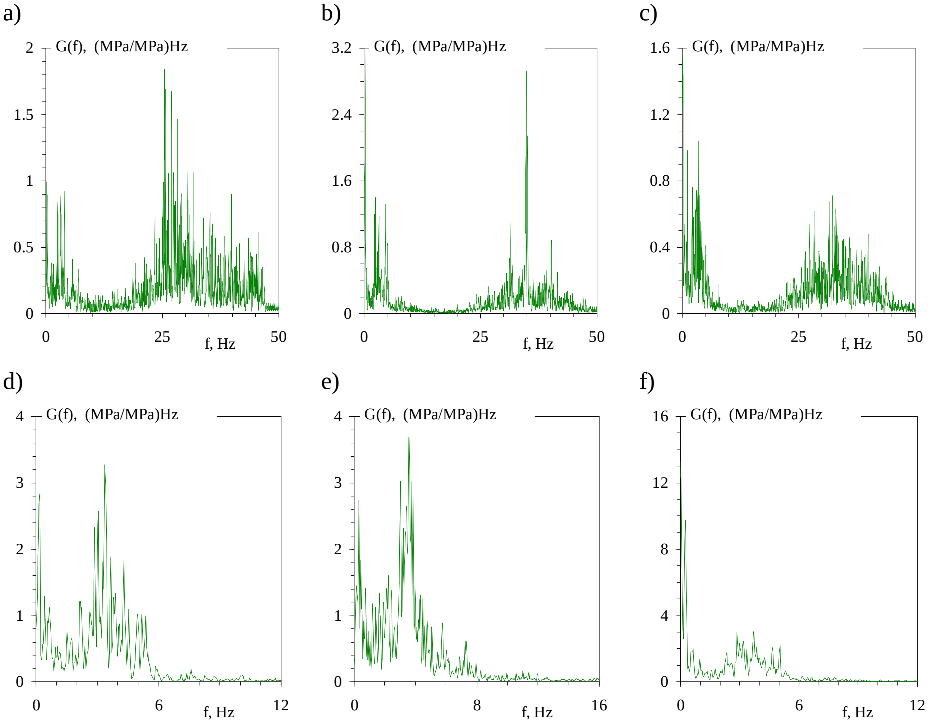


Fig. 2. Spectral density for load courses $S_i/S_{max}$: a – course A, b – course B, c – course C, d – course D, e – course E, f – course F

Rys. 2. Widmowa gęstość mocy dla przebiegów obciążeń $S_i/S_{max}$: a – przebieg A, b – przebieg B, c – przebieg C, d – przebieg D, e – przebieg E, f – przebieg F

Another way to assess the width of the load spectrum, not requiring complex statistical analysis, is to determine the value of coefficient I expressed by the following relationship:

$$I = \frac{N_i}{N_e} \tag{1}$$

In the above relationship, $N_i$ is the number of intersections of the mean value level by increasing and decreasing half-cycles, and $N_e$ is the number of local extremes that occur in the course (the sum of the minimum and maximum). This method is described in paper [5]. The determined values of coefficient I for each load are shown in Table 2.

Table 2. Statement of values of coefficient I for load courses $S_i/S_{max}$

Tabela 2. Zestawienie wartości współczynnika I dla przebiegów obciążeń $S_i/S_{max}$

| | Load course | | | | | |
|---|---|---|---|---|---|---|
| | A | B | C | D | E | F |
| Coefficient I | 0.8411 | 0.7588 | 0.7003 | 0.4455 | 0.3828 | 0.2570 |

The scope of the changes is contained in I = 0.8411 (for Course A) and I = 0.2570 (for Course F). Based on the values of coefficient I, the assumed load courses can be classified into a group with a broad spectrum. The formulated conclusion is consistent with the findings of statistical analysis.

## Block load spectrum

Load courses were determination based on set of sinusoidal cycles. The following cycle-counting methods were chosen: *peak counting method* (PCM), *simple-range counting method* (RCM), *full-cycle counting method* (FCM), *range-pair counting method* (RPM), and *rainflow counting method* (RFM) [2, 3].

Designated data sets contained sinusoidal cycles with parameters of $S_{ai}/S_{max}$ and $S_{mi}/S_{max}$ in the range of the variation of cycle asymmetry coefficient $-\infty < R < 1.0$, which is typical for a broad load spectrum. Based on the collected data sets, cycles have been developed characterising the value $S_m = 0$ and substitute amplitude $S_{az}$, which was determined on the basis of a two-parameter fatigue characteristic.

### Method for determining the substitute load spectrum

The basis for determining the substitute amplitude $S_{az}$ is the two-parameter fatigue characteristic (TFC). For the purposes of this paper, two-parameter fatigue characteristics were adopted as an IM model described in [6] (Fig. 3).

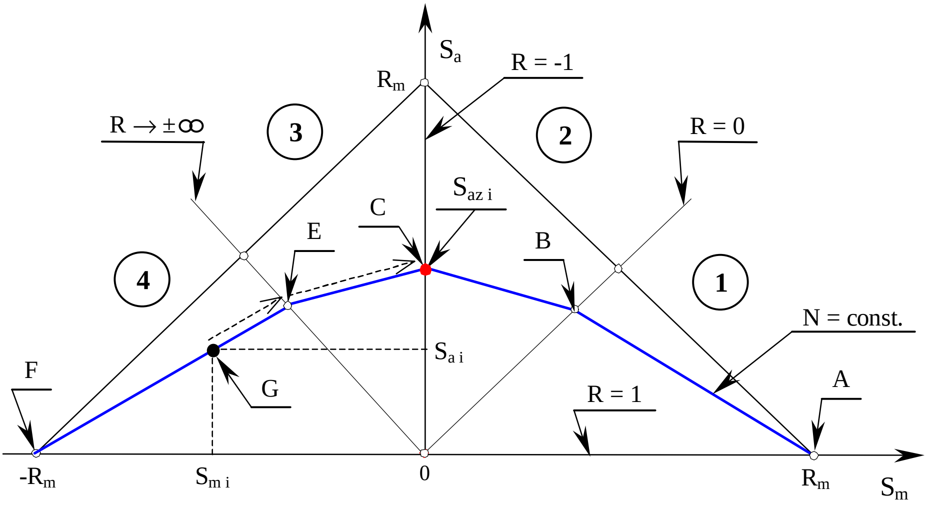


Fig. 3. Schematic approach to the mathematical model IM characteristic TFC
Rys. 3. Ujęcie schematyczne matematycznego modelu IM charakterystyki TFC

Formulas describing constants lines of durability (N = const.) for each TFC range are marked in the Figures with numbers from 1 to 4 are as follows:

– Range 1

$$N = N_0\left[\frac{S_{f(-1)}\cdot(R_m + S_{a(R)}^{(T)} - S_{m(R)}^{(T)})}{S_{a(R)}^{(T)}\cdot R_m\cdot(1+\psi_N)}\right]^{m(-1)} \quad \text{for} \quad 0 < R < 1 \qquad (2)$$

– Range 2

$$N = \frac{N_0\cdot\left(S_{f(-1)}\right)^{m(-1)}}{\left(S_{a(R)}^{(T)} + \psi_N\cdot S_{m(R)}^{(T)}\right)^{m(-1)}} \quad \text{for} \quad -1 \le R \le 0 \qquad (3)$$

– Range 3

$$N = \frac{N_0\cdot\left(S_{f(-1)}\right)^{m(-1)}}{\left(S_{a(R)}^{(T)} - \psi_N\cdot S_{m(R)}^{(T)}\right)^{m(-1)}} \quad \text{for} \quad -\infty < R < -1 \qquad (4)$$

– Range 4

$$N = N_0\left[\frac{S_{f(-1)}\cdot(R_m + S_{a(R)}^{(T)} + S_{m(R)}^{(T)})}{S_{a(R)}^{(T)}\cdot R_m\cdot(1+\psi_N)}\right]^{m(-1)} \quad \text{for} \quad 1 < R < +\infty \qquad (5)$$

Specified areas of TFC are characterised by the variability range of cycle asymmetry coefficient R.

A method for determining the substitute amplitude $S_{az\,i}$ is to determine constant lines of durability (N = const.) based on formulas (2), (3), (4) and (5) for any sinusoidal cycle load with the parameters $S_{m\,i}$ and $S_{a\,i}$ (indicated by point G in Fig. 3). The appropriate formula used is related to the range in which the load cycle is considered. Then point G is interpolated to the point of intersection of the constant line of durability with ordinate axis of coordinate system TFC (point C), for which the average stress is $S_m = 0$. In this way, the substitute load cycle is determined with coefficient R = -1. It should be underlined that all the load cycles lying on the constant line of durability (N = const.) have the same amplitude substitute $S_{az\,i}$ regardless of which TFC area is located.

## Block load spectrum

Using the method of determining the substitute amplitude $S_{az}$ described in Section "Method for determining the substitute load spectrum", the block load spectrum was determined for set of sinusoidal cycles determined as the result counting cycles (chosen five methods) for six load courses. The TFC used was developed for steel C45, which is characterised by the following parameters [7]:

- Under static conditions: $E = 211029$ MPa, $R_m = 682$ MPa, $R_e = 458$ MPa,
- Under cyclic conditions: $m_{(-1)} = 9.80$, $C_{(-1)} = 1.054\cdot10^{29}$, $S_{f(-1)} = 223.5$ MPa, $k = 0.1378$, $\psi_N = N^{-k}$.

The block load spectrums used in the calculation of durability were determined separately for the taken $S_{max}$ values of the range of variation of 200 MPa ≤ $S_{max}$ ≤ 600 MPa. Due to the volume of this paper, the block load spectrum is fixed as the value of $S_{max}$ = 500 MPa, only for changes in stress courses marked as A (Fig. 4) and

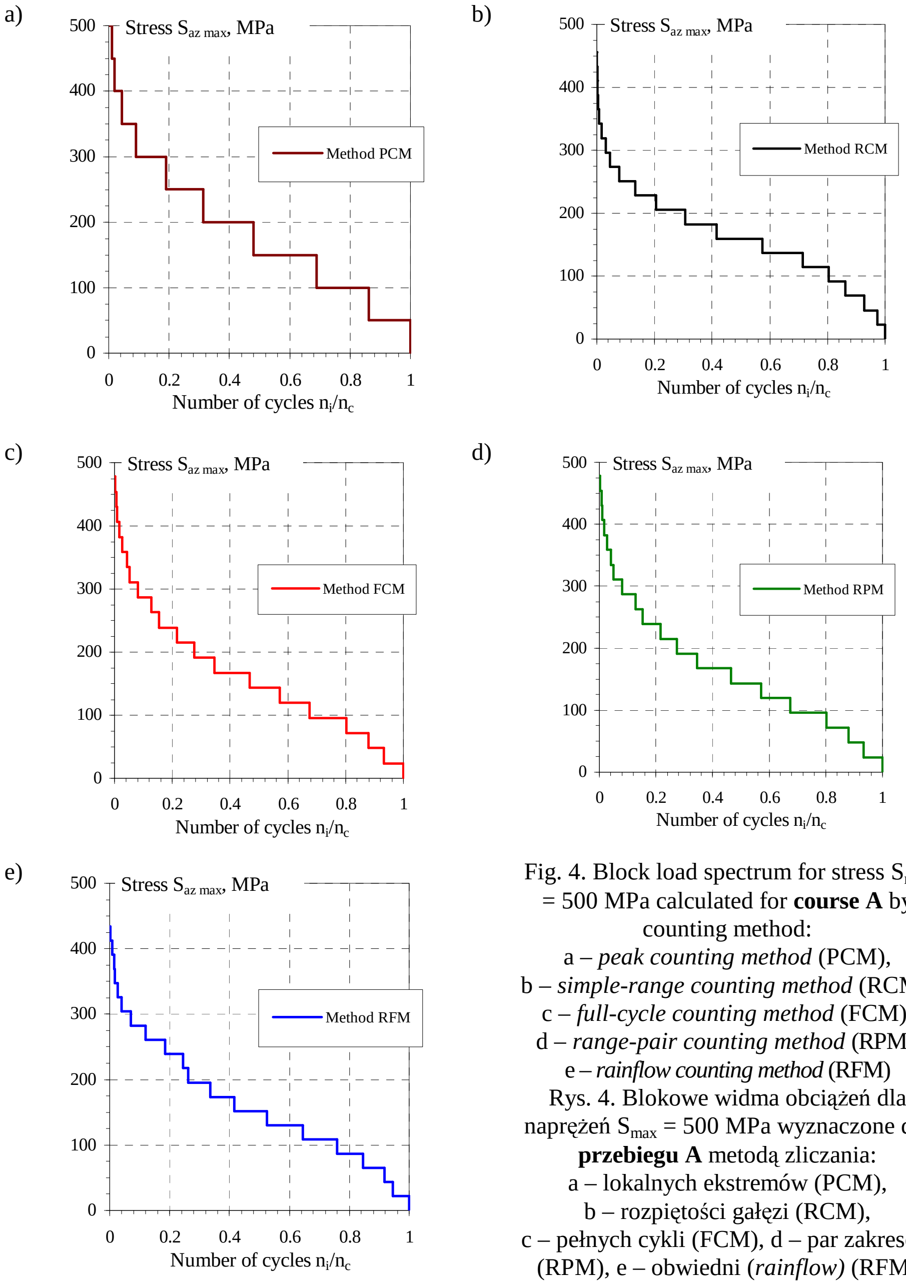
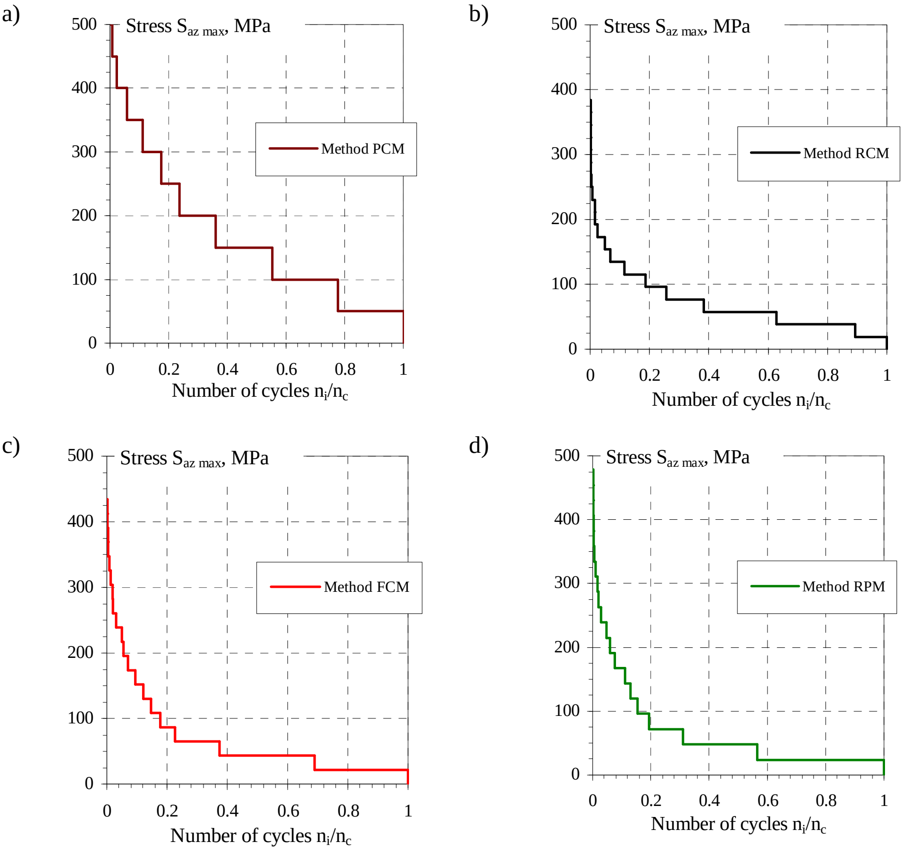
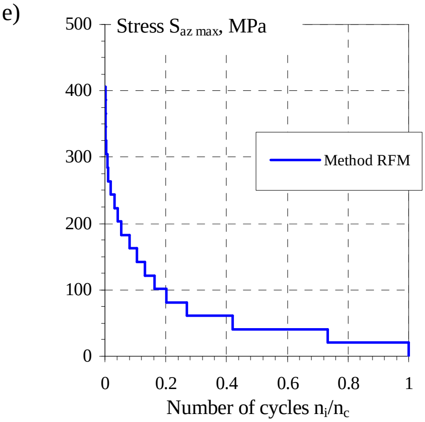


Fig. 4. Block load spectrum for stress $S_{max}$ = 500 MPa calculated for **course A** by counting method:
a – *peak counting method* (PCM),
b – *simple-range counting method* (RCM),
c – *full-cycle counting method* (FCM),
d – *range-pair counting method* (RPM),
e – *rainflow counting method* (RFM)
Rys. 4. Blokowe widma obciążeń dla naprężeń $S_{max}$ = 500 MPa wyznaczone dla **przebiegu A** metodą zliczania:
a – lokalnych ekstremów (PCM),
b – rozpiętości gałęzi (RCM),
c – pełnych cykli (FCM), d – par zakresów (RPM), e – obwiedni (*rainflow)* (RFM)

F (Fig. 5). In addition, the table contains values that characterise the block load spectrum: Table 3 - ξ spectrum filling factor, Table 4 - the value of the maximum substitute amplitude $S_{az\ max}$.





Rys. 5. Block load spectrum for stress $S_{max}$ = 500 MPa calculated for **course F** by counting method: a – *peak counting method* (PCM), b – *simple-range counting method*, (RCM), c – *full-cycle counting method* (FCM), d – *range-pair counting method* (RPM), e – *rainflow counting method* (RFM)

Rys. 5. Blokowe widma obciążeń dla naprężeń $S_{max}$ = 500 MPa wyznaczone dla **przebiegu F** metodą zliczania:
a – lokalnych ekstremów (PCM),
b – rozpiętości gałęzi (RCM), c – pełnych cykli (FCM), d – par zakresów (RPM),
e – obwiedni (*rainflow)* (RFM)

Table 3. Value summary of spectrum filling factor ζ
Tabela 3. Zestawienie wartości współczynnika wypełnienia widma ζ

| Load course | Counting cycle method | | | | |
|---|---|---|---|---|---|
| | PCM | RCM | FCM | RPM | RFM |
| 1 | 2 | 3 | 4 | 5 | 6 |
| A | 0.3696 | 0.3555 | 0.3341 | 0.3340 | 0.3686 |
| B | 0.3157 | 0.3006 | 0.2663 | 0.2663 | 0.2789 |
| C | 0.2701 | 0.2337 | 0.2204 | 0.2200 | 0.2598 |
| D | 0.2355 | 0.1469 | 0.1569 | 0.1378 | 0.1615 |
| E | 0.2838 | 0.1892 | 0.1546 | 0.1501 | 0.1531 |
| F | 0.3306 | 0.1831 | 0.1556 | 0.1372 | 0.1638 |

Table 4. Listing of the maximum substitute amplitude $S_{az\ max}$
Tabela 4. Zestawienie wartości maksymalnej amplitudy zastępczej $S_{az\ max}$

| Load course | Counting cycle method | | | | |
|---|---|---|---|---|---|
| | PCM | RCM | FCM | RPM | RFM |
| 1 | 2 | 3 | 4 | 5 | 6 |
| A | 500.0 | 456.1 | 478.0 | 478.0 | 434.1 |
| B | 500.0 | 428.1 | 478.0 | 478.0 | 450.0 |
| C | 500.0 | 456.1 | 478.0 | 478.0 | 406.1 |
| D | 500.0 | 450.0 | 428.1 | 478.0 | 406.1 |
| E | 500.0 | 400.0 | 456.1 | 478.0 | 456.1 |
| F | 500.0 | 384.1 | 434.1 | 478.0 | 406.1 |

Analysis of the values contained in Tables 3 and 4 are the result of using the chosen methods for counting cycles to estimate sets of data which differ in load amplitude value $S_{a\ i}/S_{max}$ and being part of individual cycles (especially for high values of $S_{a\ i}/S_{max}$) in the spectrum range for individual loads. The indication is that the value of spectrum filling factor ζ (Table 3) and maximum value of the substitute amplitude $S_{az\ max}$ were obtained by the *peak counting method*. For courses A, B and C, the spectrum was determined by the other methods and have similar values of ζ compared to the PCM method; however, they differ significantly comparing the values of $S_{az\ max}$. For courses D, E, and F, the major differences concern both the value and ζ and $S_{az\ max}$.

## Calculation results

Fatigue life calculations were carried out in terms of tensile strength, using fatigue-life Wöhler diagram determined experimentally in terms of a load R = -1 [7]. The following equation was used:

$$\log S_a = -\frac{1}{9.80}\log N + 2.9611 \tag{6}$$

The following is adopted for the calculation of Palmgren-Miner's linear fatigue damage aggregation hypothesis presented in equation:

$$D = \sum_{i=1}^{p}\frac{n_i}{N_i} = 1.0 \tag{7}$$

The durability results are presented in the form of load blocks, which were calculated from the following equation:

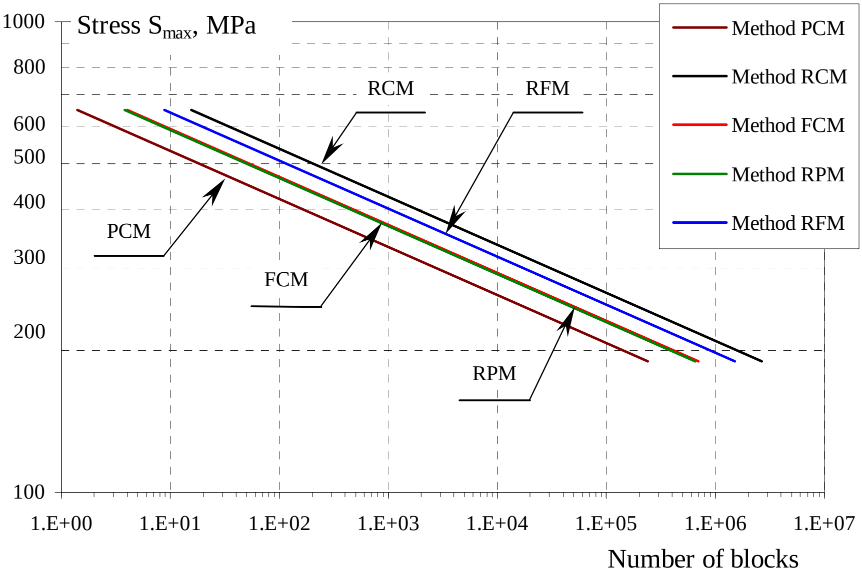
$$\lambda = \frac{1}{D} \tag{8}$$

The calculations for the substitute load spectrum allowed us to determine the fatigue life graphs in the system $S_{max} = f(\lambda)$, where $S_{max}$ is the maximum value in the load course. Figure 6 shows the results of the calculation in the form of graphs only for course A (Fig. 6a) and F (Figure 6b). However, Table 5 contains values of parameters of a straight-line equation (9) for all calculation cases:

$$\log S_{max} = -\frac{1}{m}\log\lambda + b \tag{9}$$

Preliminary evaluation of the calculation results are presented in the form of graphs (Fig. 6), and the parameters of the straight line equation (Table 5) indicates that the lowest load spectrum is obtained by the designated PCM method, which is a consequence of the high value of spectrum filling factor $\zeta$ and substitute amplitude $S_{az\,max}$. The highest fatigue life was obtained, depending on the load course, for the RCM method and/or RFM. In the case of load C, the highest (equal to) results of durability were achieved for the RCM ($\zeta$ = 0.2337, $S_{az\,max}$ = 456.1 MPa) and RFM ($\zeta$ = 0.2598, $S_{az\,max}$ = 406.1 MPa). However, for course D, the highest durability was obtained for the RFM method ($\zeta$ = 0.1615, $S_{az\,max}$ = 406.1 MPa) and close (lower values compared to RFM) to the value of durability for RCM ($\zeta$ = 0.1469, $S_{az\,max}$ = 450.0 MPa) and FCM ($\zeta$ = 0.1569, $S_{az\,max}$ = 428.1 MPa). For all other cases of loads, the highest durability was obtained for the RCM method.

a)



b)

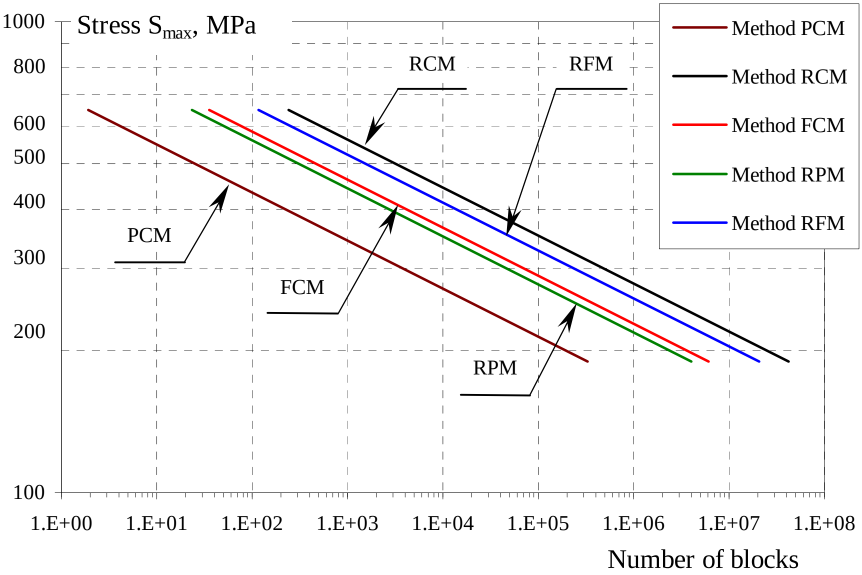


Fig. 6. Life fatigue calculation results: a – Course A, b – Course F
Rys. 6. Wyniki obliczeń trwałości zmęczeniowej dla: a – przebiegu A, b – przebiegu F

Table 5. List of m and b values as result of fatigue life calculations
Tabela 5. Zestawienie wartości parametrów m i b uzyskanych w wyniku obliczeń trwałości zmęczeniowej

| Load course | Cycle-counting method | Parameters of the equation of a straight line | |
|---|---|---|---|
| | | m | b |
| 1 | 2 | 3 | 4 |
| A | PCM | 9.80 | 2.8277 |
| | RCM | | 2.9345 |
| | FCM | | 2.8747 |
| | RPM | | 2.8721 |
| | RFM | | 2.9093 |
| B | PCM | 9.80 | 2.8574 |
| | RCM | | 2.9568 |
| | FCM | | 2.8951 |
| | RPM | | 2.8920 |
| | RFM | | 2.9334 |
| C | PCM | 9.80 | 2.8670 |
| | RCM | | 2.9529 |
| | FCM | | 2.9150 |
| | RPM | | 2.9169 |
| | RFM | | 2.9529 |
| D | PCM | 9.80 | 2.9378 |
| | RCM | | 2.9934 |
| | FCM | | 2.9933 |
| | RPM | | 2.9599 |
| | RFM | | 3.0109 |
| E | PCM | 9.80 | 2.8840 |
| | RCM | | 3.0176 |
| | FCM | | 2.9371 |
| | RPM | | 2.9295 |
| | RFM | | 2.9599 |
| F | PCM | 9.80 | 2.8416 |
| | RCM | | 3.0556 |
| | FCM | | 2.9712 |
| | RPM | | 2.9527 |
| | RFM | | 3.0235 |

## Analysis of test results

The differences between the results of calculations obtained for the assumed load courses and the methods of counting cycles were specified on the basis of the analysis of the value difference of the relative fatigue life N calculated from the following equation:

$$\delta_{(N)} = \frac{N_{(x)} - N_{(R=-1)}}{N_{(R=-1)}} \tag{10}$$

The values of the fatigue life $N_{(x)}$ is expressed as a durability of the number of cycles determined by calculation, while the $N_{(R=-1)}$ is the number of cycles read from the Wöhler diagram for stress amplitude value $S_a$ ($S_a = S_{max}$). The points of reference in the analysis were the results read from the Wöhler graph.

The results of the fatigue life calculations presented in the form of parameters of straight lines equations (Table 5) indicate that they are parallel and to the right of the Wöhler graph (for R = -1). Relative difference values $\delta_{(N)}$ between the analysed characteristics are presented in the chart in Figure 7 and numerically in Table 6.

Detailed analysis shows that the most severe load conditions are obtained for the cycle-counting method *peak counting method* (PCM) for all analysed loads. Relative difference values are in the range of $\delta_{(N)\,F} = 50.3$ to $\delta_{(N)\,E} = 123.3$. The highest durability was obtained for the block load spectrum determined by the following cycle-counting methods: *simple-range counting method* (RCM) – courses A, B, C, E, F and *rainflow counting method* (RFM) – courses C, D. For course C, the load spectrum determined by RCM and RFM methods allowed us to obtain the same results of durability, instead of different values of spectrum

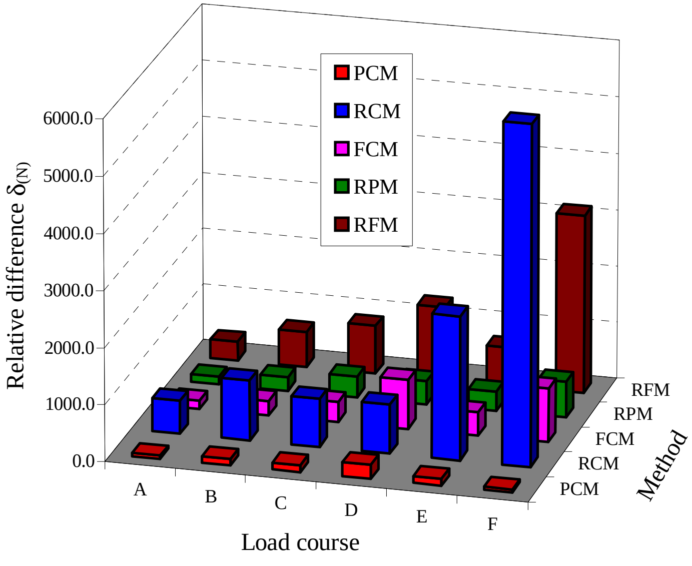


Fig. 7. Relative difference values for the assumed service load counted by chosen counting cycle methods

Rys. 7. Wartości różnic względnych dla przyjętych obciążeń eksploatacyjnych podanych zliczaniu cykli wybranymi metodami filling factor $\zeta$ and substitute amplitude $S_{az\,max}$. This is the result of a larger number of cycles of higher amplitudes in the load spectrum determined by the FCM method despite the smaller $S_{az\,max}$ value, based on the spectrum determined by RCM.

Table 6. Table of relative difference values $\delta_{(N)}$
Tabela 6. Zestawienie wartości różnic względnych $\delta_{(N)}$

| Counting cycle method | Load course | | | | | |
|---|---|---|---|---|---|---|
| | A | B | C | D | E | F |
| PCM | 51.8 | 111.4 | 121.9 | 248.0 | 123.3 | 50.3 |
| RCM | 587.0 | 1058.8 | 851.9 | 871.6 | 2538.8 | 6508.7 |
| FCM | 151.4 | 262.0 | 361.8 | 869.5 | 411.3 | 953.6 |
| RPM | 142.6 | 244.3 | 377.9 | 409.0 | 346.0 | 628.4 |
| RFM | 332.0 | 623.5 | 851.9 | 1295.4 | 687.7 | 3152.4 |

In the case of the rainflow counting method (RFM), relative difference results are in the range of $\delta_{(N)\,A} = 332$ to $\delta_{(N)\,F} = 3152.4$. The durability results obtained by RFM were significantly higher than the results obtained by PCM, FCM, and RPM methods for all the analysed loads.

Analysis of the results $\delta_{(N)}$ obtained for the full-cycle counting method (FCM) and the range-pair counting method (RPM) indicates that substitute load spectrums have similar forms, which is supported by the value of coefficient $\zeta$ and substitute amplitude $S_{az\,max}$. This concerns the results shown for the courses marked as A, B, C, and E. In the case of D and F loads, higher durability was obtained for the spectrum determined by FCM method.

The presented results of relative values $\delta_{(N)}$ also enable the determination of relationships among cycle-counting methods. It enables their more comprehensive evaluation that will be presented in a separate paper.

Block load spectrums are determined by the results of counting load cycles from chosen load courses characterised by spectrum filling factor $\zeta$ among others. Figure 8 shows the results of calculations in the form of durability charts for each method of counting cycles referring to the values of factor $\zeta$ determined for substitute spectrums. For the PCM method (Fig. 8a), durability diagrams are arranged in accordance with the value of the coefficient, except for the courses C and E, which occur in the reverse order. Impact on the form of the results is that more cycles in the range of mean values $S_{az\,i}$ for spectrums were determined for course C, which results from a comparative analysis of the block load spectrum.

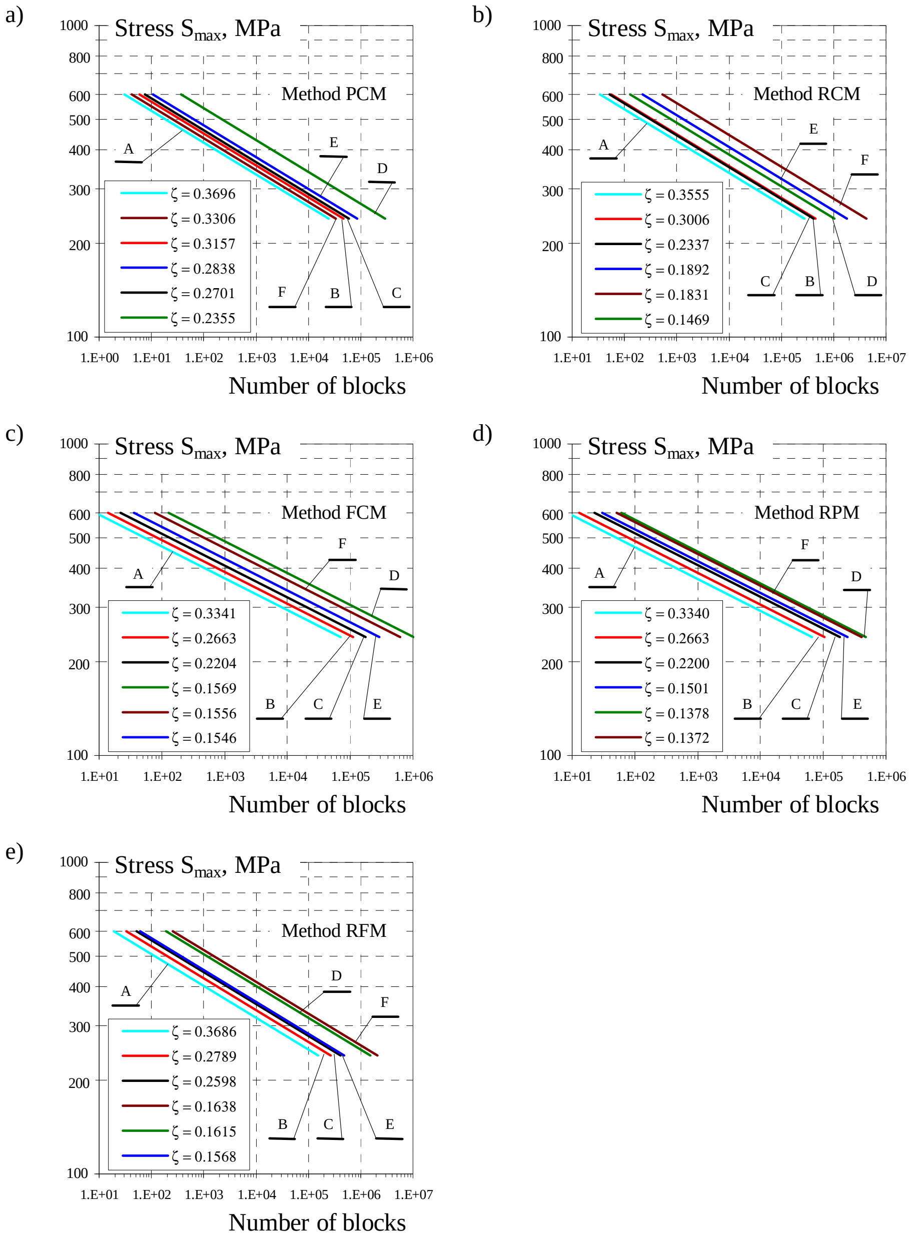


Fig. 8. List of the results of fatigue life calculations due to the factor value ζ for the block load spectrum determined by counting methods: a – *peak counting method* (PCM), b – *simple-range counting method* (RCM), c – *full-cycle counting method* (FCM), d – *range-pair counting method* (RPM), e – *rainflow counting method* (RFM)

Rys. 8. Zestawienie wyników obliczeń trwałości zmęczeniowej ze względu na wartość współczynnik ζ dla blokowych widm obciążeń wyznaczonych metodą zliczania: a – lokalnych ekstremów (PCM), b – rozpiętości gałęzi (RCM), c – pełnych cykli (FCM), d – par zakresów (RPM), e – obwiedni (*rainflow*) (RFM)

Lower durability was obtained for course D in relation to the value indicated by coefficient $\zeta$ for the load spectrum determined based on RCM cycle-counting method (Fig. 8b).

In the case of the spectrum determined by FCM (Fig. 8c) and RFM (Fig. 8e) for courses D, E and F, there are small differences in the values of the coefficient $\zeta$, while durability results differ significantly. Only for RPM method sequence diagrams of fatigue life were consistent with the values of the spectrum filling factor.

## Summary

The calculations of fatigue life under service loads showed differences in the development of the load spectrum by cycle-counting methods: *peak counting method, simple-range counting method, full-cycle counting method, range-pair counting method,* and *rainflow counting method.*

Depending on the nature of the changes in the stress courses, the range of variation of the relative difference values for each method for counting cycles was as follows:

| | |
|---|---|
| – Peak counting method | $\delta_{(N)} = 50.3 - 248.0$, |
| – Simple-range counting method | $\delta_{(N)} = 587.0 - 6508.7$, |
| – Full-cycle counting method | $\delta_{(N)} = 151.4 - 953.6$, |
| – Range-pair counting method | $\delta_{(N)} = 142.6 - 628.4$, |
| – Rainflow counting method | $\delta_{(N)} = 332.0 - 3152.4$. |

The presented data shows that the lowest durability of the load spectrum was obtained by the PCM method, and highest value was determined for the RCM method. The results for the other methods of counting cycles were in the range determined by the PCM and RCM methods.

Relative differences of the fatigue life are related to the algorithm for the determination of cycles for each cycle-counting method. This mainly concerns the determination of cycles with high amplitude values that have a significant impact on the calculations of the durability results. This is confirmed by the given examples of the block load spectrum determined for the maximum stress $S_{max}$ = 500 MPa in the load course. The maximum substitute amplitude $S_{az\,max}$ for the cycles indicated that the chosen methods have different values (for example course F) and developed spectrums, on the basis of a set of cycles, characterised by different rates of spectrum filling factor $\zeta$.

Based on these calculations a practical recommendation can be formulated on the necessity of analysing of loading spectra determined with several counting methods that enables one to define the range of changes of the estimated fatigue life.

## References

[1] Bendat J.S., Piersol A.G.: Metody analizy i pomiarów sygnałów losowych, PWN, Warszawa, 1976.

[2] Kocańda S., Szala J.: Fundamentals of fatigue calculations, (in Polish), PWN, Warszawa, 1997.

[3] Norma ASTM: Standard Practices for cycle counting in fatigue analysis, ASTM Designation: E 1049-85 (Reapproved 1990).

[4] Ligaj B., Szala G.: Experimental verification of two-parametric models of fatigue characteristics by using the tests of S355J0 steel as an example, Polish Meritime Research, no.1 (2010), pp. 39–50.

[5] Szala J.: Loads and fatigue life of machine elements, (in Polish), University of Technology and Agriculture, Bydgoszcz 1989.

[6] Szala G., Ligaj B.: Description of cyclic properties of steel in variability conditions of mean values and amplitudes of loading cycles, Materials Science Forum, Fatigue Failure and Fracture Mechanics vol. 726, 2012, pp. 69–76.

[7] Szala G.: Analiza teoretyczna i doświadczalna dwuparametrycznych charakterystyk zmęczeniowych stali konstrukcyjnych, opracowanie monograficzne, część I zbioru monografii pt.: Dwuparametryczne charakterystyki zmęczeniowe stali konstrukcyjnych i ich eksperymentalna weryfikacja, Wydawnictwo Instytutu Technologii Eksploatacji – Państwowego Instytutu Badawczego, Radom 2011.

*This research work has been financially supported by Polish Ministry of Science and Higher Education in the frame of the research project No. 2221/B/T02/2010/39.*

### Analiza wpływu metod zliczania cykli na obliczenia trwałości zmęczeniowej stali

Streszczenie

W pracy zostały przedstawione zagadnienia obliczeń trwałości zmęczeniowej w warunkach obciążeń eksploatacyjnych na przykładzie przebiegów zmian naprężeń pochodzących z pomiarów obciążeń czopa zwrotnicy samochodu osobowego. Przyjęte przebiegi zostały poddane opracowaniu prowadzącemu do wyznaczenia zbioru cykli sinusoidalnych następującymi metodami: *peak counting method*, *simple-range counting method*, *full cycles counting method*, *range-pair counting method* i *rainflow counting method*. Na podstawie zbiorów cykli, o zmiennych parametrach $S_{m\,i}$ $S_{a\,i}$, opracowano blokowe widma obciążeń dla amplitud zastępczych $S_{az}$ wyznaczonych na podstawie autorskiej metody, w której zastosowano dwuparametryczną charakterystykę zmęczeniową. Wynikiem pracy jest porównanie wyników trwałości zmęczeniowej dla widm obciążeń wyznaczonych przyjętymi metodami zliczania cykli dla obciążeń losowych o szerokim widmie, charakteryzujących się zmiennymi wartościami współczynnika I.