SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
2 (166) 2011

MARIUSZ WAŻNY*

An outline of the method for determining the density function of changes in diagnostic parameter deviations with the use of the Weibull distribution

K e y w o r d s

A diagnostic parameter, the Weibull distribution, density function, reliability.

S ł o w a k l u c z o w e

Parametr diagnostyczny, rozkład Weibulla, funkcja gęstości, niezawodność.

S u m m a r y

Due to the nature of tasks performed by an aircraft, one of the most essential criteria determining the quality of its maintenance process is the reliability of devices and systems installed on an aircraft. The assurance of the reliability of aircraft devices at an adequately high level minimises the causes of failures. Unfortunately, the influence of destructive factors connected, among other things, with the impact of changing ambient conditions, overload effects, and the influence of ageing processes, causes the technical parameters of devices deteriorate. Methods for describing diagnostic parameter changes due to the effects of destructive factors have been presented in the literature [6, 7, 8, 9, 11]. This article constitutes a new attempt of an analytical description of the changes in diagnostic parameter values describing the technical state of devices based on the method

* Military University of Technology, General Sylwester Kaliski 2 Street, 00-908 Warsaw 49, Poland.

for determining the density function of changes in diagnostic parameter deviations with the use of the Weibull distribution.

Introduction

One of the technical objects whose maintenance process is subject to a special supervision is an aircraft, which is connected with numerous factors related mainly to the environment in which it is operated, i.e. airspace. The changeability of environmental parameters in which an aircraft moves and the influence of various external and internal destructive forces cause that the maintenance process of an aircraft should be as complete as possible, i.e. the process should provide conditions enabling the performance of a scheduled task. Depending on the purpose of an aircraft, tasks can be divided into different elements. Nonetheless, considering a general model of its operation, we can assume that the basic function involves the execution of the flying function, i.e. take-off, flight to a target destination, and landing. This function can be executed in a safe way only when there is an appropriate and complex model for the performance of the maintenance process.

The framework of the scope and character of the maintenance process for a newly constructed aircraft is developed along with the launch of works on the concept of an object.

The main determinant orienting the thought process involves defining two groups of terms, i.e. [3]:

- Aircraft properties, and
- Aircraft qualities.

Aircraft properties constitute a set of functions determined at the stage of design and construction. They concern such parameters as aircraft measurements and mass, the drive unit structure, resistance to fatigue processes, fuel weight, construction-related conditions of operation, maintenance and repair, operational potential, and payload.

On the other hand, aircraft qualities are determined by such parameters as functionality, reliability, readiness, suitability, durability, service life, and susceptibility.

Each property and quality, defined in accordance with the criterion assumed, can be examined in three dimensions connected with theory, practice, and undertakings aimed at the modification of parameter values describing the properties and qualities [3].

One of the most important parameters mentioned above is reliability, referring both to devices and systems installed on an aircraft and also to the aircraft itself. The reliability parameter is inextricably linked with the safety parameter.

Testing the reliability and safety of aircraft in the maintenance process is related to the prediction of the technical state -- devices, systems, and aircraft. Destructive processes manifesting themselves in the form of overload, friction, vibrations, wear, etc. have a crucial effect on technical state changes in aircraft devices.

Today, the limits determining maintenance time intervals are not directly connected with the stage of either putting a device into operation or withdrawing a device from operation. The up-to-date knowledge acquired through the maintenance and operation of a technical device enables us to develop the maintenance process at the conceptual stage while analysing the feasibility of a given project. It is clearly depicted in broadly defined aeronautical engineering, where an iterative process is used to develop a product, which in turn is used to obtain certain benefits/profits. Fig. 1 shows the independence graph of the "life" cycle of a technical device [4].

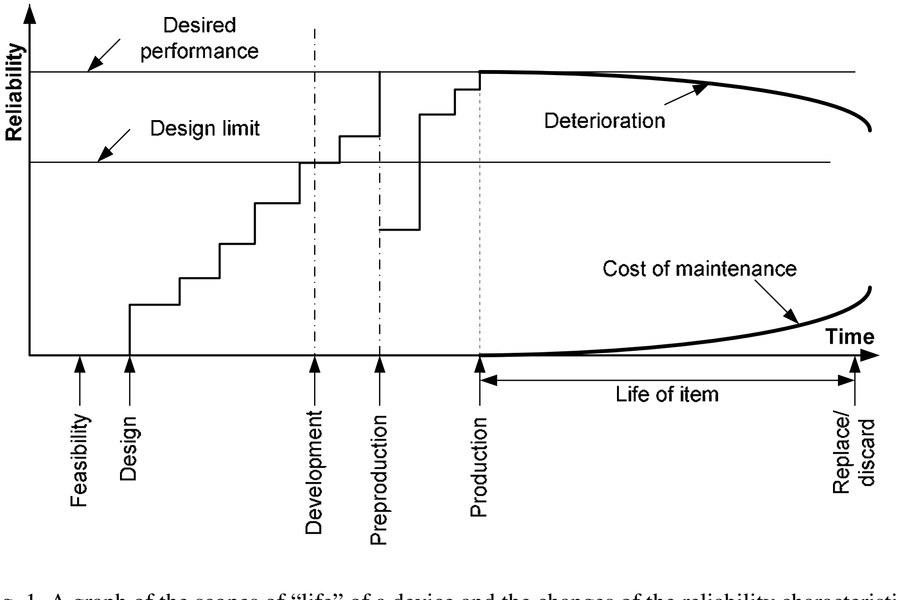

Fig. 1. A graph of the scopes of "life" of a device and the changes of the reliability characteristics in the maintenance process
Rys. 1. Wykres zakresów „życia" urządzenia oraz zmian charakterystyki niezawodnościowej w procesie eksploatacyjnym

The technical state of an aircraft device is mainly evaluated through a set of diagnostic parameters. The effect of destructive processes manifests itself in the change of diagnostic parameter values causing a rise in the deviation from the nominal values of these parameters. The values of deviations from the nominal values are used to estimate the reliability of a device.

The effect of destructive processes on the change of diagnostic parameter values can be divided into three groups:

- Elements of a device that have strongly correlated values of diagnostic parameter deviations with time or the workload of a device;
- Elements of a device that have weakly correlated values of diagnostic parameter deviations with time or the workload of a device; and,
- Elements of a device that lack correlation with time or the workload of a device.

Due to the diversity of components, real devices usually comprise all the above-mentioned groups. However, sometimes it can be shown that some of the groups are dominant, and they can be used to predict the reliability and durability of elements and devices. The development of probabilistic models based on the above-mentioned processes and the use of their results for determining the reliability and durability of elements and devices are important issues. This article is an attempt to build a model enabling the determination of the reliability characteristics for elements subjected to the effect of destructive processes.

The analytical description of device reliability starts with making initial assumptions concerning the developed method [2, 6, 7]. The following findings and assumptions were made for the purpose of developing the model:

- The technical state of a device is determined by one dominant diagnostic parameter. Its current value is denoted by "x."
- The change of a diagnostic parameter value due to the destructive effect of ageing processes occurs with the passing of calendar time.
- The deviation of a diagnostic parameter from the nominal value is

$$z = \left| x_p - x_n \right|,$$

where:

x_p – the measured value of a diagnostic parameter,

x_n – the nominal value of a diagnostic parameter.

- If $z \in [0, z_d]$, then an element of a device is regarded as operable; otherwise, an element of a device is regarded as inoperable.
- An increase of a diagnostic parameter deviation in the function of calendar time satisfies the relationship

$$\frac{dz}{dt} = c \qquad (1)$$

where:

c – the mean value, a variable velocity depending on ageing processes,

t – the calendar time.

Determining the density function of changes in values of diagnostic parameter deviations

One of the elements describing device reliability is the density function of changes in diagnostic parameter deviations. For the purpose of determining the function, it was assumed that the intensity of the growth in deviation has the form (2)

$$\lambda(t) = \frac{\alpha}{\theta} t^{\alpha-1} \tag{2}$$

where:

α i θ – the constants in the Weibull distribution with the following denotations:
α – the shape factor,
θ – the scale factor.

The random dynamics of changes of diagnostic parameter values, including the deviation, is described by the difference equation. Let $U_{z,\,t}$ denote the probability that at the time t, the value of a diagnostic parameter deviation adopts the value "*z*".

The differentiated equation has the following form:

$$U_{z,\,t+\Delta t} = \left(1 - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\right) U_{z,\,t} + \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\, U_{z-\Delta z,\,t} \tag{3}$$

where:

Δz – the increase in deviation of a diagnostic parameter over the time interval Δt.

By converting Equation (3) into the function notation, we obtain the Equation (4) as follows:

$$u(z, t + \Delta t) = \left(1 - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\right) u(z, t) + \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\, u(z - \Delta z, t) \tag{4}$$

where:

$u(z, t)$ – the density function of a diagnostic parameter deviation;
$\left(1 - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\right)$ – the probability that over the time interval Δt there is no parameter deviation;

$\frac{\alpha}{\theta} t^{\alpha-1} \Delta t$ – the probability that over the time interval Δt there is the increase in the parameter deviation "Δz";

and the following condition is met

$$\frac{\alpha}{\theta} t^{\alpha-1} \Delta t \le 1.$$

In order to determine the density function of changes in diagnostic parameter deviations, we shall convert the function notation of Equation (4) into a partial differential equation. We assume the following approximation:

$$\begin{gathered} u(z, t+\Delta t) = u(z, t) + \frac{\partial u(z, t)}{\partial t} \Delta t, \\ u(z-\Delta z, t) = u(z, t) - \frac{\partial u(z, t)}{\partial z} \Delta z + \frac{1}{2} \frac{\partial^2 u(z, t)}{\partial z^2} (\Delta z)^2 \end{gathered} \tag{5}$$

By substituting the relations presented in (5) into Equation (4), we obtain the following relations:

$$u(z, t) + \frac{\partial u(z, t)}{\partial t} \Delta t =$$

$$= \left(1 - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\right) u(z, t) + \frac{\alpha}{\theta} t^{\alpha-1} \Delta t \left(u(z, t) - \frac{\partial u(z, t)}{\partial z} \Delta z + \frac{1}{2} \frac{\partial^2 u(z, t)}{\partial z^2} (\Delta z)^2 \right),$$

$$\frac{\partial u(z, t)}{\partial t} \Delta t = -\frac{\alpha}{\theta} t^{\alpha-1} \Delta t u(z, t) +$$

$$+ \frac{\alpha}{\theta} t^{\alpha-1} \Delta t u(z, t) - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t \Delta z \frac{\partial u(z, t)}{\partial z} + \frac{1}{2} \frac{\alpha}{\theta} t^{\alpha-1} \Delta t (\Delta z)^2 \frac{\partial^2 u(z, t)}{\partial z^2}.$$

Hence,

$$\frac{\partial u(z, t)}{\partial z} = -\frac{\alpha}{\theta} t^{\alpha-1} \Delta z \frac{\partial u(z, t)}{\partial z} + \frac{1}{2} \frac{\alpha}{\theta} t^{\alpha-1} (\Delta z)^2 \frac{\partial^2 u(z, t)}{\partial z^2} \tag{6}$$

For the purpose of further transformations, it was assumed that

$$\frac{\Delta z}{\Delta t} = c \quad \Rightarrow \Delta z = c\Delta t \quad \underset{\Delta t=1}{\Rightarrow} \bar{c}$$

which specifies that we examine the increase of parameter deviation per time unit (when $\Delta t = 1$), where $\bar{c}$ denotes the deviation increase per a unit of time.

By substituting the above assumption into Equation (6), we obtain its final form

$$\frac{\partial u(z,t)}{\partial z} = -\underbrace{\frac{\alpha\bar{c}}{\theta} t^{\alpha-1}}_{\gamma(t)} \frac{\partial u(z,t)}{\partial z} + \frac{1}{2} \underbrace{\frac{\alpha\bar{c}^2}{\theta} t^{\alpha-1}}_{\beta(t)} \frac{\partial^2 u(z,t)}{\partial z^2} \tag{7}$$

As it can be seen in Equation (7), the form of the coefficients depends on the parameter values α.

For $\alpha = 1$, the coefficients has the following form:

$$\gamma(t) = \frac{\bar{c}}{\theta}; \ \beta = \frac{\bar{c}^2}{\theta}.$$

For $\alpha = 2$, the coefficients has the following form:

$$\gamma(t) = \frac{2\bar{c}}{\theta} t; \ \beta(t) = \frac{2\bar{c}^2}{\theta} t.$$

The solution of Equation (7) has the following form:

$$u(z,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(z-B(t))^2}{2A(t)}} \tag{8}$$

where:

$B(t)$ – the average value of a parameter deviation for the time of the service life t,

$$B(t) = \int_0^t \gamma(t) dt \tag{9}$$

A(t) – the value of the variance of a diagnostic parameter deviation for the time of the service life *t*.

$$A(t) = \int_0^t \beta(t)dt \tag{10}$$

Calculating the Integrals (9) and (10), we obtain the following:

$$B(t) = \int_0^t \frac{\alpha\overline{c}}{\theta} t^{\alpha-1} dt = \frac{\alpha\overline{c}}{\theta} \int_0^t t^{\alpha-1} dt = \frac{\alpha\overline{c}}{\theta} \frac{1}{\alpha} t^{\alpha} \Big|_0^t = \frac{\overline{c}}{\theta} t^{\alpha} - 0 = \frac{\overline{c}}{\theta} t^{\alpha} \tag{11}$$

$$A(t) = \int_0^t \frac{\alpha\overline{c}^2}{\theta} t^{\alpha-1} = \frac{\alpha\overline{c}^2}{\theta} \frac{1}{\alpha} t^{\alpha} \Big|_0^t = \frac{\overline{c}^2}{\theta} t^{\alpha} \tag{12}$$

Hence, Relationship (8) has the following form:

$$u(z,t) = \frac{1}{\sqrt{2\pi \frac{\overline{c}^2}{\theta} t^{\alpha}}} e^{-\frac{\left(z - \frac{\overline{c}}{\theta} t^{\alpha}\right)^2}{2\frac{\overline{c}^2}{\theta} t^{\alpha}}} \tag{13}$$

Relationship (13) presents the density function of a diagnostic parameter deviation from the nominal value.

Let

$$\frac{\overline{c}}{\theta} = b \qquad \text{and} \qquad \frac{\overline{c}^2}{\theta} = a \,.$$

Hence, relationship (13) has the following form:

$$u(z,t) = \frac{1}{\sqrt{2\pi a t^{\alpha}}} e^{-\frac{\left(z - b t^{\alpha}\right)^2}{2 a t^{\alpha}}} \tag{14}$$

By using the density function (14), we can determine the relationship for the reliability of a device in terms of an examined diagnostic parameter. This relationship has the form of Equation (15)

$$R(t) = \int_{-\infty}^{z_d} u(z, t) dz \tag{15}$$

where:

z_d – the permissible deviation value of the diagnostic parameter a $u(z, t)$ is determined by relationship (14).

Summary

The operation of technical devices installed on an aircraft depends on the influence of changeable conditions, both atmospheric conditions and mechanical ones that are connected, including in-flight overload effects. These factors cause the accumulation of destructive elements that in turn cause certain elements of technical systems to lose nominal operating parameters. The notation presented in this paper enables one to determine the reliability of devices and constitutes the basis for further analyses aimed at determining the density function of the time of exceeding the limit state by the diagnostic parameter being discussed due to the influence of destructive factors. By using the density function of the time of exceeding the limit state by the diagnostic parameter, we are able to determine the residual durability of the device being considered in this paper, which will constitute the direction of further studies. The finding obtained in this way can be used for the modification of the maintenance process of a given technical object.

Scientific work funded by the National Centre for Research and Development in 2011–2013 as a research project.

References

[1] Abezgauz G.: Rachunek probabilistyczny. Poradnik. Wydawnictwo Ministra Obrony Narodowej, 1973.
[2] Gniedenko B., Bielajew J., Sołowiew A.: Metody matematyczne w teorii niezawodności. Wydawnictwo Naukowo-Techniczne, 1968.
[3] Lewitowicz J., Kustroń K.: Podstawy eksploatacji statków powietrznych. Własności i właściwości eksploatacyjne statku powietrznego. Wydawnictwo ITWL, 2003.
[4] Murthy D.N.P., Xie Min, Jiang R.: Weibull Models, John Wiley & Sons, Inc., Hoboken, New Jersey 2004.
[5] Tomaszek H., Wróblewski M.: Podstawy oceny efektywności eksploatacji systemów uzbrojenia lotniczego. Dom Wydawniczy Bellona, 2001.
[6] Tomaszek H., Żurek J., Loroch L.: Zarys metody oceny niezawodności i trwałości elementów konstrukcji lotniczych na podstawie opisu procesów destrukcyjnych. Zagadnienia Eksploatacji Maszyn 2004, Zeszyt 3 (139): 73–85.

[7] Tomaszek H., Szczepanik R.: Metoda określania rozkładu czasu do przekroczenia stanu granicznego. Zagadnienia Eksploatacji Maszyn 2005, Zeszyt 4 (144): 35–44.
[8] Ważny M., Jasztal M.: Zarys metody określania prawdopodobieństwa zdatnej pracy urządzenia w aspekcie zmian wartości parametru diagnostycznego. Eksploatacja i Niezawodność – Maintenance and Reliability 2007, 2: 44–50.
[9] Ważny M.: Metoda wyznaczania czasookresu przebywania wybranego urządzenia system nawigacyjno-celowniczego w systemie użytkowania. Eksploatacja i Niezawodność – Maintenance and Reliability 2008, 2: 4–11.
[10] Ważny M., Wojtowicz K.: Analiza systemu eksploatacji wojskowego statku powietrznego w aspekcie jego modernizacji. Eksploatacja i Niezawodność – Maintenance and Reliability 2008, 3: 4–11.
[11] Ważny M.: Metoda oceny trwałości resztkowej wybranych urządzeń systemu awionicznego. Eksploatacja i Niezawodność – Maintenance and Reliability 2009, 3: 55–64.

Zarys metody określenia funkcji gęstości zmian odchyłek parametrów diagnostycznych z wykorzystaniem rozkładu Weibulla

S t r e s z c z e n i e

Z uwagi na charakter zadań realizowanych przez statek powietrzny, jednym z podstawowych kryterium określającym jakość jego procesu eksploatacyjnego jest niezawodność urządzeń i systemów zainstalowanych na jego pokładzie. Zapewnienie wartości niezawodności urządzeń statku powietrznego na odpowiednio wysokim poziomie minimalizuje przyczyny występowania uszkodzeń. Niestety oddziaływanie czynników destrukcyjnych związanych m.in. z wpływem zmiennych warunków otoczenia, w którym następuje ruch statku powietrznego, oddziaływania przeciążeń czy też wpływ procesów starzeniowych powoduje, że parametry techniczne charakteryzujące pracę urządzeń ulegają pogorszeniu. Metody opisu zmian wartości parametrów diagnostycznych w wyniku oddziaływania czynników destrukcyjnych przedstawiane były w pozycjach literaturowych [6, 7, 8, 9, 11]. Niniejszy artykuł jest próbą analitycznego opisu zmian wartości parametrów diagnostycznych opisujących stan techniczny urządzenia w oparciu o metodę określania funkcji gęstości zmian odchyłek parametrów diagnostycznych urządzeń technicznych z wykorzystaniem rozkładu Weibulla.