SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
2 (166) 2011

KRZYSZTOF KOŁOWROCKI*
ZBIGNIEW SMALKO**

Safety and reliability of a three-state system at variable operation conditions

K e y w o r d s

Three-state system, operation, reliability, safety, risk, prediction.

S ł o w a k l u c z o w e

System trójstanowy, eksploatacja, niezawodność, bezpieczeństwo, ryzyko, predykcja.

S u m m a r y

There is proposed the method of reliability analysis of a three-state system at variable operations conditions. Introduced are the notions of the conditional and unconditional the three-state system reliability functions, the mean values of system lifetimes in the reliability state subsets and in the particular reliability states, the system risk function, and the moment when the system risk function exceeds its permitted level. These characteristics are determined for an exemplary three-state system operating at four operation states.

* Gdynia Maritime University, Morska 81-87 Street, 81-225 Gdynia, Poland.
** Air Force Institute of Technology, Księcia Bolesława 6 Street, 01-494 Warsaw, Poland.

Introduction

Most real technical systems are structurally very complex, and they often have complicated operation processes. Large numbers of components and subsystems and their operating complexity make the evaluation and prediction of their reliability, availability, and safety difficult. The time dependent interactions between the systems' operation processes, changing of operation states and the systems' structures and their component reliability and the changing of safety states and processes are evident features of most real technical systems. The common reliability or safety and operation analysis of these complex technical systems is of great value in the industrial practice.

Thus, taking into account the importance of the safety and the operating process effectiveness of real technical systems, it seems reasonable to expand the two-state approach [2, 3] into multi-state approach [2, 4, 6-7] in their reliability and safety analysis. The assumption that the systems are composed of multi-state components with reliability states or safety states degrading in time [2, 4, 6, 7] allows the possibility for more precise analysis of their reliability, safety, and effectiveness of operational processes. This assumption allows us to distinguish a system reliability or critical state of safety to be exceeded, which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Therefore, an important system reliability or safety characteristic is the time to the moment of exceeding the system reliability or critical safety state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function and the system multi-state safety function, which are the basic characteristics of the multi-state system.

In the particular case of a three-state system reliability analysis, for instance, the best reliability state ensuring full safety of a system operation, the worse permissible critical system reliability state and the worst non-permissible system reliability state can be distinguished.

In the reliability analysis of real systems, we find systems with complicated operation processes, also considering human factors, which have a significant influence on their reliability and safety. Many technical systems belong to the class of systems changing their reliability parameters at their variable operation conditions.

A convenient approach to the solution of his problem is modelling those systems operation processes using semi-Markov processes [1, 5], together with a multistate approach to their reliability analysis [2, 3]. This approach allows us to construct a joint general model of systems reliability related to their operation processes [3, 4, 6, 7].

Reliability of a three-state system at variable operation conditions

In the multistate reliability analysis to define the system with degrading components, we assume the following:

- n is the number of the system components;
- E_i, $i = 1, 2,..., n$, are components of a system;
- All components and a system under consideration have the reliability state set $\{0, 1,..., z\}$; $z \geq 1$,
- The reliability states are ordered, and the reliability state 0 is the worst and the reliability state z is the best;
- $T_i(u)$, $i = 1, 2,... ,n$, are independent random variables representing the lifetimes of components E_i in the reliability state subset $\{u, u + 1,..., z\}$, while they were in the reliability state z at the moment $t = 0$;
- $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u, u + 1,..., z\}$ while it was in the reliability state z at the moment $t = 0$;
- The system states degrades with time t;
- $E_i(t)$ is a component E_i reliability state at the moment t, $t \in < 0, \infty)$, given that it was in the reliability state z at the moment $t = 0$; and,
- $S(t)$ is a system S reliability state at the moment t, $t \in < 0, \infty)$, given that it was in the reliability state z at the moment $t = 0$.

The above assumptions mean that the reliability states of the system with degrading components may be changed in time only from better to worse [3, 6, 7]. The way in which the components and the system reliability states change is illustrated in Figure 1.

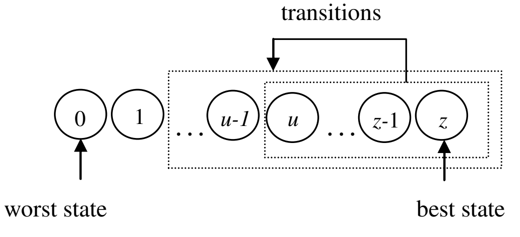

Fig. 1. Illustration of a system and components reliability states changing
Rys. 1. Ilustracja zmian stanów niezawodnościowych starzejącego się system wielostanowego

In the particular case of a three-state system, we assume that the system can stay in the following reliability states:

2 – a reliability state ensuring full safety of system operation,
1 – a critical reliability state ensuring less permissible system operation safety,
0 – a reliability state non-ensuring permissible system operation safety.

Moreover, we assume that the changes of the system operation process have an influence on its reliability, and we distinguish the following four operation states: z_1, z_2, z_3, z_4.

Under those assumptions, we define a system conditional three-state reliability function as follows:

$$[\boldsymbol{R}(t,\cdot)]^{(b)} = [[\boldsymbol{R}(t,0)]^{(b)}, [\boldsymbol{R}(t,1)]^{(b)}, [\boldsymbol{R}(t,2)]^{(b)}], \ t \in <0,\infty), b = 1, 2, 3, 4 \quad (1)$$

where

$$[\boldsymbol{R}(t,0)]^{(b)} = P(T^{(b)}(0) > t \mid Z(t) = z_b) = 1 \text{ for } t \in <0,\infty), b = 1, 2, 3, 4,$$

$$[\boldsymbol{R}(t,1)]^{(b)} = P(T^{(b)}(1) > t \mid Z(t) = z_b) \text{ for } t \in <0,\infty), b = 1, 2, 3, 4,$$

$$[\boldsymbol{R}(t,2)]^{(b)} = P(T^{(b)}(2) > t \mid Z(t) = z_b) \text{ for } t \in <0,\infty), b = 1, 2, 3, 4,$$

and $T^{(b)}(0)$ is the conditional system lifetime in the reliability state subset {0, 1, 2}, $T^{(b)}(1)$ is the conditional system lifetime in the reliability state subset {1, 2}, $T^{(b)}(2)$ is the conditional system lifetime in the reliability state subset {2}, while the system is at the operation state z_b, $b = 1, 2, 3, 4$.

In the case when the system operation time is sufficiently large, the unconditional system reliability function is as follows:

$$\boldsymbol{R}(t,\cdot) = [\boldsymbol{R}(t,0), [\boldsymbol{R}(t,1), [\boldsymbol{R}(t,2)], \ t \in <0,\infty), b = 1, 2, 3, 4 \quad (2)$$

where

$$\boldsymbol{R}(t,0) = P(T(0) > t) = 1 \text{ for } t \in <0,\infty),$$

$$\boldsymbol{R}(t,1) = P(T(1) > t) \text{ for } t \in <0,\infty),$$

$$\boldsymbol{R}(t,2) = P(T(2) > t) \text{ for } t \in <0,\infty),$$

and $T(0)$ is the unconditional system lifetime in the reliability state subset {0, 1, 2}, $T(1)$ is the unconditional system lifetime in the reliability state subset {1, 2}, $T(2)$ is the unconditional system lifetime in the reliability state subset {2} is given by the vector [6], [7]

$$\boldsymbol{R}(t,\cdot) = [1, [\boldsymbol{R}(t,1), [\boldsymbol{R}(t,2)], \ t \in <0,\infty) \quad (3)$$

where

$$\boldsymbol{R}(t,1) \cong \sum_{b=1}^{4} p_b [\boldsymbol{R}(t,1)]^{(b)} \text{ for } t \geq 0,$$

$$\boldsymbol{R}(t,2) \cong \sum_{b=1}^{4} p_b [\boldsymbol{R}(t,2)]^{(b)} \text{ for } t \geq 0,$$

whereas, p_1, p_2, p_3, p_4 are the limit values of the instantaneous probabilities of the system operation process $Z(t)$ staying at the particular operation states z_1, z_2, z_3, z_4.

Applying the above formulae, it is possible to determine [3, 7] the mean values $\mu(1)$, $\mu(2)$ of the unconditional lifetimes $T(1)$, $T(2)$ of the system staying, respectively, in the reliability state subsets {1, 2}, {2}

$$\mu(1) \cong \sum_{b=1}^{4} p_b \mu_b(1), \quad \mu(2) \cong \sum_{b=1}^{4} p_b \mu_b(2) \tag{4}$$

where $\mu_b(1)$, $\mu_b(2)$, b = 1, 2, 3, 4, are the mean values of the conditional system lifetimes In the reliability state subsets {1,2}, {2}, while the system is at the operation state z_b, b = 1, 2, 3, 4, determined by the following formulae:

$$\mu_b(1) = \int_0^\infty [\boldsymbol{R}(t,1)]^{(b)} dt, \;\; \mu_b(2) = \int_0^\infty [\boldsymbol{R}(t,2)]^{(b)} dt, \; b = 1, 2, 3, 4 \tag{5}$$

Having at our disposal the system mean unconditional lifetimes in the reliability state subsets, it is possible to determine the mean values $\bar{\mu}(1)$, $\bar{\mu}(2)$ of the system unconditional lifetimes in particular reliability states 1,2, namely

$$\bar{\mu}(1) = \mu(1) - \mu(2), \;\; \bar{\mu}(2) = \mu(2) \tag{6}$$

Moreover, it is possible to determine the system risk function defined by

$$\boldsymbol{r}(t) = P(T(1) < t) = 1 - \boldsymbol{R}(t,1), \; t \in <0, \infty) \tag{7}$$

and the moment when the system risk function exceeds the permitted level δ, given by

$$\tau = \boldsymbol{r}^{-1}(\delta) \tag{8}$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of the system risk function $\boldsymbol{r}(t)$.

An exemplary system at variable operation conditions reliability and risk prediction

We consider the three-state system that is operating at four-operation states z_1, z_2, z_3, z_4, respectively, with the probabilities as follows:

$$p_1 = 0.214,\ p_2 = 0.038,\ p_3 = 0.293,\ p_4 = 0.455.$$

We assume that its conditional reliability functions in the reliability state subsets {1,2} and {2}, respectively are as follows:

$$[\boldsymbol{R}(t,\cdot)]^{(1)} = [1, [\exp[-0.00207t], \exp[-0.00213]],\ t \in <0,\infty),$$

$$[\boldsymbol{R}(t,\cdot)]^{(2)} = [1, [\exp[-0.00144t], \exp[-0.00154t]],\ t \in <0,\infty),$$

$$[\boldsymbol{R}(t,\cdot)]^{(3)} = [1, [\exp[-0.00261t], \exp[0.00270t]],\ t \in <0,\infty),$$

$$[\boldsymbol{R}(t,\cdot)]^{(4)} = [1, [\exp[-0.00394t], \exp[0.00422t]],\ t \in <0,\infty).$$

Under those assumptions, according to (3), the system unconditional reliability function is

$$\boldsymbol{R}(t,\cdot) = [1, [\boldsymbol{R}(t,1), [\boldsymbol{R}(t,2)],\ t \in <0,\infty),$$

where

$$\boldsymbol{R}(t,1) \cong 0.214\exp[-0.00207t] + 0.038\exp[-0.00144t]$$

$$+ 0.293\exp[-0.00261t] + 0.455\exp[-0.00394t] \text{ for } t \geq 0,$$

$$\boldsymbol{R}(t,2) \cong 0.214\exp[-0.00213t] + 0.038\exp[-0.00154t]$$

$$+ 0.293\exp[-0.00270t] + 0.455\exp[-0.00422t] \text{ for } t \geq 0.$$

Using the above formulae (4)-(5), it is possibile to determine the mean values $\mu(1), \mu(2)$ of the system unconditional lifetimes $T(1)$, $T(2)$ in the reliability state subsets {1,2}, {2} that amount

$$\mu(1) \cong 0.214 \cdot 484 + 0.038 \cdot 694 + 0.293 \cdot 383 + 0.455 \cdot 254 \cong 358,$$

$$\mu(2) \cong 0.214 \cdot 469 + 0.038 \cdot 649 + 0.293 \cdot 370 + 0.455 \cdot 237 \cong 341.$$

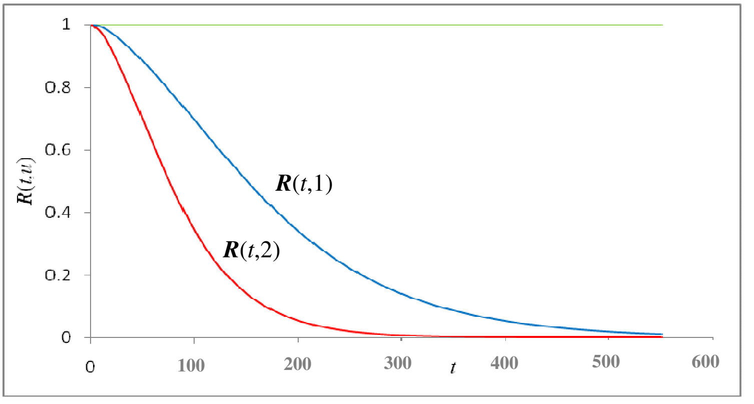

Fig. 2. The graph of the exemplary three-state system reliability function $\boldsymbol{R}(t,\cdot)$ co-ordinates
Rys. 2. Wykres składowych funkcji niezawodności $\boldsymbol{R}(t,\cdot)$ system przykładowego trójstanowego

Hence and from (6), the mean values $\bar{\mu}(1)$, $\bar{\mu}(2)$ of the system unconditional lifetimes in the particular reliability states 1, 2 are

$$\bar{\mu}(1) \cong 358 - 341 = 17,\ \ \bar{\mu}(2) = 341.$$

Additionally, considering (7) and (8), the system risk function is given by

$$\boldsymbol{r}(t) \cong 1 - 0.214\exp[-0.00217t] - 0.038\exp[-0.00144t]$$

$$- 0.293\exp[-0.00261t] - 0.455\exp[-0.00394t] \text{ for } t \geq 0, t \in <0,\infty),$$

whereas, the moment when the system risk function exceeds the permitted level $\delta = 0.05$ is

$$\tau \cong 70.$$

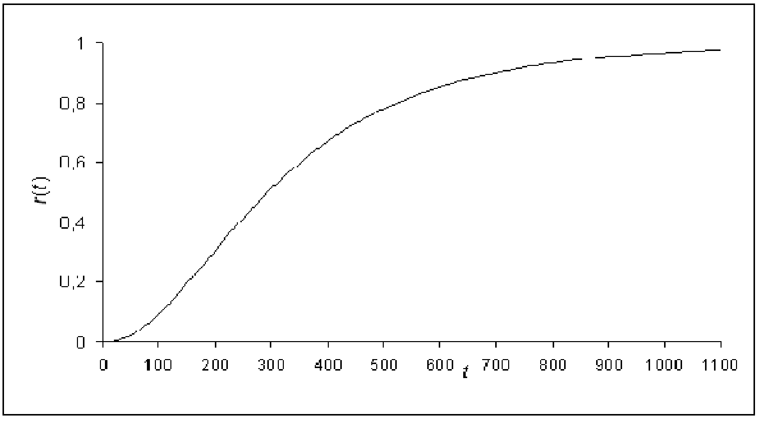

Fig. 3. The graph of the exemplary three-state system risk function $r(t)$
Rys. 3. Wykres funkcji ryzyka $r(t)$ trójstanowego systemu przykładowego

Summary

In this paper, the three-state approach to the reliability analysis of technical systems is presented. The possibilities of this approach are illustrated by an exemplary system for which the basic characteristics like the system unconditional reliability function, the mean values of the unconditional lifetimes in the reliability state subsets and in the particular reliability states, the system risk function, and the moment when the system exceeds the permitted level are determined. The proposed procedure for complex technical system reliability evaluation may be successfully applied to determine the real technical systems reliability and their reliability and operation costs optimisation.

References

1. Grabski F.: Semi-markowskie modele niezawodności i eksploatacji. Monografia, Polska Akademia Nauk, Instytut Badań Systemowych PAN, Warszawa 2002.
2. Kołowrocki K.: Reliability of Large Systems. Elsevier 2004.
3. Kołowrocki K.: Reliability and risk evaluation of complex systems in their operation processes. International Journal of Materials & Structural Reliability, Vol. 4, No 2, 2006, 129-147.
4. Kołowrocki K., Soszyńska J.: Reliability and safety of complex technical systems and processes: modeling – identification – prediction – optimization. Springer 2011.
5. Limnios N., Oprisan G.: Semi-Markov Processes and Reliability. Birkhauser, Boston 2001.
6. Soszyńska J.: Analiza niezawodności systemów w zmiennych warunkach eksploatacji. Rozprawa doktorska, Polska Akademia Nauk, Instytut Badań Systemowych PAN, Warszawa 2007.
7. Soszyńska J.: Systems reliability analysis in variable operation conditions. International Journal of Reliability, Quality and Safety Engineering. System Reliability and Safety, Vol. 14, No 6, 2007, 617-634.

Bezpieczeństwo i niezawodność systemu trójstanowego w zmiennych warunkach eksploatacji

S t r e s z c z e n i e

Zaproponowana jest metoda analizy niezawodności systemu trójstanowego w zmiennych warunków eksploatacji. Wprowadzone są pojęcia warunkowych i bezwarunkowych trójstanowych funkcji niezawodności systemu, wartości średnie bezwarunkowych czasów przebywania systemu w podzbiorach stanów oraz w poszczególnych stanach niezawodnościowych, funkcja ryzyka systemu oraz moment przekroczenia dopuszczalnego poziomu funkcji ryzyka. Charakterystyki te wyznaczone są dla przykładowego systemu trójstanowego eksploatowanego w czterech stanach eksploatacyjnych.