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An outline of a method for determining the density function of the time of exceeding the limit state with the use of the weibull distribution

Key words

A diagnostic parameter, the Weibull distribution, destructive processes, reliability, probability.

Slowa kluczowe

Parametr diagnostyczny, rozkład Weibulla, procesy destrukcyjne, niezawodność, prawdopodobieństwo.

Summary

This article presents an attempt at an analytical description of technical state changes within a selected group of technical objects. The occurring changes of the technical state of these objects are identified by diagnostic parameter values. The changes are identified by diagnostic parameter values. The technical state of a device deteriorates with the time of its maintenance due to the effect of numerous destructive factors. The conducted studies are based on the assumption that the intensity of changes of the deviation of diagnostic parameter values adopts the Weibull constants. The dynamics of changes of diagnostic parameter values is described by the difference equation that was transformed into a differential equation. Its solution in the form of a density function enables one to determine the reliability of a device in terms of an examined diagnostic parameter. The density function of the time of exceeding the limit state by a diagnostic parameter was determined using material from the literature [9], whose continuation is this article.

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Introduction

Aeronautical engineering obliges design engineers, manufacturers and users to meet requirements connected with maintaining high values of safety and reliability parameters. Examining the safety and reliability of an aircraft in the maintenance process involves the prediction of the technical state of its particular devices and systems and the aircraft itself as a platform combining all the above-mentioned elements. Analysing an aircraft as an object whose task, for example, is to ensure the transportation of passengers and cargo, we can assume that the maintenance conditions are of special importance compared with other popular means of transport [4]. The influence of a series of factors causes that the values of the parameters describing the technical state of an aircraft change over time. Destructive processes manifesting themselves in the form of overload, friction, vibrations, wear, etc. have a crucial effect on technical state changes in aircraft devices.

The technical state of aircraft devices is mainly evaluated through a set of diagnostic parameters. The effect of destructive processes manifests itself in the change of diagnostic parameter values causing a rise in the deviation from the nominal values of these parameters. The values of deviations from the nominal values are used to estimate the reliability of a device.

The classifications of the correlation between the effect of destructive processes and the change of diagnostic parameter values are presented in the paper [9]. In this article, the density function of changes of diagnostic parameter deviations is determined based on the following assumptions:

- The technical state of a device is determined by one dominant diagnostic parameter. Its current value is denoted by "*x*".
- The changes of a diagnostic parameter value due to the destructive effect of ageing processes occurs with the passing of calendar time.
- The deviation of a diagnostic parameter from the nominal value is

$$z = \left| x_p - x_n \right| \tag{1}$$

where:

x_p – the measured value of a diagnostic parameter,

x_n – the nominal value of a diagnostic parameter.

- If $z \in [0, z_d]$, then an element of a device is regarded as operable; otherwise, an element of a device is regarded as inoperable,
- An increase of a diagnostic parameter deviation in the function of calendar time satisfies the following relationship:

$$\frac{dz}{dt} = c ,$$

where:

c – the mean value, a variable velocity depending on ageing processes,
t – the calendar time.

This determines the density function of changes in values of diagnostic parameter deviations.

Determining the density function of changes in values of diagnostic parameter deviations

It is assumed that the intensity of the increase in deviations has the following form:

$$\lambda(t) = \frac{\alpha}{\theta} t^{\alpha-1} \tag{2}$$

where:

α i θ – the constants in the Weibull distribution with the following denotations:
α – the shape factor,
θ – the scale factor.

The random dynamics of changes of diagnostic parameter values including the deviation is described by the difference equation. Let $U_{z,t}$ denote the probability that at the time t, the value of a diagnostic parameter deviation adopts the value "z".

The differentiated equation has the following form:

$$U_{z,\,t+\Delta t} = \left(1 - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\right) U_{z,\,t} + \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\, U_{z-\Delta z,\,t} \tag{3}$$

where:

Δz – the increase in deviation of a diagnostic parameter over the time interval Δt.

Equation (3) has the following form in function notation (4):

$$u(z, t + \Delta t) = \left(1 - \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\right) u(z, t) + \frac{\alpha}{\theta} t^{\alpha-1} \Delta t\, u(z - \Delta z, t) \tag{4}$$

where:

$u(z,t)$ – the density function of a diagnostic parameter deviation;

$\left(1-\frac{\alpha}{\theta}t^{\alpha-1}\Delta t\right)$ – the probability that over the time interval Δt there is no parameter deviation;

$\frac{\alpha}{\theta}t^{\alpha-1}\Delta t$ – the probability that over the time interval Δt there is the increase in the parameter deviation "Δz";

and the following condition is met $\frac{\alpha}{\theta}t^{\alpha-1}\Delta t \le 1$.

We transform Equation (4) into a partial differential equation. We assume the following approximation:

$$
\begin{aligned}
u(z,t+\Delta t) &= u(z,t)+\frac{\partial u(z,t)}{\partial t}\Delta t,\\
u(z-\Delta z,t) &= u(z,t)-\frac{\partial u(z,t)}{\partial z}\Delta z+\frac{1}{2}\frac{\partial^2 u(z,t)}{\partial z^2}(\Delta z)^2
\end{aligned}
\tag{5}
$$

We substitute the relationships expressed in (5) into Equation (4) and obtain equation (6).

$$
\frac{\partial u(z,t)}{\partial z}=-\frac{\alpha}{\theta}t^{\alpha-1}\Delta z\frac{\partial u(z,t)}{\partial z}+\frac{1}{2}\frac{\alpha}{\theta}t^{\alpha-1}(\Delta z)^2\frac{\partial^2 u(z,t)}{\partial z^2}
\tag{6}
$$

We examine the increase of a parameter deviation per unit of time (when $\Delta t = 1$), so

$$
\frac{\Delta z}{\Delta t}=c\,,\quad \Rightarrow \Delta z=c\Delta t\,,\quad \underset{\Delta t=1}{\Rightarrow}\bar{c}\,,
$$

where: $\bar{c}$ denotes the deviation increase per a unit of time.

The final form of Equation (6) is as follows:

$$
\frac{\partial u(z,t)}{\partial z}=-\underbrace{\frac{\alpha\bar{c}}{\theta}t^{\alpha-1}}_{\gamma(t)}\frac{\partial u(z,t)}{\partial z}+\frac{1}{2}\underbrace{\frac{\alpha\bar{c}^2}{\theta}t^{\alpha-1}}_{\beta(t)}\frac{\partial^2 u(z,t)}{\partial z^2}
\tag{7}
$$

As it can be seen in Equation (7), the form of the coefficients depends on the parameter values α. For $\alpha = 1$, the coefficients have the following form:

$$\gamma(t) = \frac{\bar{c}}{\theta}; \qquad \beta = \frac{\bar{c}^2}{\theta}.$$

For $\alpha = 2$, the coefficients have the following form:

$$\gamma(t) = \frac{2\bar{c}}{\theta} t; \qquad \beta(t) = \frac{2\bar{c}^2}{\theta} t.$$

The solution of Equation (7) has the following form:

$$u(z,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(z-B(t))^2}{2A(t)}} \tag{8}$$

where:

B(t) – the average value of a parameter deviation for the time of the service life *t*,

$$B(t) = \int_0^t \gamma(t) dt \tag{9}$$

A(t) – the value of the variance of a diagnostic parameter deviation for the time of the service life *t*.

$$A(t) = \int_0^t \beta(t) dt \tag{10}$$

We calculate integrals (9) and (10) and obtain the following:

$$B(t) = \int_0^t \frac{\alpha\bar{c}}{\theta} t^{\alpha-1} dt = \frac{\alpha\bar{c}}{\theta} \int_0^t t^{\alpha-1} dt = \frac{\alpha\bar{c}}{\theta} \frac{1}{\alpha} t^{\alpha} \Big|_0^t = \frac{\bar{c}}{\theta} t^{\alpha} - 0 = \frac{\bar{c}}{\theta} t^{\alpha} \tag{11}$$

$$A(t) = \int_0^t \frac{\alpha\bar{c}^2}{\theta} t^{\alpha-1} = \frac{\alpha\bar{c}^2}{\theta} \frac{1}{\alpha} t^{\alpha} \Big|_0^t = \frac{\bar{c}^2}{\theta} t^{\alpha} \tag{12}$$

Hence, the relationship depicted in Equation (8) has the following form:

$$u(z,t)=\frac{1}{\sqrt{2\pi\frac{\bar{c}^2}{\theta}t^\alpha}}e^{-\frac{\left(z-\frac{\bar{c}}{\theta}t^\alpha\right)^2}{2\frac{\bar{c}^2}{\theta}t^\alpha}} \tag{13}$$

The relationship depicted in Equation (13) presents the density function of a diagnostic parameter deviation from the nominal value.

Let

$$\frac{\bar{c}}{\theta}=b \qquad \text{and} \qquad \frac{\bar{c}^2}{\theta}=a\,.$$

Hence, Equation (13) has the following form:

$$u(z,t)=\frac{1}{\sqrt{2\pi a t^\alpha}}e^{-\frac{\left(z-bt^\alpha\right)^2}{2at^\alpha}} \tag{14}$$

By using the density function (14), we can determine the relationship for the reliability of a device in terms of an examined diagnostic parameter. This relationship has the form of Equation (15) as follows:

$$R(t)=\int_{-\infty}^{z_d}u(z,t)dz \tag{15}$$

where:

z_d – the permissible deviation value of the diagnostic parameter a $u(z, t)$ is determined by Equation (14).

Determining the distribution of time when a diagnostic parameter exceeds the permissible state

Using the deviation density function, we can write down the probability of exceeding the deviation value of a diagnostic parameter in the following form:

$$Q(t,z_d)=\int_{z_d}^{\infty}\frac{1}{\sqrt{2\pi a t^\alpha}}e^{-\frac{\left(z-bt^\alpha\right)^2}{2at^\alpha}}dz \tag{16}$$

The density function for the distribution of time of exceeding the permissible value of the diagnostic parameter z_d equals

$$f(t)=\frac{\partial}{\partial t}Q(t, z_d) \tag{17}$$

If we consider (16), this equation takes the form

$$f(t)=\frac{\partial}{\partial t}\int_{z_d}^{\infty}\frac{1}{\sqrt{2\pi at^{\alpha}}}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}dz$$

Hence,

$$f(t)=\int_{z_d}^{\infty}\left\{\frac{\partial}{\partial t}\left[\frac{1}{\sqrt{2\pi at^{\alpha}}}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}\right]\right\}dz \tag{18}$$

We search for the time derivative of the integrand of this relationship (18) and obtain the following:

$$\frac{\partial}{\partial t}\left[\frac{1}{\sqrt{2\pi at^{\alpha}}}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}\right]=\left(\frac{1}{\sqrt{2\pi at^{\alpha}}}\right)'e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}+\frac{1}{\sqrt{2\pi at^{\alpha}}}\cdot\left(e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}\right)' \tag{19}$$

We can then calculate the component derivatives of Equation (19) as follows:

$$\left(\frac{1}{\sqrt{2\pi at^{\alpha}}}\right)'=\frac{0-\frac{1}{2}\left(2\pi at^{\alpha}\right)^{-\frac{1}{2}}\cdot 2\pi a\alpha t^{\alpha-1}}{2\pi at^{\alpha}}=-\frac{\pi a\alpha t^{\alpha-1}}{2\pi at^{\alpha}\left(2\pi at^{\alpha}\right)^{\frac{1}{2}}}=-\frac{\alpha}{2t\sqrt{2\pi at^{\alpha}}} \tag{20}$$

$$\left(e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}\right)'=e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}\left[-\frac{2\left(z-bt^{\alpha}\right)\left(-b\alpha t^{\alpha-1}\right)2at^{\alpha}-\left(z-bt^{\alpha}\right)^2 2a\alpha t^{\alpha-1}}{\left(2at^{\alpha}\right)^2}\right]=$$

$$=-\frac{2\left(z-bt^{\alpha}\right)\left(-b\alpha t^{\alpha-1}\right)2at^{\alpha}-\left(z-bt^{\alpha}\right)^2 2a\alpha t^{\alpha-1}}{\left(2at^{\alpha}\right)^2}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}= \tag{21}$$

$$=\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha-1}+\left(z-bt^{\alpha}\right)^2\alpha\cdot\frac{1}{t}}{2at^{\alpha}}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}$$

We then substitute the above-defined formulas into Equation (19):

$$\frac{\partial}{\partial t}\left[u(z,t)\right]=-\frac{\alpha}{2t\sqrt{2\pi a t^{\alpha}}}e^{-\frac{\left(z-bt^{\alpha}\right)^{2}}{2at^{\alpha}}}+\frac{1}{\sqrt{2\pi a t^{\alpha}}}\cdot\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha-1}+\left(z-bt^{\alpha}\right)^{2}\frac{\alpha}{t}}{2at^{\alpha}}e^{-\frac{\left(z-bt^{\alpha}\right)^{2}}{2at^{\alpha}}}=$$

$$=-\frac{\alpha}{2t}u(z,t)+\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha-1}+\left(z-bt^{\alpha}\right)^{2}\frac{\alpha}{t}}{2at^{\alpha}}u(z,t)=$$

$$=\left(-\frac{\alpha}{2t}+\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha-1}+\left(z-bt^{\alpha}\right)^{2}\frac{\alpha}{t}}{2at^{\alpha}}\right)u(z,t)= \tag{22}$$

$$=\left(-\frac{\alpha}{2t}+\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha}+\left(z-bt^{\alpha}\right)^{2}\alpha}{2at^{\alpha+1}}\right)u(z,t)$$

From this relationship (18), we obtain the following:

$$f(t)=\int_{z_d}^{\infty}\left\{\frac{\partial}{\partial t}\left[\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{\left(z-bt^{\alpha}\right)^{2}}{2at^{\alpha}}}\right]\right\}dz=$$

$$=\int_{z_d}^{\infty}\left[\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha}+\left(z-bt^{\alpha}\right)^{2}\alpha}{2at^{\alpha+1}}-\frac{\alpha}{2t}\right]u(z,t)dz \tag{23}$$

In order to calculate the integral (23), we need to determine an antiderivative. We assume the following form of the antiderivative of the integrand in the relationship presented in (23):

$$w(z,t)=u(z,t)\theta(z,t) \tag{24}$$

The derivative of the indefinite integral with respect to the variable "z" is equal to the integrand of the relationship depicted in (23).

Hence,

$$\frac{\partial u(z,t)}{\partial z}\theta(z,t)+u(z,t)\frac{\partial\theta(z,t)}{\partial z}=\left[\frac{2\left(z-bt^{\alpha}\right)b\alpha t^{\alpha}+\left(z-bt^{\alpha}\right)^{2}\alpha}{2at^{\alpha+1}}-\frac{\alpha}{2t}\right]u(z,t) \tag{25}$$

We calculate the derivative $\frac{\partial u(z,t)}{\partial z}$ as follows:

$$\frac{\partial u(z,t)}{\partial z}=\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}\left[-\frac{2(z-bt^{\alpha})}{2at^{\alpha}}\right]=u(z,t)\left[-\frac{(z-bt^{\alpha})}{at^{\alpha}}\right] \tag{26}$$

By substituting (26) into (25), we obtain the following:

$$\begin{aligned}&u(z,t)\left(-\frac{(z-bt^{\alpha})}{at^{\alpha}}\right)\theta(z,t)+u(z,t)\frac{\partial\theta(z,t)}{\partial z}=\\&=\left[\frac{2(z-bt^{\alpha})b\alpha t^{\alpha}+(z-bt^{\alpha})^2\alpha}{2at^{\alpha+1}}-\frac{\alpha}{2t}\right]u(z,t)\end{aligned} \tag{27}$$

$$u(z,t)\left[\underbrace{-\frac{(z-bt^{\alpha})}{at^{\alpha}}\theta(z,t)}_{I-L}+\underbrace{\frac{\partial\theta(z,t)}{\partial z}}_{II-L}\right]=u(z,t)\left[\underbrace{\frac{2(z-bt^{\alpha})b\alpha t^{\alpha}+(z-bt^{\alpha})^2\alpha}{2at^{\alpha+1}}}_{I-P}\underbrace{-\frac{\alpha}{2t}}_{II-P}\right] \tag{28}$$

By using the relationship (28), we can determine the function $\theta(z,t)$ in such a way that the left side of the relationship (28) equals the right side. So

$$I-L=I-P \quad \rightarrow \quad \theta(z,t)=-\frac{(2b\alpha t^{\alpha}+\alpha(z-bt^{\alpha}))}{2t} \tag{29}$$

$$II-L=II-P \quad \rightarrow \quad \frac{\partial\theta(z,t)}{\partial z}=-\frac{\alpha}{2t} \tag{30}$$

After reducing this expression, we obtain the following:

$$\theta(z,t)=-\frac{\alpha(2bt^{\alpha}+z-bt^{\alpha})}{2t}=-\frac{\alpha(z+bt^{\alpha})}{2t} \tag{31}$$

The antiderivative has the following form:

$$w(z,t)=u(z,t)\left(-\frac{\alpha(z+bt^{\alpha})}{2t}\right) \tag{32}$$

where:

$$u(z,t)=\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z-bt^{\alpha})^2}{2at^{\alpha}}}$$

We can then calculate the integral as follows:

$$f(t)_{z_d}=u(z,t)\left(-\frac{\alpha(z+bt^{\alpha})}{2t}\right)\Bigg|_{z_d}^{\infty}=u(z,t)\left(\frac{\alpha(z_d+bt^{\alpha})}{2t}\right) \tag{33}$$

where:

$$u(z_d,t)=\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z_d-bt^{\alpha})^2}{2at^{\alpha}}} \tag{34}$$

Hence, the density function of the time of exceeding the permissible value of the diagnostic parameter “ z_d ” has the following form:

$$f(t)_{z_d}=\frac{\alpha(z_d+bt^{\alpha})}{2t}\cdot\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z_d-bt^{\alpha})^2}{2at^{\alpha}}} \tag{35}$$

We need to check whether the integral (36) is equal to 1.

$$\int_0^{\infty}\frac{\alpha(z_d+bt^{\alpha})}{2t}\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z_d-bt^{\alpha})^2}{2at^{\alpha}}}dt\overset{?}{=}1 \tag{36}$$

The above relationship can be written down in the form of Equation (37)

$$\underbrace{\int_0^{\infty}\frac{\alpha z_d}{2t}\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z_d-bt^{\alpha})^2}{2at^{\alpha}}}dt}_{A}+\underbrace{\int_0^{\infty}\frac{\alpha b t^{\alpha}}{2t}\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z_d-bt^{\alpha})^2}{2at^{\alpha}}}dt}_{B}\overset{?}{=}1 \tag{37}$$

We then calculate the integral A as follows:

$$A=\int_0^{\infty}\frac{\alpha z_d}{2t}\frac{1}{\sqrt{2\pi a t^{\alpha}}}e^{-\frac{(z_d-bt^{\alpha})^2}{2at^{\alpha}}}dt=\frac{\alpha z_d}{2t}\frac{1}{\sqrt{2\pi a}}\int_0^{\infty}\frac{1}{t\sqrt{t^{\alpha}}}e^{-\frac{z_d{}^2-2z_d bt^{\alpha}+b^2t^{2\alpha}}{2at^{\alpha}}}dt \tag{38}$$

In order to determine the above-mentioned integral, we perform the following substitution:

$$u = t^{\alpha}$$

$$du = \alpha t^{\alpha-1} dt \quad \Rightarrow \quad dt = \frac{du}{\alpha t^{\alpha-1}}$$

Hence,

$$A = \frac{\alpha z_d}{2} \frac{1}{\sqrt{2\pi a}} \int_0^{\infty} \frac{1}{t\sqrt{u}} \cdot e^{-\frac{{z_d}^2 - 2z_d bu + b^2u^2}{2au}} \frac{du}{\alpha t^{\alpha-1}} =$$

$$= \frac{\alpha z_d}{2} \frac{1}{\sqrt{2\pi a}} \frac{1}{\alpha} \int_0^{\infty} \frac{1}{\underbrace{t \cdot t^{\alpha-1}}_{u} \sqrt{u}} \cdot e^{-\frac{\left({z_d}^2 - 2z_d bu + b^2u^2\right)}{2au}} du = \tag{39}$$

$$= \frac{z_d}{2} \frac{1}{\sqrt{2\pi a}} \int_0^{\infty} \frac{1}{u\sqrt{u}} e^{-\frac{1 - 2z_d bu\frac{1}{{z_d}^2} + \frac{b^2u^2}{{z_d}^2}}{2\frac{au}{{z_d}^2}}} du$$

We then perform one more substitution:

$$\omega = \frac{b}{z_d} u \quad \Rightarrow \quad u = \frac{z_d}{b} \omega$$

$$d\omega = \frac{b}{z_d} du \quad \Rightarrow \quad du = \frac{z_d}{b} d\omega$$

As a result, we get the following relationship:

$$A = \frac{z_d}{2} \frac{1}{\sqrt{2\pi a}} e^{\frac{2z_d b \cdot \frac{1}{{z_d}^2}}{2\frac{a}{{z_d}^2}}} \int_0^{\infty} \frac{1}{\frac{z_d}{b}\omega \sqrt{\frac{z_d}{b}\omega}} e^{-\frac{1+\omega^2}{2\frac{a}{z_d b}\omega}} \frac{z_d}{b} d\omega =$$

$$= \frac{z_d}{2} \frac{1}{\sqrt{2\pi a}} e^{\frac{bz_d}{a}} \cdot \frac{1}{\frac{z_d}{b}\sqrt{\frac{z_d}{b}}} \frac{z_d}{b} \underbrace{\int_0^{\infty} \frac{1}{\omega\sqrt{\omega}} e^{-\frac{1+\omega^2}{2q\omega}} d\omega}_{D} \tag{40}$$

where:

$$q = \frac{a}{z_d b}.$$

Based on the patterns posted on the table of integrals [3], we can write:

$$D = \frac{\sqrt{2q\pi}}{\sqrt[q]{e}}.$$

After further transformations, the solution of Equation (40) has the following form:

$$A = \frac{z_d}{2} \cdot \frac{1}{\sqrt{2\pi a}} e^{\frac{bz_d}{a}} \frac{1}{\sqrt{\frac{z_d}{b}}} \frac{\sqrt{2q\pi}}{\sqrt[q]{e}} = \frac{z_d}{2} \frac{1}{\sqrt{2\pi a}} e^{\frac{bz_d}{b}} \cdot \frac{1}{\sqrt{\frac{z_d}{b}}} \frac{\sqrt{\frac{2a\pi}{z_d b}}}{e^{\frac{\frac{1}{a}}{z_d b}}} =$$

$$= \frac{z_d}{2} \cdot \frac{1}{\sqrt{2\pi a}} e^{\frac{bz_d}{a}} \cdot \frac{1}{\sqrt{\frac{z_d}{b}}} \frac{\frac{\sqrt{2a\pi}}{\sqrt{z_d b}}}{e^{\frac{z_d b}{a}}} = \frac{z_d}{2} \frac{1}{\sqrt{\frac{z_d}{b}}} \cdot \frac{1}{\sqrt{z_d b}} = \frac{z_d}{2} \cdot \frac{1}{\sqrt{\frac{z_d}{b} z_d b}} = \frac{z_d}{2} \cdot \frac{1}{z_d} = \frac{1}{2} \tag{41}$$

Before calculating integral B, we write it down as Equation (42):

$$B = \int_0^\infty \frac{\alpha b t^\alpha}{zt} \cdot \frac{1}{\sqrt{2\pi a t^\alpha}} e^{-\frac{(z_d - bt^\alpha)^2}{2at^\alpha}} dt = \frac{\alpha b}{\sqrt{2\pi a}} \int_0^\infty \frac{t^\alpha}{2t} \cdot \frac{1}{\sqrt{t^\alpha}} e^{-\frac{(z_d - bt^\alpha)^2}{2at^\alpha}} dt \tag{42}$$

We then make the following substitution:

$$u = t^\alpha$$

$$du = \alpha t^{\alpha-1} dt \quad \Rightarrow \quad dt = \frac{du}{\alpha t^{\alpha-1}}$$

$$B = \frac{\alpha b}{2\sqrt{2\pi a}} \int_0^\infty t^{\alpha-1} \frac{1}{\sqrt{u}} e^{-\frac{(z_d - bu)^2}{2au}} \cdot \frac{1}{\alpha t^{\alpha-1}} du = \frac{b}{2\sqrt{2\pi a}} \int_0^\infty \frac{1}{\sqrt{u}} e^{-\frac{(z_d - bu)^2}{2au}} du \tag{43}$$

$$B=\frac{b}{2\sqrt{2\pi a}}\cdot\int_0^\infty\frac{1}{\sqrt{u}}e^{-\frac{z_d{}^2-2z_dbu+b^2u^2}{2au}}du=\frac{b}{2\sqrt{2\pi a}}\cdot\int_0^\infty\frac{1}{\sqrt{u}}e^{-\frac{\left(1-2\frac{z_db}{z_d{}^2}u+\frac{b^2}{z_d{}^2}u^2\right)}{2\frac{a}{z_d{}^2}u}}du=$$
$$=\frac{b}{2\sqrt{2\pi a}}e^{\frac{2\frac{z_db}{z_d{}^2}}{2\frac{a}{z_d^2}}}\int_0^\infty\frac{1}{\sqrt{u}}e^{-\frac{1+\frac{b^2}{z_d{}^2}u^2}{2\frac{a}{z_d{}^2}u}}du=\frac{b}{2\sqrt{2\pi a}}e^{\frac{z_db}{a}}\int_0^\infty\frac{1}{\sqrt{u}}e^{-\frac{1+\frac{b^2}{z_d{}^2}u^2}{2\frac{a}{z_d{}^2}u}}du \tag{44}$$

We then substitute again as follows:

$$\omega=\frac{b}{z_d}u\ \Rightarrow\ u=\frac{z_d}{b}\omega$$
$$d\omega=\frac{b}{z_d}du\ \Rightarrow\ du=\frac{z_d}{b}d\omega$$

As a result, we obtain the following:

$$B=\frac{b}{2\sqrt{2\pi a}}e^{\frac{z_db}{a}}\int_0^\infty\frac{1}{\sqrt{\frac{z_d}{b}\omega}}e^{-\frac{1+\omega^2}{2\frac{a}{z_d{}^2}\frac{z_d}{b}\omega}}\cdot\frac{z_d}{b}d\omega=\frac{b}{2\sqrt{2\pi a}}e^{\frac{z_db}{a}}\cdot\sqrt{\frac{z_d}{b}}\underbrace{\int_0^\infty\frac{1}{\sqrt{\omega}}e^{-\frac{1+\omega^2}{2q\cdot\omega}}d\omega}_{E} \tag{45}$$

where:

$$q=\frac{a}{z_db}$$

Similar to the dependency expressed in Equation (40), using patterns posed in [3], we can write

$$E=\frac{\sqrt{2q\pi}}{\sqrt[q]{e}}$$

As a result of the above transformations, the solution of integral B has the form of the following relationship (46):

$$B=\frac{b}{2\sqrt{2\pi a}}\cdot e^{\frac{z_db}{a}}\sqrt{\frac{z_d}{b}}\cdot\frac{\sqrt{2\frac{a}{z_db}\pi}}{e^{\frac{1}{\frac{a}{z_db}}}}=\frac{b}{2\sqrt{2\pi a}}\cdot\frac{e^{\frac{z_db}{a}}}{e^{\frac{z_db}{a}}}\cdot\frac{\sqrt{2\pi a}}{\sqrt{z_db}}\cdot\sqrt{\frac{z_d}{b}}=\frac{1}{2} \tag{46}$$

The above results indicate that the relationship depicted in (36) is true, i.e. $f(t)_{z_d}$ is the density function for the distribution of the time of exceeding the permissible state.

Summary

Determining the density function for the distribution of the time of exceeding the limit state is an extremely significant issue. Based on the above-mentioned function, we can determine the residual durability of a device, which will constitute the subject of further analyses. A significant element of this paper involves the utilisation of the Weibull distribution to determine both the density function of changes in diagnostic parameter deviations and the density function of the time of exceeding the limit state by a diagnostic parameter.

Contemporary aircraft are equipped with various electronic devices supporting both its functions and flight. These devices undergo periodic inspections during which the values of diagnostic parameters are recorded. The monitoring of diagnostic parameter values is contingent upon the destructive effect of factors deteriorating the technical state of devices and systems. Such changes are often described by means of the Weibull distribution. Therefore, the utilisation of the Weibull distribution seems to be justified.

The description presented in this paper may be used not only in the field of aeronautical engineering but also in all other fields where the technical state of devices is determined on the basis of analysing the changes of diagnostic parameters.

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Zarys metody określenia funkcji gęstości czasu przekroczenia stanu dopuszczalnego z wykorzystaniem rozkładu Weibulla

S t r e s z c z e n i e

W artykule podjęto próbę analitycznego opisu zmiany stanu technicznego wybranej grupy obiektów technicznych. Zachodzące zmiany stanu technicznego tychże obiektów identyfikowane są za pomocą wartości parametrów diagnostycznych. W wyniku oddziaływania licznej grupy czynników destrukcyjnych stan techniczny urządzeń wraz z upływem czasu ich eksploatacji ulega pogorszeniu. Podstawą przeprowadzonych rozważań było przyjęcie założenia, że intensywność zmian odchyłki wartości parametrów diagnostycznych przyjmuje stałe o rozkładzie Weibulla. Dynamikę zmian wartości parametrów diagnostycznych opisano za pomocą równania różnicowego, dla którego, po przekształceniu do postaci równania różniczkowego, wyznaczono rozwiązanie w postaci funkcji gęstości umożliwiającej określenie niezawodności urządzenia ze względu na rozpatrywany parametr diagnostyczny. Posiłkując się materiałem zamieszczonym w [9], której niniejszy artykuł jest kontynuacją, wyznaczono funkcję gęstości czasu przekroczenia stanu dopuszczalnego przez parametr diagnostyczny.