SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
3 (167) 2011

MICHAŁ STYP-REKOWSKI*, LESZEK KNOPIK*, EUGENIUSZ MAŃKA*

Probabilistic formulation of steel cables durability problem

Key words

Steel cable, durability, probabilistic method.

Słowa kluczowe

Lina stalowa, trwałość, metoda probabilistyczna.

Summary

This paper introduces a procedure for defining the probability of achievement of quantity value received as a state symptom. The results of magnetic investigations of steel cables confirmed the usefulness of the presented procedure in monitoring of the object state, in this case, of a cable mechanism.

Introduction

The durability of steel cables is determined by many factors [1, 2] creating a multielement set. The significance of individual factors that exist in the set depends on the kind of mechanism in which the cable is used. It is different for

* University of Technology and Life Sciences in Bydgoszcz, Prof. S. Kaliski Avenue 7, 85-796 Bydgoszcz, Poland.

stay-cables of some constructions (mast, chimney), and different for the cable of hoists and lifts (shaft, ski). However, in each case, extortion influence on the cable has a random character. Therefore, the cable durability is considered a random problem, in which probabilistic methods are used. This paper presents a method that makes it possible to estimate the probability of the cable reaching the acceptable border state.

Verifying the usefulness of the proposed method is the aim of this work. In practical conditions, it may to contribute to an increase in the time of the exploitation of the cables and the whole mechanism with the assurance of indispensable safety.

Statistical analysis of magnetic investigations results

The presented analytic research results were conducted on the basis of results received in magnetic investigations of cables [3]. Because these signals are of a random character, statistical analysis is indispensable.

The first step was the choice of hypothetical distributions of empirical investigation results. The programs STATGRAPHICS and STATISTICA were used to study and introduce research results, both analytic and empirical, and they contain a spacious collection of distributions. As a result of their review and analysis, the following distributions were accepted:

- Gamma, for which the frequency function of variable x can be expressed by formula:

$$f(x) = \frac{1}{b^p \Gamma(p)} x^{p-1} e^{-x/b} \tag{1}$$

  for $p, b, x > 0$; and,

- Weibull, with the frequency function of variable x expressed with equation:

$$f(x) = a \cdot b^{-a} \cdot x^{a-1} \cdot e^{\left(\frac{x}{b}\right)^{a}} \tag{2}$$

In both cases, variables belong to the range $x \in (0, +\infty)$.

The preliminary, estimated opinion of the usefulness of the chosen distribution indicated agreement with empirical results.

To check whether the studied population has the defined type of distribution, the tests of goodness of fit are used. In practice, the two most often used tests are *chi-square* (χ^2) and *Kolmogorov* (λ)[4].

The *chi-square* test is used for both continuous and step distributions. The populations are divided by class of value and for each class from the hypothetical distribution of theoretical sizes and compared the empirical values by means of the suitable statistics (χ^2). The sample size of population is limited applying this test. It has to be large because its elements are divided by class of value, which should be sufficiently numerous. It is assumed that each class of value should contain at least 8 test results.

In second test, *λ Kolmogorov*, empirical and hypothetical distribution functions are compared. If the general population has a distribution concordant with the hypothesis, then the value of empirical and hypothetical distribution functions in all studied points should be close to each other. The continuity of hypothetical distribution function is the condition that essentially limits the applicability of this test.

In the results of preliminary analysis of the obtain results of empirical investigations, *chi-square* (χ^2) and *Kolmogorov* (λ) [5] were proven to be indispensable for the analytic research of cables, for the following reasons:

- The sample size of the population from over 300 tests is sufficient to apply *chi-square* test.
- The distribution functions of received hypothetical distributions (the gamma and Weibull) are continuous.

The statistical analysis used n = 326 values of measurements of the amplitudes of magnetic recorder plotter inclinations, which were recorded for transmission of tape recorder movement p_R = 20 mm/m. A comparison of results for transmission values 10 and 20 mm/m indicated that higher value had greater accuracy. The number of signal values resulted from the fact that twenty measuring sections were tested, which permitted to register mentioned to be above the number of peaks. Average value of all registered amplitudes is $\overline{x} = 4.02$, and the standard deviation s = 2,04. The value of the coefficient of changeability, expressed by the quotient $v = s/\overline{x} = 0.51$, shows that the analysed statistical data have the comparatively large dispersion of values.

The preliminary analysis indicates that the distribution of probability of the studied statistical feature is asymmetrical. The attempt of adjustment of theoretical distribution to empirical data distribution indicates that, among the considered ones, the gamma distribution is better; therefore, it received the following analysis. The gamma distribution has a frequency function according to Formula (1), in which expression $\Gamma(p)$ is gamma function is described by the following equation:

$$\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx \tag{3}$$

Average value of the random variable X in the gamma distribution is obtained from the following formula:

$$EX = p \cdot b, \tag{4}$$

And variance is determined by

$$D^2X = p \cdot b^2 \tag{5}$$

Formulae (4) and (5) are the basis to determining the initial values of parameters *p* and *b* using empirical data. $\overline{X}$ and S^2 were marked as the estimators of average the values and variances, which were obtained from following formulae:

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{6}$$

$$S^2 = \frac{1}{n}\sum_{1=1}^{n} (x_i - \overline{x})^2 \tag{7}$$

If by $\hat{p}$ and $\hat{b}$ there are marked the estimators of parameters respectively *p* as well as *b*, then the equation (4) and (5) for moments method obtain form:

$$\overline{X} = \hat{p}\hat{b} \tag{8}$$

$$S^2 = \hat{p}\,\hat{b}^2 \tag{9}$$

From Equations (8) and (9), the estimators $\hat{p}$ and $\hat{b}$ of parameters *p* as well as *b* of distribution (1) were determined, according to formulae:

$$\hat{b} = S^2/\overline{X} \tag{10}$$

$$\hat{p} = S^2/(\overline{X})^2 \tag{11}$$

Parameter *p* is the parameter of form (shape), and it does not depend on the unit in which random variable *X* is measured; however, *b* is the parameter of scale. Defined with formulae (10) and (11) notes are treated as preliminary notes (initial) of the values of parameters *p* and *b*, in process of the exact values estimation. To get the more exact notes of parameters p and b, in this work the method of the largest credibility was applied. In this case statistical pack of program STATISTICA was used.

Results of investigations and their analysis

The procedure was applied to the results of experimental investigations of steel cable. The results are recorded in graphic form on a recorder tape. An exemplary fragment of the defectogram is shown in Fig. 1. Obtained results, with the division of individual classes of value, are shown in Table 1.

Table 1. Statement of measurements results – power of a set in classes
Tabela 1. Zestawienie wyników pomiarów – liczebności w klasach

$\langle x_i; x_{i+1})$	⟨0; 1)	⟨1; 2)	⟨2; 3)	⟨3; 4)	⟨4; 5)	⟨5; 6)	⟨6; 7)	⟨7; 8)	⟨8; 9)	⟨9;10)
n_i	19	43	61	71	56	32	18	13	8	5

The analysis of data in Table 1 presents that the studied empirical distribution expansion is not symmetrical. Class ⟨ 3; 4) has the largest size, so one should accept that the modal value of distribution belongs to this class. Moreover, one can notice that the median and modal values do not coincided to the average value of the analysed distribution.

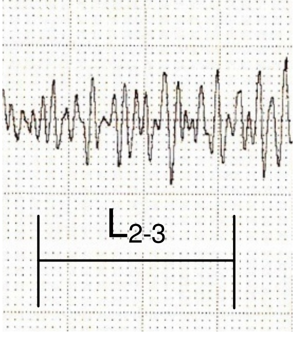

Fig. 1. Fragment of recorded magnetic investigations
Rys. 1. Fragment zapisu wyników badań magnetycznych

The highest probabilities of data from measurements gave the following values for the distribution parameters:

$$p = 3.2455 \tag{12a}$$

$$b = 1.1960 \tag{12b}$$

For gamma distribution with frequency Function (1), the skew is described by the following formula:

$$b_1 = \frac{1}{\sqrt{p}} \tag{13}$$

For analysed data, $b_1 = 0{,}555$. This result confirms fact that the skew of the studied distribution is visible (comparatively large).

Therefore, for examining goodness of fit of empirical distribution and received the gamma distribution with parameters (12), the tests χ^2, as well as test *λ-Kolmogorov*, were applied. For test χ^2, the calculated value is

$$\chi^2_{obl} = 4{,}83 \tag{14a}$$

However, from statistical tables, for significance level $\alpha = 0.05$, the value of this statistic is

$$\chi^2_{tabl} = 14{,}07 \tag{14b}$$

The value of parameter p calculated for (14a), is 0.68. The obtained results confirm the goodness of fit of the gamma distribution with the empirical one.

For test *λ-Kolmogorov*, the λ_{obl} value is equal to 0.40, and value λ_{tab} received from statistical tables, for significance level $\alpha = 0.05$, is equal to 1.36. These values testify to the very good goodness of fit of the gamma distribution with the empirical distribution.

In images below present the results of statistical analyses in graphic form. Graphs of the empirical and theoretical distribution functions are shown in Fig. 2. They are very close, which testifies to the good adjustment of the results of experiments to the assumed hypothetical distribution.

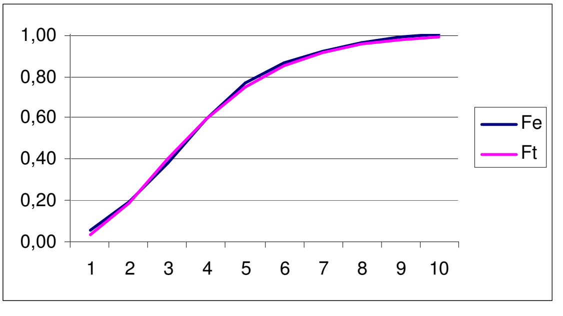

Fig. 2. Graphs of distribution functions: ***Fe*** – empirical, ***Ft*** – theoretical
Rys. 2.Wykresy dystrybuant ***Fe*** – empirycznej i ***Ft*** – teoretycznej

In Fig. 3 the graphs of probability frequency functions were introduced for both distributions. The compatibility of the probability frequency functions, empirical *fe* and theoretical *ft,* of the analysed distributions is visible.

Tested distributions differ at points of maximum probability density. This means that modes of distribution, theoretical and empirical, are different, but only slightly.

Table 2 indicates that the probabilities do not cross the value of the random variable, which was accepted as the admissible (boundary) value.

Table 2. Probability of boundary values
Tabela 2. Prawdopodobieństwo nieprzekroczenia wartości progowej

Boundary value	Probability	Boundary value	Probability
8	0.95101	14	0.99901
9	0.97335	15	0.99951
10	0.98580	16	0.99976
11	0.99256	17	0.99988
12	0.99616	18	0.99994
13	0.99804		

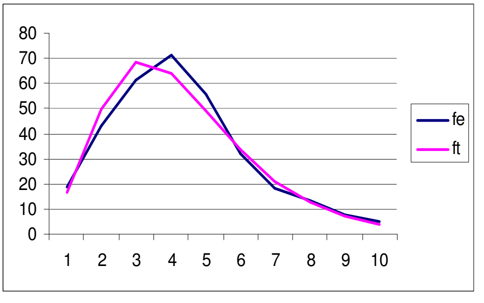

Fig. 3. Graphs of frequency functions: ***fe*** – empirical, ***ft*** – theoretical
Rys. 3. Wykresy gęstości: ***fe*** – empirycznej i ***ft*** – teoretycznej

Fig. 4 presents the graphs of the dependence of probability on the value of the threshold, e.g. value accepted as boundary operational safety. It is a visualisation of data contained in Table 2.

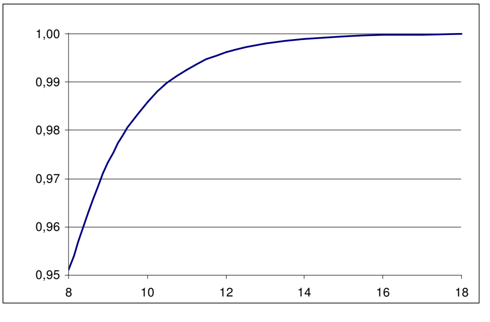

Fig. 4. Relation between probability and assumed boundary value
Rys. 4. Zależność prawdopodobieństwa od przyjętej wartości progowej

Presented in Table 2 and in Fig. 4 results of calculations of probability of not crossing of boundary value indicate, that for boundary values greater than 11 probabilities have crossed 0,99 and they are not differ more than several pro miles.

On this basis, one can formulate a practical conclusion that observing recorded magnetic investigations makes it possible to estimate the probability of the occurrence of cable weakness limits in regard to safety limits of cables operating in a shaft hoist.

Closure

These investigations have cognitive and practical aspects. Cognitive element is that, in statistical categories, the results of the magnetic investigations of cables have a distribution very closed to the gamma distribution.

The confirmation of the possibility of application of described procedure to continuous diagnostic investigations is the practical element. Monitoring the change of the signal which has direct relationship with cable operational features – its weakness, one can react in suitable moment, e.g. to reduce working load of devices or to stop its, what will permit to avoid breakdown generated extensive damages.

References

[1] Mańka E., Skrok T., Styp-Rekowski M.: Eksploatacyjne czynniki determinujące proces zużywania lin górniczych wyciągów szybowych. Materiały XVI Międzynarodowej Konferencji N-T TEMAG'08. Politechnika Śląska, Gliwice – Ustroń 2008, s. 227–237.
[2] Hansel J.: Podstawy teorii i inżynierii bezpieczeństwa systemów maszynowych transportu pionowego. Materiały VI Międzynarodowej Konferencji „Bezpieczeństwo pracy urządzeń transportowych w górnictwie". Centrum Badań i Dozoru Górnictwa Podziemnego, sp. z o.o. Lędziny – Ustroń 2010, s. 20–28.
[3] Kwaśniewski J.: Badania magnetyczne lin stalowych. Wydawnictwo AGH, Kraków 2010.
[4] Bendat J.S., Piersol A.G.: Metody analizy i pomiaru sygnałów losowych. PWN, Warszawa 1976.
[5] Benjamin J.R., Cornel C.A.: Rachunek prawdopodobieństwa, statystyka matematyczna i teoria decyzji dla inżynierów. WNT, Warszawa 1977.

Probabilistyczne ujęcie zagadnienia trwałości lin stalowych

S t r e s z c z e n i e

W artykule przedstawiono procedurę określania prawdopodobieństwa osiągnięcia określonej wartości wielkości przyjętej jako symptom stanu. Wykorzystując wyniki magnetycznych badań lin, potwierdzono jej przydatność w monitorowaniu stanu obiektu, w tym przypadku mechanizmu linowego.