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Classification of phase trajectory portraits in the process of recognition in the changes in the technical condition of monitored machines and constructions

K e y w o r d s

Phase trajectory, attractor, recurrence, diagnostics.

S ł o w a k l u c z o w e

Trajektoria fazowa, atraktor, rekurencja, diagnostyka.

S u m m a r y

A methodology of the functioning correctness control of machines and structural components is described in the article. The proposed new approach to the construction of a vibration-based monitoring system is presented. A methodology based on the quantitative analysis of attractor, phase trajectory and recurrence quantification analysis (RQA) is presented in detail.

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Introduction

Performing the review of systems – functioning in the industrial practice – monitoring changes in a machine and structure technical conditions, it is possible to generate several synthesising statements and conclusions of a general nature being the assessment of their solutions.

- Monitoring systems, which trace changes of the special numerical estimates (e.g. *effective, peak and average values of the measuring signal or their mutual combinations)* as well as the determined functional patterns *(meaning: shaft neck motion trajectory in a bearing sleeve, spectrum density function, correlation, coherence, cepstrum, envelopes etc.*), ensure control of the state of the machine.
- Criteria values for the monitored diagnostic symptoms are determined by the appropriate standards, regulations, and findings resulting either from maintenance experiences or from the assumption of acceptable projections of object damages.
- Structural and exploitation features of the monitored object are not taken into consideration to a satisfying degree in the process of building monitoring systems.

These assessments are not pretending to list all problems occurring in the construction of monitoring systems. However, they can constitute an inspiration in searching for new methodological guidelines for the monitoring systems that without limitations as shown in the presented synthesis.

The aim of this paper is to indicate some possibilities in this scope. It seems that the quantitative analysis of certain geometrical features, which are graphical signal representations, can be a good tool for the realisation of such tasks. Such an analysis can help in finding new diagnostic symptoms related to the analysis of the monitored object dynamics.

Description of the system dynamics in the phase space

The phase space of a dynamic system is a mathematical space of orthogonal coordinates representing all variables necessary for the determination of the instantaneous state of the system. The total description of the system dynamics in the phase space can also be obtained when the system attractors are known. An *attractor* is a certain set in the phase space towards which the trajectories initiated in various domains of the phase space (i.e. trajectories for various initial conditions) are heading as the time progressed.

An attractor can be a point, a closed curve, or a fractal. The possibilities of using the attractor in a form of a boundary cycle are described in Section 3. The possibilities of utilising a quantitative analysis of trajectory in the case when the point is the attractor are given in Section 4, and the quantitative analysis of the trajectory recurrence are given in Section 5.

Both the attractor and phase trajectory are multidimensional curves. The trajectory projection on a certain plane, formed by two perpendicular axes of the phase space, can be analysed without loosing the generalities of considerations [1].

The most obvious coordinates of such plane used in the topological analysis of vibrations are velocity and displacement. Instead of analysing velocity as a function of displacement, it is possible to analyse displacement as a function of velocity. Apart from advantages due to fewer time series integrations (time and cost of calculations), another benefit is the fact that they can be determined directly on the object being under diagnostics. The authors, in their investigations, were using a speedometer VS80 produced by Brüel & Kjær and accelerometer type PCB 356A16 of the PCB Pizotronics Company.

Another way to construct the projection plane is to use the delay method as known from chaos theory. It is enough in this method to determine one time series (e.g. vibration acceleration) and on its basis determine the whole phase space.

The trajectory reconstruction from the individual time series requires the creation of additional variables. In searching for new variables, the Takens Theorem can be helpful, since it states that each point in the phase space $a(n)$ is represented by a sequence of time series values.

$$a(n) = [\,y(n),\ y(n+\tau), \ldots,\ y(n+(m-1)\tau)] \qquad (1)$$

where: m is a phase space dimension, τ – delay time.

The most often applied procedure of selecting the phase space dimension m is the method of the False Nearest Neighbours (FNN).

The criterion of the time delay selection, which utilises non-linear dependencies between observations, is the method of Mutual Information. The number of mutual information $I(x_i,T)$ is determined from the following dependence [11]:

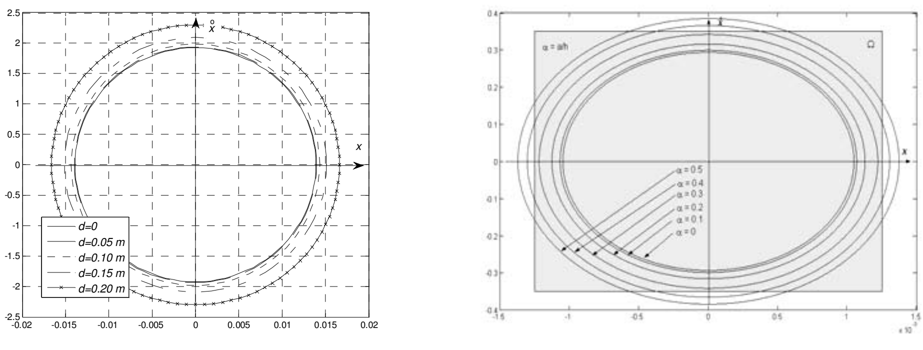
$$I(x_i,T) = \frac{1}{N}\sum_n \left[\log_2(p(x_i(n), x_i(n+T)) - \log_2(p(x_i(n)) - \log_2 p(x_i(n+T))\right] \qquad (2)$$

where: $p(x_i(n))$ – the probability density function of the analysed series, T – the delay time, $p(x_i(n),x_i(n+T))$ – combined probability density, N – number of samples in the time series.

According to this method for the time delay τ, the smallest time value T in Equation (2) for which the mutual information function obtains the local minimum should be assumed.

Qualitative analysis of the attractor in a form of the boundary cycle

The proposed diagnostics method is based on the determination of displacement and velocity for the arbitrary selected point of the investigated system, where vibrations originated as a result of excitation. The mono-harmonic excitation of a frequency lower than the first natural frequency of vibrations of the element undergoing diagnostics was assumed in the study model excitation. Attractors determined for the loaded beam of various axial cracking length (delamination) *d* are shown in Fig. 1a [8], and Fig. 1b shows the changes of the attractor determined for the beam with a transverse cracking of a depth – *a*. The area of allowable solutions Ω was selected in such a way as to have the crack propagation rate being equal to the determined value. Such a selection of the allowable solution area enables one to assess the time remaining to the damage of the analysed beam [2–6].



(a) (b)

Fig. 1. Attractors of the cracked beam
Rys. 1. Atraktory pękniętej belki

The diagnosed system in which trajectory exceeds the allowable solutions area Ω is not suitable, because of the condition for which the Ω area was determined.

Quantitative analysis of the phase trajectory

The first damage index proposed by the authors is related to the distance change of the point in the trajectory from the point that is the attractor of this trajectory, which is the scalar damage index (the authors are using the sum of the relative vectors difference *r*) for each time instant (of each sample *n*).

$$\mathrm{WU_r} = \frac{1}{N}\sum_{n}\frac{r_u(n) - r_z(n)}{r_z(n)} \tag{3}$$

where: r_u – vector of the distance of points in the trajectory from the attractor determined for the damaged element, r_z – vector for the not damaged element.

The second damage index is related to the Poincare map. It is constructed by the stroboscopic ‘viewing’ of the trajectory phase pattern at constant time intervals. If ‘pictures’ are taken at time intervals corresponding to the period of the first frequency of natural vibrations, the map is a straight line. When the same time intervals are applied for the formation of the Poincare map of the trajectory of the system with different inertial-elastic parameters (e.g. of a damaged object), the map will not be a straight line. The proposed damage index WU_ϕ is determined from the following equation:

$$\mathrm{WU}_\phi = \frac{1}{N}\sum_{n}\frac{\phi_u(n) - \phi_z(n)}{\phi_z(n)} \tag{4}$$

where: ϕ_u – vector of polar coordinates of the Poincare map for the damaged element, ϕ_z – vector for the not damaged element.

The simulation of the impulse response of the beam determined the example of the phase trajectory with a transverse cracking of various depths [9]. Both damage indexes as a function of a crack depth are shown in Fig. 2.

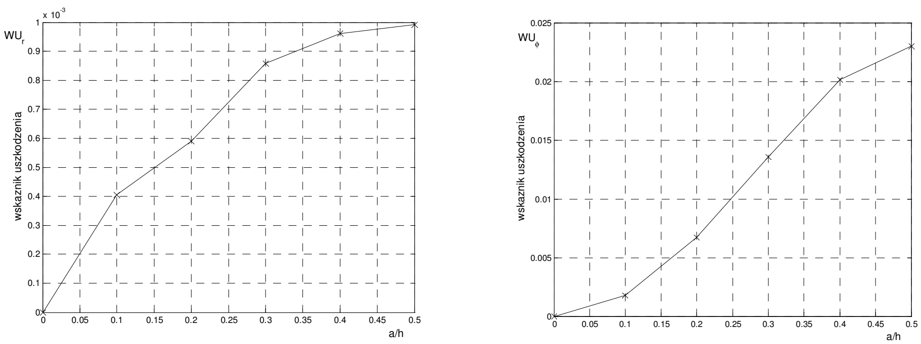


Fig. 2. Damage Indexes based on the phase trajectory analysis
Rys. 2. Wskaźniki uszkodzenia oparte o analizę trajektorii fazowej

Other examples of this diagnostic method application can be found in the authors’ papers [6, 9].

Application of recurrence diagrams

A recurrence diagram is a diagram presenting the repeatability (recurrence) of processes, effects, or system states. An important advantage of the diagram is the possibility of its application both for large and small data sets, including non-stationary ones. The diagram presents the following dependence [10]:

$$R_{i,j} = H\left(\eta - \left\| X_i - X_j \right\|\right), \quad i, j = 1, \ldots, M \tag{5}$$

where: $\mathbf{X_i}$, $\mathbf{X_j}$ – states in the space R^m, M – number of states, H – Heaviside's function, $\|\mathbf{X}\|$ – standard of vector $\mathbf{X}$ in the space R^m (the most often it is the Euclidean or maximum standard), η – non-negative real number, the so-called: cut-off parameter.

As can be seen, the basis of the diagram determined by Equation (5) is the zero-one square matrix R_{MM}. Value $R_{i,j} = 1$ is marked by a black point in the diagram, while $R_{i,j} = 0$ is marked by a white point (no point).

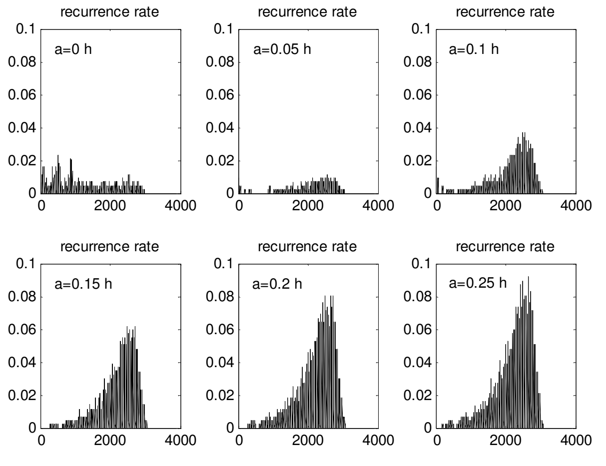


Fig. 3. Changes in recurrence rate as a function of the crack depth
Rys. 3. Przebiegi wybranych funkcji uszkodzenia w funkcji uszkodzenia

In the dependence on the nature and properties of the considered problem, the black points in the diagram form various structures. These can be individual points, points collected along curves of various lengths, or straight lines arranged horizontally, perpendicularly, or skewed. The most important values allowing one to perform the quantitative diagram analysis are Recurrence Rate, Determinism, lmax, Trend, Entropy, Laminarity, and Trapping time [10]. These and several other values characterising (describing quantitatively) the recurrent diagram can be determined for the whole or part of the recorded signal. The analysis of the waveform part is based only on the determination of the sought values in the observation window. When shifting the window by one or more samples, it is possible to determine certain functions that were assumed in this work as damage functions. In the waveforms and their changes, the symptoms related to the cracking of the beam are sought.

The waveforms of the recurrence rate that are dependent on the crack depth are shown in Fig. 3. The view in the upper left window is related to the artificial noise added to the signal.

The possibilities of using this and other functions of damages together with the analysis of measuring error influences are described in [7].

Conclusions

An utilisation of the proposed diagnostics method based on phase trajectory analysis allows for the fast and effective diagnostics of damages.

Analysis of damage indexes as a function of damage indicates a high sensitivity of the proposed method (the possibility of detecting damages in the early stage of their formation). The early detection of damages of structural elements allows for the optimisation of repairs (their necessity and scope), avoiding losses related to forced shutdowns, and decreasing costs of not needed spare parts storage and costs related to unexpected breakdowns.

All proposed indexes are also characterised by a high sensitivity in a damage function. This high sensitivity means large changes of the damage index in a function of a damage degree, which allows for the detection and analysis of the damage progressing. In other words, a comparison of the current trajectory with the trajectory from previous diagnostics allows one to check whether the crack opening is propagating or remains stationary.

The method does not filter non-linear effects or the changes of the frequency structure of the monitored diagnostics signals related to the development of damages, which can be its special advantage.

A practical application of the trajectory changes is useful as a control method for the beginning and development of damage. This can be the most distinctive feature of the method, which is easily adaptable for practical applications.

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Klasyfikacja obrazów trajektorii fazowych w procesie rozpoznawania zmian stanu monitorowanych maszyn i konstrukcji*

S t r e s z c z e n i e

W artykule omówiono metodykę nadzoru poprawności funkcjonowania maszyn i konstrukcji wsporczych, bazującą na systemach monitoringu drganiowego. W szczególności zaprezentowano metodykę opartą na ilościowej analizie graficznych reprezentacji monitorowanego sygnału w postaci atraktora, trajektorii fazowej oraz analizy ilościowej rekurencji.

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