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B-spline wavelet packets and their application in the multiresolution non-stationary signal processing

Key words

Wavelet packets, multi-resolution signal processing, faults identification.

Słowa kluczowe

Pakiety falkowe, wielorozdzielcza analiza sygnałów, identyfikacja uszkodzeń.

Summary

Increasing requirements for the technical condition of machines induce the development of novel diagnostic methods for possible fault detection and identification in an early phase. Most of these methods are based on the processing of vibration signals. The classical methods often do not give full information about the actual condition of a machine. Therefore, it is necessary developing the appropriate diagnostic methods. Some of the promising signal processing methods are the group based on the Wavelet Transform (WT), which give a possibility for the effective diagnosing of non-stationary vibration signals in the time-scale domain. The generalisation of WT, the Wavelet Packet Transform (WPT), allows the extraction of additional useful diagnostic features from the signal. However, the effectiveness of diagnostics in the case of wavelet-based methods is determined by the selection of an appropriate wavelet function. In the present study, the author introduces new wavelet packets based on B-spline wavelets. A comparative analysis of their effectiveness was performed on non-stationary synthetic signals. The B-spline wavelet packets were applied for rolling bearing condition evaluation.

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1. Introduction

The progress in modern machine construction implies the development of diagnosing systems, which use increasingly complicated algorithms of the measured signals processing. Increasing requirements, according to the durability of machines and their low-cost maintenance, demand much improvement in signal processing methods with the following criteria: The method must give the most complete information about the technical condition of a machine, the method must detect and identify faults possibly clearly and it must allow faults detection and identification in possible early stage.

As it is well known, the most informative vibration signal in rotor machinery could be registered during machine run-up or run-out. Usually, these signals are non-stationary and the classical methods like Discrete Cosine Transform (DCT) or Fast Fourier Transform (FFT) are not efficient. The methods for non-stationary signal analysis are the Short-Time Fourier Transform (STFT) and order tracking analysis [1]. However, in the case of STFT, the basic function is the sine function, and it allows for only constant-resolution analysis. An additional and widely used method of signal processing is WT, which gives the possibility of multi-resolution signal analysis and the basic function is not limited to the one type.

Previous research indicates that the Discrete Wavelet Transform (DWT) allows the extraction of more diagnostic features from the vibration signal. The comparative analysis of FFT, STFT, and DWT effectiveness was performed for the structural damage detection of polymeric composites [2]. The most crucial factor that influences the effectiveness of WT is the appropriate wavelet selection. In the case of DWT, the selected wavelet must fulfil some additional criteria, e.g. compact support or orthogonality; thus, not every available wavelet could be applied for the analysis. The most popular wavelets used in the technical diagnostics for signal processing using DWT are the Daubechie's wavelets [3], biorthogonal wavelets, symlets, and coiflets [4,5].

Katunin's research shows the higher effectiveness of B-spline wavelets in comparison with the above-mentioned ones first applied in the diagnostic signal processing problems in [6]. In [7], the comparative study for some orthogonal wavelets was carried out using the degree of scalogram density (DSD) parameter. The analysis was carried out for the most typical signal components: the harmonic, the harmonic with variable frequency, and the pulse component. In all cases, the B-spline wavelets show excellent results. Furthermore, these wavelets were applied for the detection and localisation of single [7] and multiple [8] cracks in composite beams.

An additional wavelet-based method used in the diagnostic signal processing is WPT. This method, proposed by Coifman *et al.* in [9], is a kind of generalisation of DWT, which provides full multilevel decomposition of the signal. The graphical interpretation of DWT and WPT is presented in Fig.1.

WPT found many applications in mechanical and technical diagnostics problems. Glabisz, in his monograph [11], showed the application of WPT in signal processing problems, nonlinear systems identification, and the solution of differential equations. In diagnostic problems, WPT is often used for damage identification. The entropy-based method for rolling bearing condition diagnostics was developed by Wysogład [3,12,13]. The rolling bearing condition could be analysed using other methods based on WPT, e.g. weak signature detection method [14,15]. WPT is also used in gearbox diagnostics [16], structural damage detection in multi-layered composites [17], medical applications [18] and others.

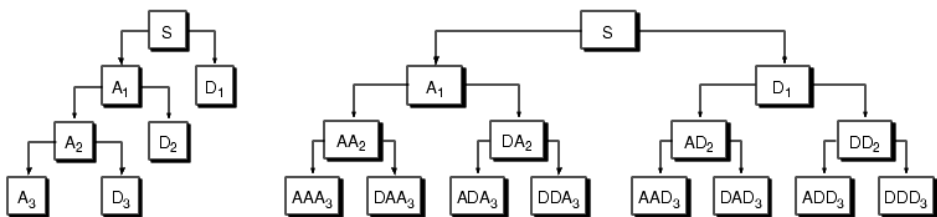


Fig. 1. The principle of multilevel decomposition using Discrete Wavelet Transform and Wavelet Packet Transform [10]

Rys. 1. Zasada wielopoziomowej dekompozycji z zastosowaniem Dyskretnej Transformaty Falkowej i Pakietowej Transformaty Falkowej [10]

Considering the excellent results for B-spline wavelets, the author decided to construct new wavelet packets based on these wavelets. Furthermore, the general order B-spline wavelet packets mathematical formulation will be proposed. Then, their effectiveness will be analysed in application for synthetic non-stationary signals based on Inverse Degree of Scalogram Density (IDSD) parameter. Finally, the B-spline wavelet packets will be applied for fault identification in rolling bearings, and the obtained results will be compared with the results obtained using other wavelet packet bases.

2. B-spline wavelets and wavelet packets

2.1. General order B-spline wavelets

The B-spline wavelets were introduced by Chui in [19]. The scaling function φ_m of the B-spline wavelet is defined by the two-scale relation:

$$\varphi_m(x) = \sum_{k=0}^m p_k \varphi_m(2x - k), \quad k \in Z \quad (1)$$

Where

$$p_k = 2^{1-m} \binom{m}{k}, \quad 0 \leq k < m \quad (2)$$

is the two-scale sequence.

The wavelet function ψ_m is also presented by the two-scale relation in the form of

$$\psi_m(x) = \sum_{k=0}^{3m-2} q_k \varphi_m(2x-k), \quad k \in Z \quad (3)$$

Where

$$q_k = (-1)^k 2^{1-m} \sum_{l=0}^m \binom{m}{l} \varphi_{2m}(k-l+1), \quad 0 \leq k < m \quad (4)$$

The decomposition using B-spline wavelets could be carried out with the following relation:

$$\varphi_m(2x-l) = \sum_k (a_{l-2k} \varphi(x-k) + b_{l-2k} \psi(x-k)), \quad l \in Z \quad (5)$$

Where

$$a_k = \frac{(-1)^{k+1}}{2} \sum_l q_{-k+2m-2l+1} c_{l,2m} \quad (6)$$

$$b_k = -\frac{(-1)^{k+1}}{2} \sum_l p_{-k+2m-2l+1} c_{l,2m} \quad (7)$$

are the decomposition sequences. The coefficient sequence $c_{k,m}$ could not be determined analytically for $m < 5$ [7]; thus, it could be calculated from the bi-infinite system of equations [19]:

$$\sum_{k=-\infty}^{\infty} c_{k,m} \varphi_m\left(\frac{m}{2} + j - k\right) = \delta_{j,0}, \quad j \in Z \quad (8)$$

The approximate solution of (8) and the method of the determination of decomposition coefficients $c_{k,m}$ could be found in [19]. The explicit form of B-spline wavelets with order of 2 – 4 could be found in [6] and with order of 5 – 7 are presented in [7]. Note that the first order B-spline wavelet is the Haar wavelet.

2.2. Construction of the general order B-spline wavelet packets

The Discrete Wavelet Packet Transform (DWPT) is a generalisation of DWT which yields time-scale decomposition by decomposing not only the approximate coefficients (as in DWT) but also the details coefficients (see the scheme in Fig. 1). It could improve the localisation of high frequency components. Following the dependencies presented by Coifman *et al.* [9] and considering two-scale relations for scaling (1) and wavelet (3) functions, the general order B-spline wavelet packets could be constructed by induction as a set of functions represented by the following recursive equations:

$$\begin{aligned} w_{2n,m}(x) &= \sum_k p_k w_{n,m}(2x-k), \quad k \in Z \\ w_{2n+1,m}(x) &= \sum_k q_k w_{n,m}(2x-k), \quad k \in Z \end{aligned} \quad (9)$$

Note that $w_{0,m} = \varphi_m$ and $w_{1,m} = \psi_m$. The B-spline wavelet packets have some unique properties, and some of them are related to B-spline wavelet properties presented, e.g. in [20]. The main properties are mentioned below:

- All of the B-spline wavelet packets are of compact support. From (9) it could be concluded that B-spline wavelet packets are compactly supported on the finite intervals. For the general order B-spline wavelet packets considering (9), we obtain:

$$\begin{aligned} \text{supp } w_{0,m} &= [0, \text{supp } \varphi_m] \\ \text{supp } w_{n,m} &= [0, \text{supp } \psi_m], \quad n, m \in Z, \quad n = 1, 2, \dots \end{aligned} \quad (10)$$

- B-spline wavelet packets are symmetric for even m and any n . In the case of odd m , they are symmetric for even n and anti-symmetric for odd n , namely:

$$\begin{cases} w_{n,m}(x) = w_{n,m}(2m-x-1), & \text{for even } m \\ w_{n,m}(x) = w_{n,m}(2m-x-1), & \text{for odd } m \text{ and even } n \\ w_{n,m}(x) = -w_{n,m}(2m-x-1), & \text{for odd } m \text{ and odd } n \end{cases} \quad (11)$$

- B-spline wavelet packets are semi-orthogonal, that is, they are orthogonal during scaling and not orthogonal during translation:

$$\langle w_{n,m}(2^i x - k_1), w_{n,m}(2^j x - k_2) \rangle = 0, \quad i \neq j, \quad k_1 \neq k_2 \quad (12)$$

$$\langle w_{n,m}(2^j x - k_1), w_{n,m}(2^j x - k_2) \rangle \neq 0, k_1 \neq k_2, m \neq 1.$$

Considering (5) and (9), the multilevel decomposition relation for discussed wavelet packets could be presented as follows:

$$w_{n,m}(2x-l) = \sum_k (a_{l-2k} w_{2n,m}(x-k) + b_{l-2k} w_{2n+1,m}(x-k)) \quad (13)$$

$$l \in Z,$$

Where a_k and b_k are the decomposition sequences given by (6) and (7), respectively.

The B-spline wavelet packets constructed from the B-spline wavelets of order 3 (quadratic) and of order 7 (sextet) are presented in Fig. 2 and Fig. 3, respectively. In both cases, the parameter n is in the range of 0 - 7. Analysing (10), one could convince that the support of a given wavelet packet of order m equals the support of the appropriate wavelet function of order m . However, the length of the parts with highest disturbances of a given function is almost stable.

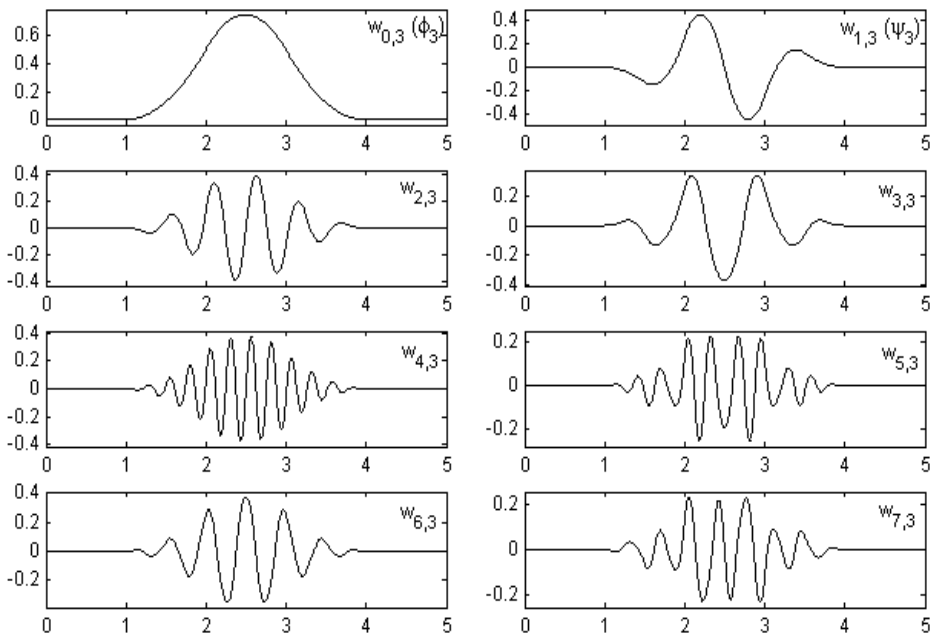


Fig. 2. Wavelet packets based on quadratic B-spline wavelet for $n = 0 - 7$
 Rys. 2. Pakiety falkowe na podstawie falki B-splajnowej rzędu 3 dla $n = 0 \div 7$

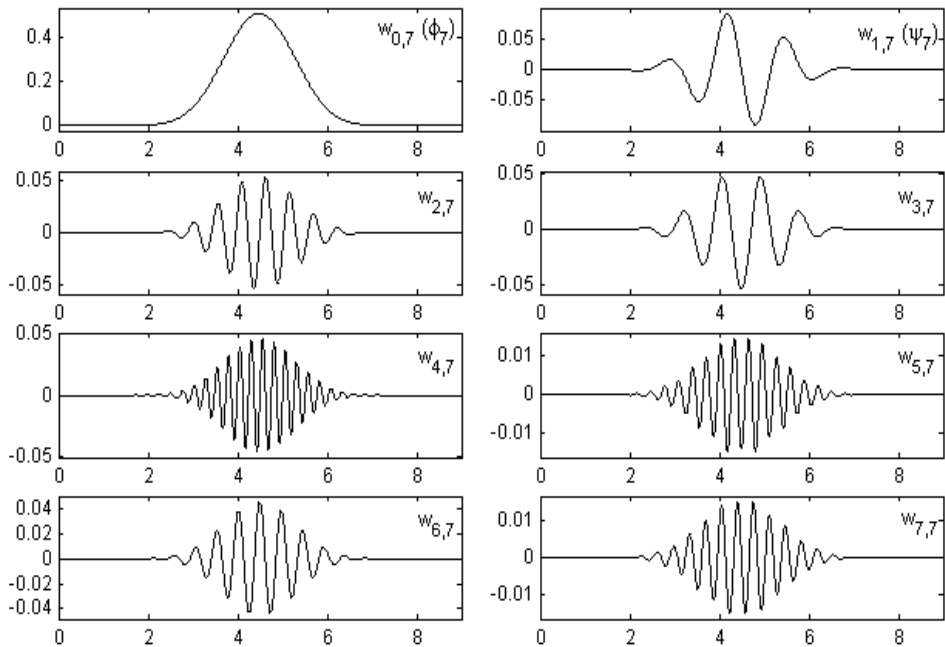


Fig. 3. Wavelet packets based on sextic B-spline wavelet for $n = 0 - 7$
 Rys. 3. Pakiety falkowe na podstawie falki B-splajnowej rzędu 7 dla $n = 0 \div 7$

3. Effectiveness of the evaluation based on synthetic signals

For the evaluation of the wavelet packet effectiveness in the signal processing problems, the analysis based on synthetic signals using IDSD was performed. The DSD parameter used in [1, 6, 7] for the evaluation of the approximation effectiveness of wavelets is a scalar value, which is based on the determination of the ratio between the number of the wavelet coefficients higher than some non-zero threshold and the number of all wavelet coefficients in the scalogram. The IDSD parameter gives the ratio between the number of the wavelet coefficients higher than the threshold and the number of all wavelet coefficients. It makes possible the analysis of disturbances inputted by the given wavelet packets.

As previously mentioned, the best application of WPT is the analysis of non-stationary signals. In this study, three types of such signals were analysed: the linear chirp, the quadratic chirp, and the logarithmic chirp. The signals were synthetically generated with the length of 1 second and sampling rate of 1/512 s with the frequency range of 10 - 250 Hz. Then, the WPT algorithm was applied with the 4th decomposition level. Obtained coefficients were normalised into the closed set [0,1]. An exemplary comparison between scalograms with the Daubechies 3 (*db3*) wavelet packet base (1st column) and the B-spline 3 (*bsp3*)

wavelet packet base (2nd column) is presented in Fig.4 for the linear (1st row), quadratic (2nd row) and logarithmic (3rd row) chirp. On the vertical axes, the wavelet packet decomposition tree numbers were shown. The order of priority of the wavelet packets was set due to the frequency order.

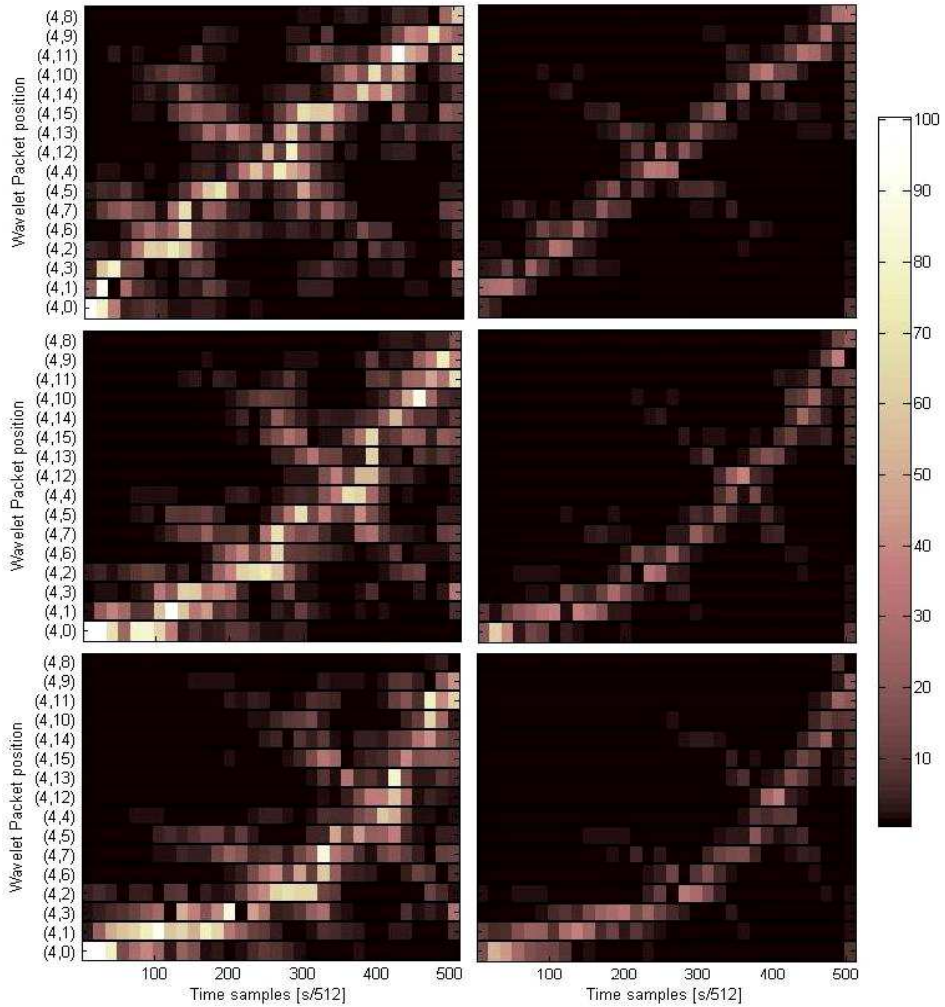


Fig. 4. Scalograms after WPT using Daubechies 3 and B-spline 3 bases
 Rys. 4. Skalogramy po pakietowej transformacji falkowej z użyciem baz Daubechies 3
 i B-splajn 3

Analysing Fig. 4 it can be noticed that the quadratic B-spline wavelet packet base gives lower disturbances in comparison with *db3* wavelet packet base for all of the investigated signal types. For determining the measure of the effectiveness of the wavelet packets, the IDSD was calculated for Daubechie's

wavelets (*db*), B-spline wavelets (*bsp*), symlets (*sym*) and coiflets (*coif*) with order from 1 to 5. Results of the calculation are presented in Table 1. The effectiveness of given wavelet packets is better when IDSD tends to the minimum.

Table 1. IDSD values for several wavelet packet bases

Tabela 1. Wartości odwrotnego stopnia zagęszczenia skalogramu dla różnych baz pakietów falkowych

	db1	db2	db3	db4	db5
Linear chirp	0.4414	0.3695	0.3090	0.2401	0.2656
Quadratic chirp	0.4531	0.3217	0.2743	0.2237	0.2344
Logarithmic chirp	0.4238	0.3419	0.2448	0.2237	0.2297
	bsp1	bsp2	bsp3	bsp4	bsp5
Linear chirp	0.4414	0.2554	0.1528	0.1014	0.0888
Quadratic chirp	0.4531	0.2339	0.1476	0.1047	0.0938
Logarithmic chirp	0.4238	0.2125	0.1389	0.1182	0.0724
	sym1	sym2	sym3	sym4	sym5
Linear chirp	0.4414	0.3695	0.3090	0.2484	0.2594
Quadratic chirp	0.4531	0.3217	0.2743	0.2434	0.2266
Logarithmic chirp	0.4238	0.3419	0.2448	0.2451	0.2000
	coif1	coif2	coif3	coif4	coif5
Linear chirp	0.3438	0.2798	0.2606	0.2394	0.2256
Quadratic chirp	0.3142	0.2619	0.2593	0.2406	0.2129
Logarithmic chirp	0.3073	0.2440	0.2327	0.2193	0.2044

Wavelet packets with the B-spline bases show the best results in comparison with other orthogonal wavelet packet bases; thus, during WPT, the B-spline wavelet packets detect non-stationary components in the signal most clearly. As observed in Table 1, the values of IDSD for wavelet bases tend to the minimum when their order grows, and the best IDSD values were obtained for B-spline bases. The linear chirp gives higher disturbances in the scalogram, while the logarithmic chirp gives lower ones. Due to the good results obtained for B-spline wavelet packet bases in tests on synthetic signals, they were applied to the real vibration signals in the fault identification problem of rolling bearings.

4. Faults identification in rolling bearings using wavelet packet decomposition

4.1. Problem description and methodology

In the present study, the rolling bearing condition was analysed using several wavelet packets. The tests were performed based on the study presented in [14]. In this study, four ZA-2115 double row bearings manufactured by Rexnord installed on a shaft driven by an AC motor and tested. The bearings

have 16 roller elements in each row, a pitch diameter D_p of 71.5 mm, roller diameter D_r of 8.4 mm, and the contact angle ψ of 15.17° . The rotation frequency f_r of the shaft was constant and equaled 33.33 Hz (2000 rpm). The measurements were carried out on the bearing housing using PCB353B33 accelerometers. The vibration data was collected by a NI DAQ Card 6062E with a sampling rate of 20 kHz. The signals for the present analysis were kept from NASA Ames Prognostics Data Repository [21].

In the present study, the signal from the first bearing with a faulty outer race was analysed. The bearing worked in heavy-load conditions over 160 hours. The characteristic frequency for the outer race defect f_d could be calculated from the following formula [15]:

$$f_d = \frac{nf_r}{2} \left(1 - \frac{D_r}{D_p} \cos \psi \right) \quad (14)$$

Where n is the number of roller elements. In the present case, we obtain the characteristic defect frequency of 293.54 Hz.

The defect detection was performed based on wavelet packet decomposition (WPD) using the following algorithm. The original signals were decomposed on the third level. The level of decomposition in most bearing damage detection problems is assumed as third or fourth [22]. Then, on the assumed decomposition level, the energy vector is determined for all of the sets of wavelet coefficients. The set with the maximum value of energy is considered in further analysis. The energy value is greater when the number of harmonics consisted in a given decomposed realisation is high. For the selected set of wavelet coefficients, the frequency spectrum and the magnitude component with a characteristic defect frequency is extracted and analysed.

4.2. Fault identification and comparative analysis of wavelet packet bases

Based on the presented algorithm, WPD was executed for the signal from faulty bearing using a *bsp5* wavelet packet base. The first set of wavelet coefficients $c_{3,0}$ was omitted because it contains information about low frequencies, which usually represent harmonics from unbalancing, misalignment, etc. [22]. Then, the energy values were determined for the rest of the wavelet coefficient sets. It could be noticed that the set with a higher energy value is $c_{3,2}$ and for this set we determine the spectrum. The energy variation for wavelet coefficient sets on the third level of decomposition and the spectrum for $c_{3,2}$ is presented in Fig. 5. In the spectrum, the component which comes from the fault is clearly identified (the highest peak). Other peaks represent the rotation frequency harmonics.

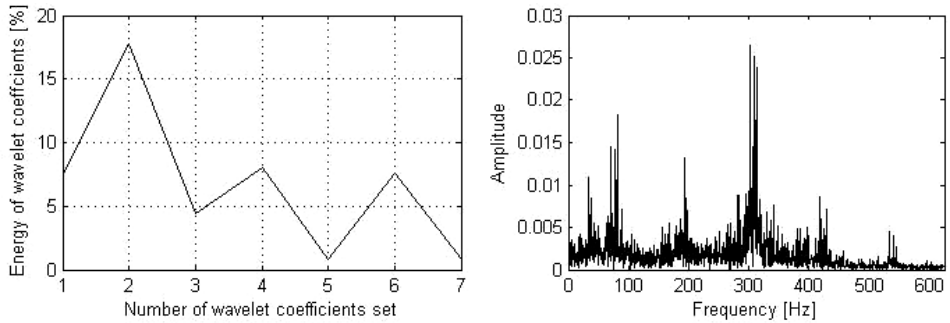


Fig. 5. Energy of wavelet coefficients set on third decomposition level and the spectrum of the best wavelet coefficients set

Rys. 5. Energia zbioru współczynników falkowych na trzecim poziomie dekompozycji oraz widmo dla najlepszego zbioru współczynników falkowych

For the comparison of different wavelet packet bases, we determine the mean value of amplitudes in the spectrum and calculate the ratio R of the fault component amplitude to the mean value. The higher value of R denotes the better sensitivity of a given base. In this analysis, we compare wavelet packet bases used in [12,13,22], some other orthogonal bases and B-spline bases. The results of a comparison are presented in Table 2, where symbols denote the bases of *bior* – biorthogonal wavelet, *rbio* – reverse biorthogonal wavelet, *dmey* – discrete Meyer wavelet.

Table 2. Comparison of different wavelet packet bases sensitivity
Tabela 2. Porównanie czułości różnych baz pakietów falkowych

haar	db3	db5	db7	db10	db12	db15
14.3372	14.5636	12.3543	12.8808	14.4217	14.5438	16.2800
sym2	sym3	sym5	sym7	sym10	sym12	sym15
15.1338	14.5636	12.9303	12.5914	15.4521	14.6641	15.4401
coif1	coif3	coif5	bior1.1	bior2.4	bior3.9	bior6.8
14.8011	15.2622	16.1934	14.3372	12.6599	14.0450	15.1294
rbio1.1	rbio2.4	rbio3.9	rbio6.8	dmey	bsp2	bsp3
14.3372	12.3030	14.2793	16.2204	15.1700	12.7392	16.9109
bsp4	bsp5	bsp7	bsp8	bsp10	bsp12	bsp15
16.8412	16.8598	15.1383	18.4434	18.2720	18.1280	17.9483

The comparison shows that the most sensitive to the fault characteristic frequency are the B-spline bases. The WPD filters constructed based on these wavelets allow the extraction of the harmonics with the best purity.

5. Conclusions

In this paper, the B-spline wavelet packets were constructed and their main properties were presented. Their effectiveness in the signal processing in diagnostic problems was investigated based on the synthetically generated non-stationary signals. The B-spline wavelet packet bases show the best purity of obtained scalograms. Then, they were applied to the fault identification problem of the rolling element bearings using wavelet packet decomposition. The characteristic fault frequency of the bearing was clearly identified. In comparison with other bases, the B-spline bases show the best purity of the identified fault component in the spectrum. The presented research confirms that the B-spline wavelet packets could be used in practical diagnostic problems as well.

References

- [1] Timofiejczuk A., Methods of analysis of non-stationary signals, Silesian University of Technology Publishing House, Gliwice, 2004 [in Polish].
- [2] Katunin A., Moczulski W., Faults detection in layered composite structures using wavelet transform, *Diagnostyka* 1(53), 2010, pp. 27-32.
- [3] Wysogład B., Bearing condition diagnostics using entropy of signal in frequency domain, *Diagnostyka* 2(50), 2009, pp. 65-68.
- [4] Douka E., Loutridis S., Trochidis A., Crack identification in beams using wavelet analysis, *International Journal of Solids and Structures*, 40, pp. 3557-3569, 2003.
- [5] Douka E., Loutridis S., Trochidis A., Crack identification in plates using wavelet analysis, *Journal of Sound and Vibration*, 270, pp. 279-295, 2004.
- [6] Katunin A., Korczak A., The possibility of application of B-spline family wavelets in diagnostic signal processing, *Acta Mechanica et Automatica*, 3(4), pp. 43-48, 2009.
- [7] Katunin A., Construction of high-order B-spline wavelets and their decomposition relations for faults detection and localization in composite beams, *Acta Mechanica et Automatica*, 2010, submitted.
- [8] Katunin A., Identification of multiple cracks in composite beams using discrete wavelet transform, *Scientific Problems of Machines Operation and Maintenance* 45, 2(162), 2010, pp. 41-52.
- [9] Coifman R., Meyer Y., Wickerhauser M.V., Size properties of wavelet packets, in: Ruskai M.B. et al. (Eds.), *Wavelets and their applications*, Jones and Bartlett, Boston, 1992, pp. 453-470.
- [10] Misiti M., Misiti Y., Oppenheim G., Poggi J., *Wavelet Toolbox®*. For use with MATLAB®, The MathWorks Inc., 1996.
- [11] Glabisz W., *Wavelet packet analysis in mechanics problems*, Wroclaw University of Technology Publishing House, Wroclaw, 2004 [in Polish].
- [12] Wysogład B., Rolling element bearing diagnosing with the use of wavelet packet transform, *Diagnostyka* 30, 2004, pp. 195-198 [in Polish].
- [13] Wysogład B., Application of entropy-based analysis of signals to identification of rolling element bearings faults, *Diagnostyka* 3(51), 2009, pp. 65-68.
- [14] Qiu H., Lee J., Lin J., Yu G., Wavelet filter-based weak signature detection method and its application on rolling element bearings prognostics, *Journal of Sound and Vibration* 289, 2006, pp. 1066-1090.

- [15] Eren L., Devaney M.J., Bearing damage detection via wavelet packet decomposition of stator current, *IEEE Transactions on Instrumentation and Measurements* 53, 2, 2004, pp. 431-436.
- [16] Fan X., Zuo M.J., Gearbox fault detection using Hilbert and wavelet packet transform, *Mechanical Systems and Signal Processing* 20, 2006, pp. 966-982.
- [17] Wei Z., Yam L.H., Cheng L., Detection of internal delamination in multi-layer composites using wavelet packets combined with modal parameter analysis, *Composite Structures* 64, 2004, pp. 377-387.
- [18] Choi S., Detection of valvular heart disorders using wavelet packet decomposition and support vector machine, *Expert Systems with Applications* 35, 2008, pp. 1679-1687.
- [19] Chui C.K., *On cardinal spline wavelets, Wavelets and their applications*, Jones and Bartlett, Boston, 1992, pp. 419-438.
- [20] Ueda M., Lodha S., *Wavelets: an elementary introduction and examples*, University of California, Santa Cruz, 1995.
- [21] Lee J., Qiu H., Yu G., Lin J., Rexnord Technical Services, Bearing Data Set, IMS, University of Cincinnati. NASA Ames Prognostics Data Repository, [<http://ti.arc.nasa.gov/project/prognostic-data-repository>], NASA Ames, Moffett Field, CA, 2007.
- [22] Nikolaou N.G., Antoniadis I.A., Application of Wavelet Packets in bearing fault diagnosis, *Proceedings of the 5th WSES International Conference on Circuits, Systems, Communications and Computers (CSCC 2001)*, Rethymno, Greece, 2001, pp. 12-19.

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**B-splajnowe pakiety falkowe i ich zastosowanie
w wielorozdzielczej analizie sygnałów niestacjonarnych**

Steszczenie

Wzrastające wymagania do stanu technicznego maszyn powodują rozwój nowych metod diagnostycznych dla detekcji i identyfikacji uszkodzeń w możliwie wczesnej fazie. Większość z tych metod bazuje na analizie sygnałów drganiowych. Klasyczne metody często nie dają pełnej informacji o aktualnym stanie maszyny. Dlatego niezbędny jest rozwój odpowiednich metod diagnostycznych. Niektórymi z obiecujących metod analizy sygnałów są metody oparte na transformacji falkowej dające możliwość efektywnej diagnostyki niestacjonarnych sygnałów drganiowych w dziedzinie czasowo-skalowej. Uogólnieniem transformacji falkowej jest pakietowa transformacja falkowa pozwalająca wydobyć z sygnału dodatkowe korzystne cechy. Jednak efektywność w przypadku metod diagnostycznych bazujących na przekształceniu falkowym jest zdeterminowana wyborem odpowiedniej funkcji falkowej. W niniejszej pracy autor wprowadza nowe pakiety falkowe bazujące na falkach B-splajnowych. Analiza porównawcza ich efektywności była przeprowadzona na niestacjonarnych sztucznych sygnałach. B-splajnowe pakiety falkowe były zastosowane do oceny stanu łożysk tocznych.

