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Method of formulating the required number of tankers for delivery aircrafts in aviation fuel

K e y w o r d s

Aircraft, tanker, supply, aviation fuel.

S ł o w a k l u c z o w e

Statek powietrzny, cysterna, zaopatrywanie, paliwo lotnicze.

S u m m a r y

An important part of the air base logistic system is the supply sub-system. In military operations the main delivery can be focused on munitions and aviation fuel. Effective management of the supply stream and the reliability of vehicles in the air base logistic system affect the quality of operations, which can be measured by on time provisions, economic factors and the reliability of tankers. At present the number of tankers in the air base logistic system is based on experiences. This paper presents a mathematical model that enables one to estimate the minimum number of tankers in dependence from the type of the aircrafts, the number of aircrafts, the length of flights and structure of the flights.

1. Introduction

The main aim of the air base logistics system is meeting the needs of military technology. The following areas create the structured elements of that system:

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– Supply,
– Transportation,
– Infrastructure,
– Furnishing of services,
– Fire protection,
– Repair and serviceability, and,
– Airborne engineering supply.

The important elements of supplying aircrafts with aviation fuel are tankers. The extract number of them influences the punctuality of the flights and the quality of air force training and operational readiness. Nowadays this number is empirical. This article presents a mathematical model of estimating the required number of tankers.

2. Mathematical model of estimating the required number of tankers

The assumptions taken to formulate the mathematical model of supplying aircraft in flight with aviation fuel are as follows:

– The tanker can, be in one of specified operational states at any time.
– The number of aircrafts participating in flights is a random variable.
– The fuel consumption index of the main tank is a random variable and takes values $K_{zu} \in \{0.33; 0.5; 0.66; 0.85\}$.
– The storage tanks can be damaged at random during operation.
– The time needed to exchange a used storage tank for an operational one is specified.
– The duration of flights is fixed.

The following variables were taken to formulate the model:

- The number of aircrafts - N_{SP};
- The capacity of main tank of the aircrafts –V_{zbsp};
- The fuel consumption index of the aircraft– K_{zu};
- The number of storage tanks - N_{CD};
- The capacity of storage tanks – V_{CD};
- The flights structure (according to the planned schedule).

The fuel balance Equation for a single squadron of flights in accordance with the set time duration can be estimated as follows:

$$\sum_{k}^{N_{sp}} {}_{k}K_{zu} \cdot V_{zbsp} = V_{elt1} \tag{1}$$

Where: k = 1,2, ..., means the aircraft number.

The used fuel balance Equation for the maximum number of flights of a tactical aviation squadron can be set as follows:

$$\sum_{k}^{N_{SP}} \sum_{l}^{{}_kN_{elt}} ({}_{lk}K_{zu}) \cdot V_{zbsp} = V_{elt\max} \tag{2}$$

Where: ${}_{lk}N_{elt}$ – the l flight of the k squadron aircraft;
${}_{lk}K_{zu}$ – the fuel consumption index for l flight of the k aircraft;
$N_{elt} \in \{1,...,8\}$ – the number of flights.

Equation (2) is based on the assumption that all aircrafts where in flights.

If not, zero must be taken for the l flight of the k aircraft. The refuelling state of the aircrafts was considered for 2 situations:

1) for zero waiting time (t_{ocz}=0);

$$t_4 = t_m + K_{zu} \times t_{et} \tag{3}$$

Where: t_4 – the time of refuelling;
t_m – the handling time (connected with the time of approaching a new storage tank and connecting the injection sprayer);
K_{zu} – fuel consumption index;
t_{et} – the time of refuelling an empty tank of the aircraft.

2) $t_{ocz} \in (40\text{min.} - t_4)$ – for the maximum flight frequency (e.g. every 40 min).

For K_{zu}=0,85 and T_0=8h, we obtain for the squadron N_{elt}=5 and the maximum amount of the used fuel is:

$$V_{elt\max} = \sum_{k=1}^{16} \sum_{l=1}^{5} K_{zu} \cdot V_{zbsp} \tag{4}$$

The balance Equations of the capacity and work time for storage tanks are described with Equations (5) and (6) as follows:

a) The capacity balance:

$$N_{CDV} \cdot V_{CD} \geq V_{elt\max} \quad \text{Hence:} \quad N_{CDV} \geq \frac{V_{elt\max}}{V_{CD}} \tag{5}$$

Where: N_{CDV} – the number of storage tanks required for the all fuel used by the squadron;
V_{CD} – the capacity of the storage tank;
V_{eltmax} – the maximum fuel consumption for the squadron while performing the task.

$$N_{CDV} \geq \begin{Bmatrix} 69.889 & dla & V_{CD-4,5} \\ 41.933 & dla & V_{CD-7,5} \end{Bmatrix}$$

b) The balance of the times: It was assumed that the time of the refuelling cycle for storage tank t_5 must fulfil the following condition:

$$t_5 \leq t_{nCDV} + t_m + t_2 + t_3 \; ; \tag{6}$$

Where: t_{nCDV} – the time of filling up the storage tank of a particular capacity dependent on its refuelling state;
t_m – the handling time, connected with approaching the depot;
t_2 – the required dwell time;
t_3 – the time of fuel quality inspection in a storage tank.

It was assumed, that the storage tank in T_0 could perform at most N_5 refuelling cycles. The Equation at the work time balance for one storage tank, which is needed for examining the feasibility of the process, can be defined as follows:

$$t_5 \cdot N_5 + t_4 \cdot N_4 \leq T_0 \tag{7}$$

$$N_5 \leq \frac{T_0}{t_5} \qquad\qquad N_4 \leq \frac{T_0}{t_4} \tag{8}$$

Where: t_5 – the time of refuelling cycle by a storage tank;
N_5 – the number of refuelling cycles;
t_4 – the time of refuelling of aircraft;
N_4 – the number of the aircraft refuelling;
T_0 – the flight duration.

If $V_{CD} \approx V_{zbsp}$, then for one storage tank $N_4 = N_5$ must be taken and the time margin appears. As a rule $t_4 \neq t_5$, but in practice $t_4 << t_5$.

The number of storage tank N_{CDV} needed by volume that provide the squadron which the all fuel must be converted to the number actually needed storage tanks N_{CD} according to the following Equation:

$$N_{CD} \geq \frac{N_{CDV}}{N_5} \tag{9}$$

Where: N_{CD} – the number of actually needed storage tanks for the flights;
N_{CDV} – the number of storage tanks needed by volume for the correct amount of fuel required in one flight;
N_5 – the number of possible refuelling cycles for a storage tank.

Based on these assumptions, we have the following Equations:

$$\sum_{k}^{N_{SP}} \sum_{l}^{{}_k N_{elt}} ({}_{lk}K_{zu}) \cdot V_{zbsp} = V_{elt\,\max} \tag{10}$$

$$N_{CDV} \geq \frac{V_{elt\,\max}}{V_{CD}} \tag{11}$$

$$N_{CD} \geq \frac{N_{CDV}}{N_5} \tag{12}$$

$$t_5 \cdot N_5 + t_4 \cdot N_4 \leq T_0 \tag{13}$$

$$N_5 \cdot V_{CD} \geq N_4 \cdot V_{zbsp} \cdot K_{zu} \tag{14}$$

We can then determine the range of feasible solutions on limited assumptions.

3. Numerical example

Basic assumption (variables):

- the number of aircrafts-8;
- the duration of the flights $T_0 = 480$ h.;
- the flights performed on Su-22 aircrafts;
- the fuel consumption index is dependent on flight duration (according to the graph 1) K_{zu}= {0.33; 0.5; 0.66);
- the flight structure according to the planned schedule (Graph. 1).

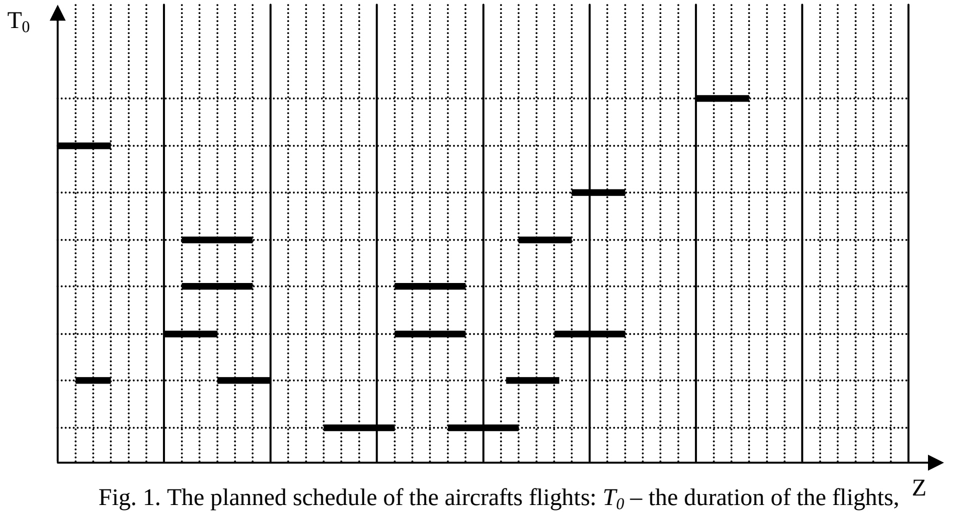

Fig. 1. The planned schedule of the aircrafts flights: T_0 – the duration of the flights, Z – the number of aircrafts

Rys. 1. Planowa tabela lotów – wariant: T_0 – czas wykonywania lotów, Z – liczba statków powietrznych

The minimum number of storage tanks (capacity 7.5 m^3) that will guarantee flight continuity must be specified.

The solution from: Equation (10) of the amount of fuel used by the aircraft in flights can be calculated as follows:

$$V_{elt} = \sum_{k}^{N_{SP}} \sum_{l}^{{}_kN_{elt}} ({}_{lk}K_{zu}) \cdot V_{zbsp} = 38991.25 \ [\mathrm{dm}^3]$$

Since the duration of the aircrafts is described with a discrete random variable for which the fuel consumption index takes values from K_{zu}= {0.33; 0.5; 0.66), the required number of storage tanks must be calculated for three cases and then the partial results summed.

Case I

The amount of the used fuel for K_{zu}= 0.66

$$V_{0,66} = 0{,}66\ (\mathbf{2}*V_{1zbsp}+2*V_{3zbsp}+2*V_{4zbsp}+1*V_{5zbsp}) = 21277.5\ [\mathrm{dm}^3]$$

The particular number of storage tanks which have a fixed capacity according to the Equation (11) must be bigger or the same as the needed amount of fuel:

$$N_{CDV} \cdot V_{CD} \geq V_{elt}$$

$$N_{CDV} \geq \frac{V_{elt}}{V_{CD}} \geq \frac{21277.5}{7500} \geq 2.83 = 3$$

The time of the refuelling cycle consist of the group of different times: The time needed for reaching the store, the handling time, the exact time of refuelling, the time of coming back to the surface of the airfield, the dwell time and quality control of the fuel. It is:

$$t_5 = 2.54 + 54 = 56.54\ [\mathrm{min}]$$

The refuelling time of the aircraft is precisely determined, because it depends on the fuel consumption index (K_{zu}), the efficiency of the tanker and the time needed to reach the aircraft by the tanker:

$$t_4 = 0.66*4625/300+5 = 15.17\ [\mathrm{min}]$$

The number of refuelling cycles both by storage tank (N_5) and the aircraft (N_4) can be calculated, according to Equations (13 and 14), by solving the following system of inequalities:

$$\begin{cases} t_5 \cdot N_5 + t_4 \cdot N_4 \leq T_0 \\ N_5 \cdot V_{CD} \geq N_4 \cdot V_{zbsp} \cdot K_{zu} \end{cases}$$

$$N_5 \le \frac{T_0}{t_5} - \frac{t_4 \cdot N_4}{t_5} \le \frac{480}{56.54} - \frac{15.17 \cdot N_4}{56.54} \le 8.48 - 0.268 N_4$$

$$N_5 \ge \frac{K_{zu} \cdot V_{zbsp} \cdot N_4}{V_{cd-7,5}} \ge \frac{0.66 \cdot 4625 \cdot N_4}{7500} \ge 0.407 N_4$$

$$8.48 - 0.268 N_4 = 0.407 N_4$$

$$0.675 N_4 = 8.48 \quad \Rightarrow \quad N_4 = 12.56$$

$$N_5 \ge 0.407 N_4 \ge 5.11 = 6$$

From Equation (11) the number of storage tanks needed for one flight can be calculated as follows:

$$N_{CDV} \ge \frac{V_{elt}}{V_{CD}} \ge \frac{21277.5}{7500} \ge 2.83 = 3$$

After substituting N_{CDV} and N_5 (12) we get:

$$N_{CD0,66} \ge \frac{N_{CDV}}{N_5} \ge \frac{3}{6} = 0.5$$

Case II

The amount of the used fuel for K_{zu}= 0.5

$V_{0.5}$= 0.5 **(2***V_{2zbsp}+V_{3zbsp}+V_{5zbsp}+V_{6zbsp} + V_{7zbsp} + V_{8zbsp})= 16187.5 [dm^3]

The particular number of storage tanks of fixed capacity must be bigger or the same as the maximum amount of fuel. It can be calculated by the following Equation:

$$N_{CDV} \cdot V_{CD} \ge V_{elt\,\max}$$

$$N_{CDV} \ge \frac{V_{elt}}{V_{CD}} \ge \frac{16187.5}{7500} \ge 2.158 = 3$$

The time t_5 of refuelling cycle by a storage tank is as follows:

t_5= 1.927 +54 = 55.927 [min]

The refuelling time of the aircraft is precisely determined, because it depends on the capacity of the aircrafts tank, the fuel consumption index, the efficiency of the tanker and the time need to reach the aircraft by the storage tank:

t_4= 0.5*4625/300+5 =12.71 [min]

The number of refuelling cycles by the storage tank *(N_5)*, as well as the number of refuelling aircraft *(N_4)*, can be calculated from the following system of inequalities:

$$\begin{cases} t_5 \cdot N_5 + t_4 \cdot N_4 \le T_0 \\ N_5 \cdot V_{CD} \ge N_4 \cdot V_{zbsp} \cdot K_{zu} \end{cases}$$

$$N_5 \le \frac{T_0}{t_5} - \frac{t_4 \cdot N_4}{t_5} \le \frac{480}{55.927} - \frac{12.71 \cdot N_4}{55.927} \le 8.58 - 0.227 N_4$$

$$N_5 \ge \frac{K_{zu} \cdot V_{zbsp} \cdot N_4}{V_{cd-7.5}} \ge \frac{0.5 \cdot 4625 \cdot N_4}{7500} \ge 0.308 N_4$$

$$8.58 - 0.227 N_4 = 0.308 N_4$$

$$0.535 N_4 = 8.58 \quad \Rightarrow \quad N_4 = 16.03$$

$$N_5 \ge 0.308 N_4 \ge 4.93 = 5$$

From Equation (11) we calculate N_{CDV} :

$$N_{CDV} \ge \frac{V_{elt}}{V_{CD}} \ge \frac{16187.5}{7500} \ge 2.15 = 3$$

After substituting N_{CDV} and N_5 we get:

$$N_{CD0.5} \ge \frac{N_{CDV}}{N_5} \ge \frac{3}{5} = 0.6$$

Case III

The amount of the used fuel for K_{zu}= 0.33

$$V_{0.33} = 0.33\ (V_{2zbsp}) = 1526.25\ [dm^3]$$

The particular number of storage tanks of fixed capacity must be bigger or equal as the needed amount of fuel. It can be calculated as follows:

$$N_{CDV} \cdot V_{CD} \ge V_{elt\,\max}$$

$$N_{CDV} \ge \frac{V_{elt}}{V_{CD}} \ge \frac{1526.25}{7500} \ge 0.2 = 1$$

The time t_5 of refuelling cycle by a storage tank is as follows:

$$t_5 = 1.27 + 54 = 55.27\ [min]$$

The time t_4 of refuelling is precisely estimated and is as follows:

t_4= 0.33*4625/300+5 =10.08 [min]

The number of refuelling cycles by the storage tank *(N_5)*, as well as the number of refuelling aircraft *(N_4)*, can be calculated from the following system of inequalities:

$$\begin{cases} t_5 \cdot N_5 + t_4 \cdot N_4 \le T_0 \\ N_5 \cdot V_{CD} \ge N_4 \cdot V_{zbsp} \cdot K_{zu} \end{cases}$$

After substituting data we get the following:

$$N_5 \le \frac{T_0}{t_5} - \frac{t_4 \cdot N_4}{t_5} \le \frac{480}{55.27} - \frac{10.08 \cdot N_4}{55.27} \le 8.68 - 0.18 N_4$$

$$N_5 \ge \frac{K_{zu} \cdot V_{zbsp} \cdot N_4}{V_{cd-7.5}} \ge \frac{0.33 \cdot 4625 \cdot N_4}{7500} \ge 0.2035 N_4$$

$$8.68 - 0.18 N_4 = 0.2935 N_4$$

$$0.3835 N_4 = 8.68 \Rightarrow N_4 = 22.63$$

$$N_5 \ge 0.2035 N_4 \ge 4.605 = 5$$

From Equation (11) we calculate N_{CDV} as follows :

$$N_{CDV} \ge \frac{V_{elt}}{V_{CD}} \ge \frac{1526.25}{7500} \ge 0.2 = 1$$

After substituting N_{CDV} and N_5 we get the following:

$$N_{CD0.33} \ge \frac{N_{CDV}}{N_5} \ge \frac{1}{5} = 0.2$$

The precisely required number of storage tanks (capacity 7.5 m^3) was achieved as a sum of the three partial -results for the particular fuel-consumption indexes *(K_{zu})* according to the following equation:

$$N_{CD-7.5} > N_{CD0.33} + N_{CD0.5} + N_{CD0.66}$$

Because 0.5 +0.6 + 0.2= 1.3 Hence $N_{CD-7.5} > 1.3 \Rightarrow \mathbf{2}$

Based on (13) and (14), the relation between the storage tank capacity V_{CD} and the number of its refuelling cycles N_4 can be estimated. The results are illustrated by Graph 2.

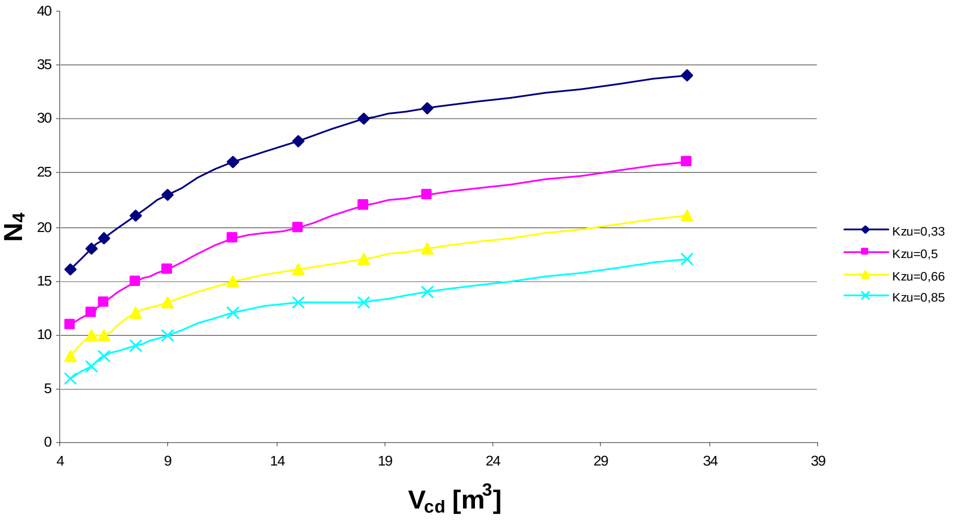

Fig. 2. The relation between the cisterns capacity and the number of refuelling cycles
Rys. 2. Wpływ pojemności cysterny na liczbę tankowań

The curves on Graph 2 show the relation between the number of refuelling cycles and the storage tank capacity. Equation (15) describes the mutual dependence:

$$N_4 = a + b * (\log cV_{cd} + d) \tag{15}$$

For a= 13.2647, b=4.705904, c=22.31486, d=-26.1808, the correlation coefficient is R^2 =98.89%, which means that the empirical curie fits well.

Summary

The article presents a method for the estimation the required number of tankers supplying aircrafts with aviation fuel, which depends on the type and number of the aircraft and the flight structure. This method is an attempt to use a mathematical model enabling the combination of theory and practice. This study is the results of two conditions. Firstly, there is no mathematical model, despite the fact that the problem is known and obvious. Secondly, the number of storage tanks is estimated by only human experience. A small defect of the proposed model might be the fact that that the calculations must be done in stages, grouped according to the flight duration (the same fuel consumption indexes)- (See the numerical example above). As a result of this method we obtained the minimum number of storage tanks of a particular capacity that are able to deliver the required amount of fuel to a certain place. This method could be

modified by introducing the excess factor (the number of storage tank will be bigger then). It will enhance the reliability of the fuel delivery system, but the costs will increase.

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Metoda obliczania niezbędnej liczby cystern do zaopatrywania statków powietrznych w paliwo lotnicze

S t r e s z c z e n i e

W czasie wykonywania lotów przez statki powietrzne zasadniczymi kwestiami są gotowość bojowa oraz mający na nią wpływ system zaopatrywania. Zasadniczymi przedmiotami zaopatrzenia w czasie prowadzenia działań są środki bojowe, energia i paliwa lotnicze. Skuteczne zarządzanie przepływem wymaganych produktów a także niezawodność pojazdów i dysprozycyjność załogi w systemie logistycznym bazy lotniczej wpływa na jakość prowadzonych działań. Jakość ta w odniesieniu do cystern dystrybutorów może być mierzona m.in. niezawodnością dostaw i czynnikami ekonomicznymi (kosztami ich eksploatacji). Liczba cystern zaopatrująca statki powietrzne w paliwa lotnicze w czasie wykonywania lotów określana jest empirycznie. W artykule przedstawiono metodę wyznaczania niezbędnej (minimalnej) liczby cystern-dystrybutorów w zależności od rodzaju statków powietrznych, ich liczby, długości lotu oraz organizacji lotów.