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Methodology for reliability estimation of systems with sliding reserve

Key words

Reliability, sliding reserve, events, reliability structure of type k out of n.

Słowa kluczowe

Niezawodność, przesuwająca się rezerwa, zdarzenie, struktura niezawodnościowa typu k z n.

Summary

The main goal for using slipping reserve in renewable machine systems is to ensure high functional reliability with a simultaneous decrease to the minimum number of reserve elements.

The development of a theoretical basis for the estimation of the exploitation reliability of systems with sliding reserve was confined, in general, for the case of one reserve object that ensures the reliable function of a selected group of basic devices.

1. Introduction

Finding a solution to the problem of reserving for renewable technical devices with optional probability distributions for proper work is difficult from the theoretical point of view. The case under consideration refers to a class of processes broader than homogeneous processes. We consider independent

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increments as streams of failures and renewals as streams without reciprocal interactions. Moreover, we assume that the parameters for the stream of failures, the stream of renewals, and the parameters for the failure of objects (which are in reserve) do not have to be constant. If those parameters are constant, then the behaviour of the system in defined by time intervals, and the probability of proper work can be determined on the basis of a system of differential equations [1, 2].

The assumption, which is often made, that the time for renewals has a constant value or is a random variable with an exponential probability distribution is not significant in this model. Very often, in practice, an assumption that streams of failures and renewals are straight streams (λ (t) = const.) is not correct. This may happen when (in spite of random factors which disturb proper work of the system) failures caused by the natural wear of elements of machines and devices start to dominate. An assumption that probability distributions for failures and renewals have optional shapes makes analytical solutions complicated. Although these problems can be solved by inserted Markow chains or methods based on differential-integral equations or Spicer identity, the results obtained in the form of complicated dependencies are often not practically useful [2].

2. Methodological assumptions

The methodological approach to the problem of reserving differs from traditional approaches in which problems with reserving are solved by adjustment of the number of reserve devices to the number of working basic devices. If the optimum number of basic devices is known and the reliable function of these devices is ensured by a single reserve object by multiplication of the calculated system, it is possible to increase the number of reserve objects for more numerous groups of technical devices. Therefore, it is necessary to search for solutions to such problems by means of simulation experiments that can be carried out in continuous or discrete times [8].

The methodological approach, which is proposed, substantially facilitates (in comparison with methods used so far) making simulation models with high practical importance [2].

The object under study is a machine system, which consists of n elements. The reliability structure of the system is a threshold structure of type k out of n [4, 5], provided that k=n-1. All elements are renewable. Elements which are marked by code numbers from 1 to k are called basic elements; whereas, the element marked by subscript n is a renewable reserve element [6, 7, 8].

A reserve element plays a role of the “sliding reserve”. Term *element* can refer to either a technical object or a part of a machine. The meaning of this word depends on the degree of generalisation made and does not play a significant role in this analysis.

The presented machine system has an excessive and coherent reliability structure described by an efficiency function, which, in turn, is a logical function of the following form:

$$F(X) = 1 \text{ for } X: f(X) \geq k,$$

$$F(X) = 0 \text{ for } X: f(X) < k.$$

The system is in an “able-to-work state” (aptitude) when F(X)=1, or in an “unable-to-work state” (non-aptitude) when F(X) = 0. It is preliminarily assumed that the introduction of the reserve device into work starts with reliability equal to 1. This assumption can be changed (weaken) if we assume that reliability is different from 1; however, this does not have a crucial significance in methodological analysis of the problem.

By the analysis of work of studied system, the following hypothetical reliability and exploitation states for system aptitude can be distinguished.

S_l – hypothetical reliability-exploitation states of technical system; l = 1,........m, for basic elements that reside in states of work, reserve, and renewal [2].

A given object has an universal character and, on this basis, it is possible to examine other cases of reserving for systems that encompass, e.g., multiple slipping reserve when reliability of switches is taken into account or the application of this type of redundancies in complex, parallel-serial reliability structures. This model can also be used for forecasting the number of exchangeable parts, which can be considered as redundancies that belong to cold reserve [3, 9].

The concept of simulation models that are presented below is based on the *transition module* [3]. Analysis of transitions in the network of events leads to the assessment of the reliability of function for the whole system. In order to characterise of the simulation model, it is necessary to present terms and concepts formulated in the work [9].

The most important terms and concepts are the following:

- Reliability-exploitation states,
- Elementary events,
- Events,
- Relations between elements within the set of events, and
- Trajectories of events.

e_j – (code numbers for system elements) residence time in reliability-exploitation states for system elements.

They are briefly characterised below [2].

Ad a) It can be assumed that the physical state of an object [8] is characterised by measurable or non-measurable features, which are specified according to the needs of the analysis. Taking into account the ability of performing operational tasks by the object, it is possible to talk about its ability-to-work or inability-to-work. However, it is equally important to determine its degree of readiness for performing these tasks when the object being repaired and when it can start working or when it immediately starts to work in the able-to-work state. As a result, it was assumed the following:

Reliability state – refers to able-to-work state or unable-to-work state,

Exploitation state – refers to state of work, state of waiting for work (state of reserve) or state of renewal (which takes into account the waiting time for renewal).

Reliability and exploitation states are jointly examined, and they are described as **reliability-exploitation states**. This means that an element at any moment is in one of reliability states (aptitude or non-aptitude) and simultaneously in one of the exploitation states (work, reserve, or renewal).

Ad b) **elementary event ($\mathbf{e_{ij?}}$)** – this term is exclusively associated with the exploitation of elements of the system and describes the shift of every element to the next moment of exploitation t (exploitation time is examined in a discrete way; the term *exploitation time* t is defined in work [8]. A set of elementary events that can occur is marked as follows:

$$\Omega = \{ \mathbf{e_{ijt}} : i = 1, 2, \dots ,k ,n ; j = 1, 2, 3 ; t = 1, 2, 3, \dots\}$$

i – number elements,

j – state of work (1), reserve (2) or renewal (3),

t – moment of exploitation.

The ability to work for the whole system depends on abilities of its elements. One non-aptitude element does not cause failure of the whole system; however, failure of two elements makes the whole system unable to fulfil operational tasks.

Ad c) **Event ($\mathbf{E_{w,t}}$)** referring to changes which occur during exploitation of the whole system describes any shift of the system to the next moment in exploitation t. Occurrence of an event is identified with a change in operation properties of the system in subsequent moments of exploitation [6, 8], which depends on the reliability-exploitation states of elements.

The ability of the system to function can be described by using the above mentioned elementary events and events.

In order to do that, the set of elementary events $\{e_{ij?}\}$ should be transformed into a set of events $\{E_{w,t}\}$ which characterises the behaviour of the whole system within the time of exploitation:

$$\Psi: \{e_{ijt}\} \xrightarrow{\Psi} \{E_{w,t}\}$$

t – index which refers to exploitation time when the system was in the specified reliability-exploitation state

$$t = (1,2,.....),$$

w – index which describes the reliability-exploitation state of the analysed technical system.

This set of events $\{E_{w,t}\}$ represents hypothetical reliability-exploitation states of system's ability-to-work, that is, whether the operating system has able-to-work (aptitude) reserve or whether it operates with an additional unable-to-work (non-aptitude) element which is being repaired or whether the whole system failed because at least two elements were damaged.

The probability for proper work of the system with a reserve aptitude element is higher than that with an additional element that is being repaired. Therefore, distinguishing between these two reliability-exploitation states has substantial significance for reliability analysis.

Ad d) Systematisation of a set of events $\{E_{w,t}\}$ is done on the base of the following binary relations [9]: simultaneity and preceding relations for events of concurrent events.

The above mentioned terms, elementary event ($e_{ij?}$), event ($E_{w,t}$) and reliability-exploitation state (S_l), successfully describe the actual situation in a quite detailed way. This approach makes the building of theoretical models very complicated; therefore, this approach to the problem was simplified. In order to do this, specified relations of preceding, simultaneity, similarity, and affinity are used.

In this model, a significant role is played by subsets of events that are connected to each other by **relations of simultaneity (J), preceding (C) and affinity of events (K).** Some of them are briefly characterised below.

At any moment of exploitation (t), the technical system is in one of specified reliability-exploitation states, i.e., one of the events specified by the simultaneity relation occurs. In trajectories of events under evaluation, only events that belong to different classes distinguished by means of simultaneity relation (J) can precede each other.

The trajectory encompasses events that belong to different classes $\{A_i\}$ [9]. These transitions have a certain probability, for instance, the transition from the class of concurrent events A_1 to the class of concurrent events A_2.

Affinity relation of events (K) plays a significant role in developing the simulation model.

Subsets of the set of affinity events cover events that occur one by one in subsequent moments of exploitation. Subsets of affinity events consist of events that occur one by one in subsequent moments of exploitation and belong to the same trajectories. Set Z_p is a one-component set. Specified events precede each other. Subsets of affinity events E_o and E_p are defined as reliability-exploitation states, S_1, S_2, ..., S_m, in which the system exists in subsequent moments of exploitation.

It is said that the event belongs to the set of events E_o when any of the elements of the technical system, but only one, undergoes renewal. It is said that the event belongs to the set of events E_r when any of the elements of the technical system, but only one, is in reserve. Any other event, which does not belong to set $E_r \cup E_o$ means that this element belongs to set E_p.

Events that belong to classes Er and E0 occur one by one in subsequent events. Each of these classes can be considered as a multiplet in an aspect of potential opportunities for the realisation of operational tasks (function) by the examined technical system. These events constitute a phase space through which proceeds the trajectory of a random process.

The number of affinity events that subsequently occur in the trajectory is associated with time needed for change in the reliability-exploitation state.

Subsequent time intervals of waiting for changes in the state of the trajectory of affinity events are non-negative random variables, which can have identical or different probability distributions during the exploitation of the system. Residence time in which the system resides in specified reliability-exploitation states and the probability of transitions to subsequent states (the occurrence of new events) are interesting from a practical point of view.

Therefore, the function that describes the probability of proper work is as follows:

$$R(t) = P(T > t).$$

In this work, reliability is concerned with the probability of keeping specified characteristics or parameters that characterised the proper work of the technical system in certain exploitation conditions and in a specified time interval.

Application of statistical studies and the theory of probability in order to assess the behaviour of the object during exploitation time is justifiable when there are problems with identification of the interaction of external factors (either the identification of what sort of factors influence correct function of objects and the strength of their interactions). This case often occurs in practice. The basic concept of reliability theory is term failure. Failure is treated as a partial or full loss of these properties of the object (technical system), which constitute the condition for proper function, according to a specified goal. According to probability theory, every complex reality can be described in categories of the occurrence or non-occurrence of certain events. By examining this reality within a specified time interval, we can assess, in statistical way, of the probability of the occurrence of certain events in subsequent moments of the exploitation of the event system [2].

Ad e) Behaviour of the technical system during exploitation can be described by means of trajectories $\{T_{zwt}\}$ defined in set $\{E_{w,t}\}$. The number of possible occurring trajectories of events is doubled in any subsequent moment of exploitation.

With reference to the goal of the study, it is important to distinguish such trajectories that characterise the system behaviour in a certain time interval. Because the trajectory consists of subsequent events, the actual problem is how to specify criteria for the selection and choice of preferred events in the analysed moment of exploitation.

If event $E_{0,t}$ occurs at moment t, it means that in subsequent moment t+1, event $E_{0,t+1,}$ or $E_{r,1}$ or $E_{p,1}$ can occur. Which event occurs depends on what principle for event selection is used. The set of events $\{E_{w,t}\}$ with trajectories $\{T_{zw,t}\}$ that were specified, according to accepted selection principle, constitutes the event system of exploitation. The selection principle can be identified with a relation specified on sets of events. The goal for the operation of a specified event system exists and is included within this system, because the set of events $\{E_{w,t}\}$ is specified according to this target. In addition, trajectories of events are considered according to this goal. It was assumed in the model described in this publication that the analysis of the processes of failures and renewals is performed in discrete time because, in this way, it is possible to use modelling based on the theory of events.

Occurrence of subsequent events can happen in a random or deterministic way. Time intervals between events can differ on the bases of length or they can be the same. They may have a random or deterministic character. The concept which is adopted in this publication is based on events of which occurrence specifically depends on “exploitation moments.” Digitisation of time, i.e. the analysis of the occurrence of subsequent events in subsequent moments of exploitation has a stationary character (time intervals are constant) and depends on actual exploitation conditions as well as on the character of the simulation experiment carried out. Residence time for the technical system in the same reliability–exploitation state is a random variable [8].

Subsequent events occur according to the random selection of transition in the “transition modules.” It is assumed that each reliability and exploitation state described by a set of relevant events, in which the system occurs, have transition characters. Possible transition to other states is determined by a relevant transition function. Residence time of the system in these states and trajectories of transitions in the space of events determine the dynamics of changes which happen in the system. The concept of transition modules makes it possible to determine probability values for the residence of the system in an able-to-work state at any moment of exploitation. The system is in an able-to-work state if basic elements perform production tasks and reserve element n is in repair (e.g. state S_1) or has just been repaired (e.g. state S_2) and is waiting to be introduced into work.

The idea of building of transition modules is illustrated on the base of the analysis of the ability-to-work state of the system as presented. It is possible to distinguish the following cases according to the results of the assessment of the able-to-work state of the system at moments from t_1 until t+1:

- At moment t, all elements of the system from 1 to k perform operation tasks and reserve element n is being repaired ($E_{0,t}$). Within a short time interval, which follows moment t through moment t+1, the situation does not change ($E_{0,t+1}$); or,
- In moments from t to t+1, all elements of the system from 1 to k work and reserve element n which was in repair at moment t was repaired within the time interval from t to t+1 and is waiting to be introduced into work ($E_{r,1}$); or,
- At moment t+1, a failure of the system ($E_{p,1}$) happens. This can happen when element n within time interval t to t+1 was not repaired and, in the meantime, another working element was damaged.

Failure of the whole technical system always occurs when an event that belongs to the class of events Ep happens. Reliability analysis of the function of the technical system should start with output event, i.e., event $E_{0,1}$.

Linearly ordered moments determine exploitation time [8] and the above mentioned events occur in stroke mode. Residence times for the system in relevant reliability-exploitation states, which are counted by means of the number of transitions (elements) of the system through subsequent exploitation moments t without changes in reliability-exploitation states are representative attributes of the quality of the whole system and its elements (physical technical objects). Indices for shares of the able-to-work state, work and renewal states, with reference to the global time of exploitation, are particularly important in system reliability assessment.

On the base of assumptions made in work [8], which were partially characterised above, principles for the analysis of transition modules and making of the simulation model are given in further parts of this work.

3. Conclusions

The concept of modelling the reliability of systems with sliding reserve type k out of n is described in detail in [2, 8]. The idea we have presented in this paper has been applied in simulation the modelling of reliability. This article is the first in a series of the forthcoming papers about this topic. The forthcoming articles will concern simulation methods of reliability estimation. The approach we have presented is unique, and it has not been proposed in literature before (except the articles of the author of this publication). It may be successfully applied to the reliability problems together with standard analytical methods.

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Metodyka szacowania niezawodności układów z przesuwającą się rezerwą

S t r e s z c z e n i e

W artykule przedstawiono metodyczne podstawy przeprowadzania oceny niezawodności układów o strukturze typu k z n. Tego typu układy są często stosowane w praktyce, ponieważ stanowią efektywny sposób zwiększenia niezawodności układów z rezerwą. Najskuteczniejszym sposobem tego typu rezerwowania jest układ podwajany, ale koszt jego stosowania jest najwyższy. Stąd też ocena, ile urządzeń pracujących powinno przypadać na jeden obiekt obiekt rezerwowy będący rezerwą przesuwającą się jest interesującym problemen z praktycznego i teoretycznego punktu widzenia.

W celu zbudowania modeli oceny niezawodności przeprowadzono analizę funkcjonowania układów z rezerwą przesuwającą się w ujęciu zdarzeniowym. Wyróżniono i zdefiniowano podstawowe pojęcia i opracowano sposób oszacowania niezawodności. Sformułowane założenia metodyczne umożliwiły zbudowanie modeli symulacyjnych szacowania niezawodności układów z rezerwa przesuwającą się, znajdujących zastosowanie do szybkiej oceny niezawodności różnych form rezerwowania.

Artykuł jest wprowadzeniem do serii opracowań na temat niezawodności układów typu k z n, z rezerwą przesuwającą się.