SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
3 (163) 2010

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Maintenance models – role of logistic support review

K e y w o r d s

Logistic support system, maintenance system modelling, spare part inventory models.

S ł o w a k l u c z o w e

System wsparcia logistycznego, modelowanie systemów obsługi, modele zapasów części wymiennych.

S u m m a r y

This paper presents an overview of some recent developments in the area of inventory planning and maintenance scheduling issues. The emphasis is on spare part inventory models, which the authors divided into four main groups of models: models of optimal spare part inventory policy for system under PM, number of spare parts optimisation models, storage reliability models, multi-echelon systems models. The first group of models is discussed in more depth. Later, the paper considers the time dependent system of systems where the system total task must be executed during the constrained time resource. The simulation approach is used to investigate the influence of parameters of the procurement process (order quantity, critical inventory level, lead-time length) on the system of systems behaviour.

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1. Introduction

The interest in the development and investigation of maintenance problems has been extensively discussed in literature since the early 1960s. The basic review in the area of maintenance modelling is prepared by Pierskalla & Voelker [1], where the authors investigated discrete time vs. continuous time maintenance models, later updated by Valdez-Flores & Feldman [2]. For other surveys see e.g. [3–11].

However, most of the maintenance models investigated in the literature on reliability theory assume that all the necessary logistic support resources, which include maintenance resources, support personnel, logistic information and data, spare and repair parts, and facilities, are immediately provided when desired. In practice, the repair capacity is not infinite, and logistic information may be unreliable. Moreover, the influence of a spare provisioning policy on the maintenance policy also cannot be ignored, since spare parts are ordered and carried in a limited quantity and the procurement lead-time is not negligible [11].

Throughout the years, the importance of the logistic support functions and logistic support management has grown. The plethora of studies, which have addressed the problem of logistic support systems modelling, can be divided into four main groups being presented in Fig. 1. A bibliography of the work done can be found in [11].

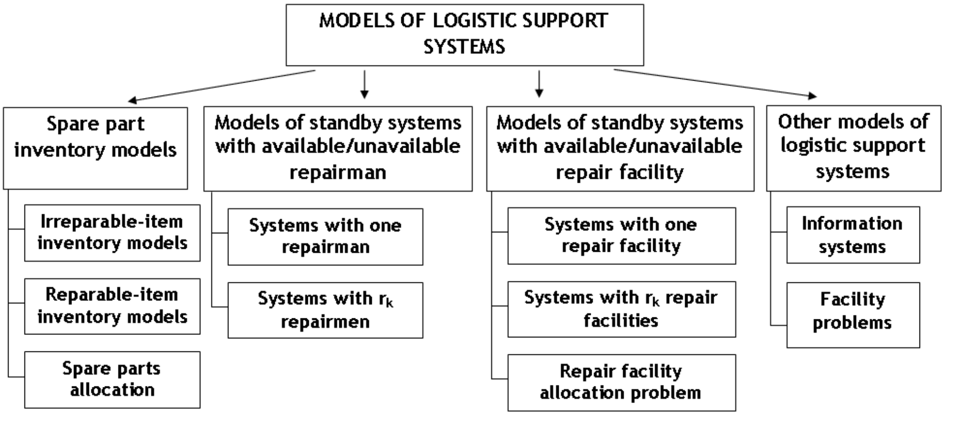

Fig. 1. Classification scheme of logistic support system models [11]
Rys. 1. Klasyfikacja modeli wsparcia logistycznego funkcjonowania systemu technicznego [11]

The problem of providing an adequate and efficient supply of spare parts, in support of maintenance and repair of operational systems, has been researched for many decades.

A sufficient inventory level of spare parts has to be maintained to keep the system in operating condition. When the spare elements are under stocked, it may lead to extended system downtime and, as a consequence, have a negative impact on the system availability level. On the other hand, maintenance of excessive spare parts can lead to large inventory holding costs. Moreover, the requirements for planning the logistics of spare elements necessary in maintenance actions of operational systems differ from those of other materials in several ways: the service requirements are higher, the effects of stock-outs may be financially remarkable, the demand for parts may be sporadic and difficult to forecast, and the prices of individual parts may be very high.

Consequently, one of the most important decisions faced by maintenance managers is the determination of optimal stocking levels, which involves finding the answer to the following questions, such that the total expected inventory costs be minimised:

- When to (re)order?
- How many items to (re)order?

There are many models in the literature on reliability theory regarding spare parts supply process optimisation. A significant portion of them are bases on a classic inventory theory, where the procurement process parameters are usually optimised taking into account the cost constraints (see e.g. [12]).

Many inventory papers that treat stock replenishment problems for stochastically failing equipment/systems are surveyed in [1] and updated by [3, 13, 14].

Following the introduction, this paper is focused on spare part inventory optimisation problems. The paper is organised as follows: In the Section 2, we present an overview of the most often applied models. We focus on one group of defined models. For more detailed investigation of other groups of models see [6, 11]. Later, we provided an example of a time dependent system of systems, where the system total task must be executed during the constrained time resource and briefly summarise.

We used a “system of systems” concept to model the interactions between the operational system and its logistic support system. According to the definition [15], *the system of systems context arises when a need or a set of needs are met with a mix of multiple systems, each of which are capable of independent operation but must interact with each other in order to provide a given capability*. More information can be found in [16, 17, 18].

2. Spare part inventory models

The reparable-item inventory problem has received much attention in the logistics literature. The general classification scheme for spare part inventory models is presented in Fig. 2.

The models presented below regard single-echelon systems. However, for advanced technical systems, such as engines, or aeroplanes, high system availability is enhanced thanks to the multi-echelon inventory system, in which usually two or more echelons are equipped with repair and stocking facilities. For more information see e.g. [11, 19, 20]

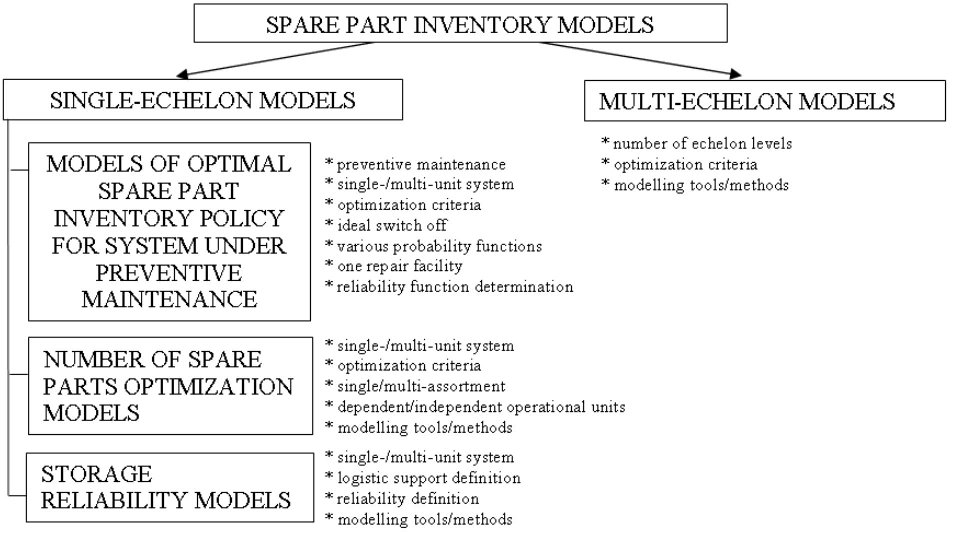

Fig. 2. Spare part inventory models classification [11]
Rys. 2. Klasyfikacja modeli sterowania zapasami obiektu technicznego podlegającego procesowi obsługiwania [11]

2.1. Models of optimal spare part inventory policy for a system under preventive maintenance

The main classification scheme of the investigated models is presented in Fig. 3.

There are two fundamental types of maintenance – preventive maintenance (PM) and corrective maintenance (CM). For PM demand for spare parts is predictable. For such maintenance, it may be possible to order parts to arrive just in time for use, and it may not be necessary to stockpile repair parts at all. In case of unplanned repair, the consequences of stock-outs regard to system unavailability has significant costs, and that is why some kinds of stock policies are necessary. It is natural in technical systems that only spare units that can be delivered by order are available for maintenance/replacement. In this case, we cannot neglect a lead-time for delivering the spare unit.

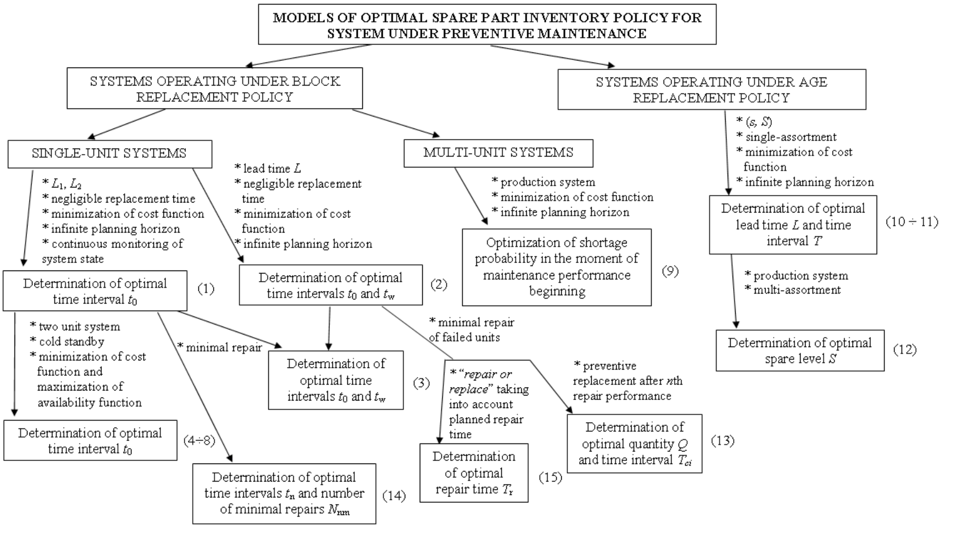

Fig. 3. Models of optimal spare part inventory policy for system under PM [11]
Rys. 3. Modele doboru optymalnych parametrów strategii sterowania zapasami, gdy strategia obsługi profilaktycznej obiektu jest zdefiniowana [11]

The first models investigated the possibility of spare part shortage due to delivery process performance regarded single-unit systems (see e.g. [21, 22]). In [21], the authors consider a one-unit system, where each failure is scrapped without repair and each spare part is only provided after a lead-time by an order. In the presented model, the following policy is adopted: An order for a spare is made at a pre-specified time instant t_o during an operating period of an original unit that is called a regular order. The lead-time entails L_2 time units. After delivery, the original unit is replaced whether it is operable or not. However, if the failure of the unit takes place before the time instant t_o, an emergency order is made at a failure time immediately. After an emergency delivery, which entails L_1 time units, a failed unit is replaced, and the process repeats itself.

Taking into account the following assumptions:

- Infinite planning horizon;
- Negligible replacement time of operating unit; and,
- The system is continuously kept under constant observation until a pre-specified time instant t_o or till the instant of failure, whichever occurs first.

The expected cost per unit time in the steady state is given by the following formula:

$$C_s(t_o) = \frac{c_m \int_0^{t_0} R(t)dt + c_{p1}F(t_0) + c_{p2}R(t_0) + c_{dw}\left[(L_1 - L_2)F(t_0) + \int_{t_0}^{t_0+L_2} F(t)dt\right]}{(L_1 - L_2)F(t_0) + L_2 + \int_0^{t_0} R(t)dt} \qquad (1)$$

Where: c_{p1} – cost of spare element expedited order which is made at time instant t,
c_{p2} – costs of spare element regular order made at time t_o,
c_{dw} – cost of system downtime per unit time,
$L_1(L_2)$ – random lead time for emergency (regular) order,
c_m – cost of system observation proportional to the expected duration of observation,
$R(t)$ – system reliability function,
$F(t)$ – cumulative distribution function of unit.

The presented model development can be found in [22], where the additional assumption is made that the operational unit replacement is made in one of two situations, whichever occurs first: when unit fails or when time of PM occurs at time instant t_w. The expected cost per unit time in a steady state and is given by the following:

$$C_z(t_0, t_w) = \frac{c_{p1}F(t_0) + c_{p2}R(t_0) + c_{dw}\int_{t_0}^{t_0+L} F(t)dt + c_h \int_{t_0+L}^{t_w} R(t)dt}{\int_0^{t_w} R(t)dt + \int_{t_0}^{t_0+L} F(t)dt} \tag{2}$$

Where: c_h – cost of holding a spare unit in a stock per unit of time,
L – random lead time.

Another work, made by Dohi et al. [23], presents a generalised order-replacement model arising in spare part inventory management, which is based on the assumptions taken in [21] and [22]. A replacement problem is considered for one-unit systems where each failed unit is scrapped and each spare part is provide, after a lead-time, by an order. If the original unit does not fail up to a pre-specified time instant $t_{o,}$ the regular order is made, and after a lead-time L_2 the spare unit is delivered. The delivered spare element is put into inventory until the moment of the original unit failure or until the moment of PM, whichever occurs first. On the other hand, if the original unit fails before the time instant t_o, the expedited order is made immediately at the failure time, and the spare part takes over its operation just after it is delivered after a lead time L_1. In this situation, the regular order is not made. The function of the expected total cost per time unit is given by the following:

$$C_z(t_0,t_w)=\left\{c_{sp}\left\{\int_0^{\infty}\int_0^{t_0}L_1dF(t)dE_1(L_1)+\int_0^{\infty}\int_{t_0}^{t_0+L_2}(t_0+L_2-t)dF(t)dE_2(L_2)\right\}+c_{p1}F(t_0)+\right.$$
$$+c_{p2}R(t_0)+c_h\int_0^{\infty}\int_{t_0+L_2}^{t_0+L_2+t_w}(t-t_0-L_2)dF(t)dE_2(L_2)+$$
$$\left.+c_h\int_0^{\infty}\int_{t_0+L_2+t_w}^{\infty}t_wdF(t)dE_2(L_2)\right\}\times\left(\int_0^{\infty}\int_0^{t_0}(t+L_1)dF(t)dE_1(L_1)+\int_0^{\infty}\int_{t_0}^{t_0+L_2}(t_0+L_2)dF(t)dE_2(L_2)+\right. \tag{3}$$
$$\left.+\int_0^{\infty}\int_{t_0+L_2}^{t_0+L_2+t_w}tdF(t)dE_2(L_2)+\int_0^{\infty}\int_{t_0+L_2+t_w}^{\infty}(t_0+L_2+t_w)dF(t)dE_2(L_2)\right)^{-1}$$

Where: c_{sp} – shortage cost per unit of time,
$E_i(t)$ – cumulative distribution function of L_i (i = 1, 2).

The problem of optimal spare part ordering policies for two-unit cold standby redundant system with two dissimilar units is considered in [24]. The replacement problem is defined as follows: Unit 1 begins working and unit 2 is in standby at time 0, and the planning horizon is infinitive. If unit 1 does not fail up to a pre-specified time t_o, the regular order for spare parts of both units 1 and 2 is made at time t_o. After a lead-time L_2 the spare parts are delivered, and at the time t_o+L_2, all original units are replaced correctively/preventively by spares, irrespective of the states of original ones, since the two units are not identical. The order for two spare parts is always needed. On the other hand, if the unit 1 fails before the time t_o, the operation is switched to the unit 2, and the expedited order for spare parts of both units is immediately made at the failure time. All original units are replaced by spares just after delivery, which lasts a lead-time L_1. The switchover is assumed to be perfect and instantaneous. The state in which both units fail before delivery of spare units implies the system down.

To obtain the optimal ordering policy parameter, the expected cost per unit time in the steady state and the stationary availability are developed.

The expected inventory cost function for one cycle is given by the following:

$$C_z(t_0)=c_{sp}\left\{\int_0^{t_0}\int_0^{L_1}(L_1-s)dF_2(s)dF_1(t)+\int_{t_0}^{t_0+L_2}\int_0^{t_0+L_2-t}[(t_0+L_2)-(t+s)]dF_2(s)dF_1(t)\right\}+$$
$$+c_{rz}\left\{\int_0^{t_0}\int_{L_1}^{\infty}(s-L_1)dF_2(s)dF_1(t)+\right. \tag{4}$$
$$\left.+\int_{t_0}^{t_0+L_2}\int_{t_0+L_2-t}^{\infty}[(t+s)-(t_0+L_2)]dF_2(s)dF_1(t)+\int_{t_0+L_2}^{\infty}[t-(t_0+L_2)]dF_1(t)+\int_{t_0+L_2}^{\infty}\int_0^{\infty}sdF_2(s)dF_1(t)\right\}+$$
$$+c_h\left\{\int_0^{t_0}tdF_1(t)+\int_{t_0}^{t_0+L_2}tdF_1(t)+\int_{t_0+L_2}^{\infty}(t_0+L_2)dF_1(t)\right\}+c_{p1}F_1(t_0)+c_{p2}R_1(t_0)$$

Where: c_{rz} – cost per unit time incurred for the residual lifetime of the original unit, which is still operable.

The expected time for one cycle is defined as:

$$E[T_{c_j}(t_o)] = \int_0^{t_0} (t + L_1) dF_1(t) + \int_{t_0}^{\infty} (t_0 + L_2) dF_1(t) \tag{5}$$

Applying the renewal theorem, the expected cost per unit time in the steady state is given by the following:

$$C_s(t_o) = \frac{C_{sj}(t_o)}{E[T_{cj}(t_o)]} \tag{6}$$

Moreover, the stationary availability $A(t_o)$, defined as the probability that a system is operative in the steady state, is given by the following:

$$A(t_0) = \frac{E[T_{oj}(t_0)]}{E[T_{c_j}(t_0)]} \tag{7}$$

Where: $T_{oj}(t_o)$ – effective time of a system for j-th cycle, given by the following formula.

$$E[T_{o_j}(t_0)] = \int_0^{t_0}\int_0^{L_1} (t+s) dF_2(s) dF_1(t) + \int_0^{t_0}\int_{L_1}^{\infty} (t + L_1) dF_2(s) dF_1(t) + \int_{t_0}^{t_0+L_2}\int_0^{t_0+L_2-t} (t+s) dF_2(s) dF_1(t) + t_0 + \\ + \int_{t_0}^{t_0+L_2}\int_{t_0+L_2-t}^{\infty} (t_0 + L_2) dF_2(s) dF_1(t) + \int_{t_0+L_2}^{\infty} (t_0 + L_2) dF_1(t) \tag{8}$$

Another very interesting problem regarding spare parts inventory planning in order to keep a production system in operating condition is considered. An example of such a system, consisting of n identical and stochastically independent production machines in k-out-of-n reliability structure, is given in [25]. The operational process of the presented system includes planned machine shutdown, during which all of the failed elements in that maintenance cycle are replaced in order to increase system reliability.

The problem considered in the presented paper regards the a priori planning of spare part inventories required for maintenance during a phased mission. In the system, it is possible to replace failed elements only during overhauls performed between two phases. The replacement of the failed elements at the end of phase k may be done by spare parts remaining unused from the proceeding overhauls and by S_k items planned to become available at time point t_k. the failed elements, after replacement, are repaired and put into stock (inventory with returns system). The shortage can occur when the demand exceeds the number of spare elements available from stock, and then spare parts are obtained by an emergency order or by borrowing, a penalty cost is paid, and the mission continues. The problem of spare part planning is to find S_k for which stock out of probabilities p_{spk} at time point t_k are smaller than the specified numbers α_{spk}:

$$\begin{cases} \min C_z = \sum_{k=1}^{K} c_z S_k \\ p_{spk}(S_1,...,S_K) \le \alpha_{spk} \end{cases} \tag{9}$$

Where: C_z – function of expected total purchase and holding cost,
c_z – total purchase and holding cost per unit per unit of time,
t_k – moments of planned overhauls, $k = 1, 2, .., K$,
S_k – planned number of spare elements at t_k,
$p_{spk}(S_1,..., S_K)$ – probability of stock out at t_k,
α_{spk} – maximal level of stock out probability in one maintenance cycle.

The model is solved with the use of a Markov process whose states are determined by the number of available spare parts and the following assumptions:

- Perfect maintenance conditions, and
- Elements of the system are identical and identically distributed.

Many works that address the problem of determining the optimal ordering policy parameters for technical systems operating under a block replacement policy are based on using simulation processes (see e.g. [26, 27, 28]). Those models give the solutions to define optimal ordering policy parameters (e.g. order placement moments) and define optimal PM parameters (moments of maintenance performance).

Another group of models where the problem of spare inventory optimisation is investigated regards the age replacement policy. The problem of age replacement policy with inventory restrictions can be found in, e.g. [29], where authors investigated two inventory policies (s,S).

According to the (1,1) inventory policy, operating element will be replaced in one of two situations: at age T or whenever the minimal repair cost C_{nm} is greater than some predetermined value $C_{nm}^{\max}$, whichever occurs first. When the unit must be replaced, it will be ordered and delivered after a lead-time L. During the time of waiting for spare part delivery, the system is in downstate.

Optimisation of the following parameters: lead time L and the age T when a system must be replaced base on the minimisation of total maintenance and inventory cost, is defined by the following formula:

$$C_s(L,T)=\frac{c_d(L)+\left(c_{w1}-c_{w2}+\int_0^{C_{nm}^{kr}} x\,dF_{cn}(x)\left(1-F_{cn}(C_{nm}^{\max})\right)^{-1}\right)\left[1-\left(R(t)\right)^{1-p_{cnm}}\right]+c_{w2}}{L+\int_0^T \left(R(u)\right)^{1-p_{cnm}}du} \tag{10}$$

Where: $F_{cw}(x)$ – cost of repair distribution function,

p_{cnm} – probability that defined minimal repair cost is greater than $C_{nm}^{\max}$,

$c_d(L)$ – function of costs associated with delivery performance (e.g. ordering cost, cost of lost production).

This kind of model might correspond to some very critical but expensive piece of equipment where one backup is provided [29].

The second investigated inventory policy called (2,2) is a modification of described (1,1) inventory models. In this model, the system will always contain one unit in operation and one additional unit either in inventory or on order. According to the model assumptions, when an operating unit fails, one of two possible situations can happen:

- An additional unit being in inventory is immediately available for replacement,
- An additional unit is on delivery – then the failed unit is repaired at all cost.

No system downtime is ever allowed.

The total cost function is defined as follows:

$$C_s(L,T)=\left\{c_d(L)-\ln R(L)C_{nm}^{L}+\left[1-\left(R(L)\right)^{1-p_{cnm}}\left(R(T)\right)^{1-p_{cnm}}\right]\times\left(\int_0^{C_{nm}^{kr}} x\,dF_{cn}(x)\left(1-F_{cn}(C_{nm}^{\max})\right)^{-1}+c_{w1}-c_{w2}\right)+c_{w2}\right\}\times \\ \times\left(L+\left(R(L)\right)^{1-p_{cnm}}\int_L^T \left(R(\tau)\right)^{1-p_{cnm}}d\tau\right)^{-1} \tag{11}$$

Where: C_{nm}^{L} – expected repair cost function during a lead time.

For more complicated problem investigation (see e.g. [30]) simulation processes, dynamic programming, integer programming, and nonlinear programming are the main tools suggested.

Lots of models for the joint optimisation of an optimal age-dependent inventory policy and PM policy regarding production systems subjected to random machine breakdowns (see e.g. [31, 32]) have been investigated. An interesting inventory problem is investigated in [33], where authors developed the optimal number of inventories $S = S_1, S_2, \ldots, S_r$ when

- The system performs under age replacement policy, and
- The system failure rate increase with its age.

The optimisation problem is stated as follows:

$$\begin{cases} \max \prod_{q=1}^{n_q} p_q(S_q) \\ \sum_{q=1}^{n_q} H_q s_q \le C_z^{\max} \end{cases} \tag{12}$$

Where: n_q – number of types of spare elements in production machine,
$p_q(S_q)$ – probability that there will be no stock out of spare elements type q during the overhaul,
s_q – initial inventory level,
$C_z^{\max}$ – maximal allowed level of inventory costs.

The solution of the stated optimisation problem is received with the use of dynamic programming.

However, in real life systems, the failed element can be replaced or repaired, which needs to give an answer to the following questions:

- When should the unit be repaired instead of replaced?
- How many spare parts should be ordered in order to meet demand while keeping the ordering and inventory costs minimal?

One of the models that tries to answer these questions is presented in [34]. In this paper, a joint stocking and replacement model with minimal repair at failure is considered. The authors assumed that Q units are purchased per order, operation unit is replaced after using for time interval T_{ci}, if inventory level is (i-1), and minimal repair is performed for any intervening failures. The problem is to select optimal order quantity Q and replacement intervals T_{ci}, so as to minimise the total maintenance and inventory cost per unit time, given by the following formula:

$$C_s(Q,T_{ci}) = \frac{c_o + c_{wz}Q + c_{nm}\sum_{i=1}^{Q} H(T_{ci}) + c_h \sum_{i=1}^{Q}(i-1)T_{ci}}{\sum_{i=1}^{Q} T_{ci}} \tag{13}$$

Where: c_o – cost of an order placing,
c_{wz} – replacement cost per unit,
i – inventory level.

The investigated problem of optimal ordering and maintenance policy parameter definition is continued in many recent papers (see e.g. [35]). In this paper, the authors defined optimal ordering point t_o and the optimal number of minimal repair N_{nm} before PM performance for a single-unit system. Assumptions made in this model are the same as presented in [21].

An order for a spare element is placed before the *n*th failure of the operational unit occurrence (moment t_n), and lasts a lead time L. If the operational unit fails before t_n moment occurrence, the system is in a downstate until the moment t_n+L. However, if the unit fails after spare element delivery (*k*th failure), the system is immediately replaced. Other failures are minimally repaired in time $(0, t_k)$.

Optimisation of parameters is performed with the minimisation of the total expected cost function given by the following:

$$\begin{aligned} C_s(t_n, N_{nm}) = \Bigg\{ & (N_{nm}-1)c_{nm} + c_o + \left(\int_0^{\infty} \int_x^{x+L} f_{t_n,t_k}(x,y)\,dy\,dx \right) \times \left(c_{sp} \int_0^{\infty} \int_x^{x+L} (x+L-y) f_{t_n,t_k}(x,y)\,dy\,dx \right) + \\ & + \left(\int_0^{\infty} \int_{x+L}^{\infty} f_{t_n,t_k}(x,y)\,dy\,dx \right) \left(c_h \int_0^{\infty} \int_{x+L}^{\infty} (y-x-L) f_{t_n,t_k}(x,y)\,dy\,dx \right) \Bigg\} \times \\ & \times \Bigg\{ \left(\int_0^{\infty} \int_x^{x+L} f_{t_n,t_k}(x,y)\,dy\,dx \right) \left(\int_0^{\infty} \sum_{j=1}^{t_n-1} \frac{\exp(-R(x))(R(x))^j}{j!} dx + L \right) + \\ & + \left(\int_0^{\infty} \int_{x+L}^{\infty} f_{t_n,t_k}(x,y)\,dy\,dx \right) \left(\int_0^{\infty} \sum_{j=1}^{k_{nm}-1} \frac{\exp(-R(x))(R(x))^j}{j!} dx \right) \Bigg\}^{-1} \end{aligned} \tag{14}$$

Where: $f_{t_n,t_k}(x,y)$ – probability density function of t_n and t_k.

Another interesting solution of the problem ‘replace or repair’ is given in [36], where a simple repair-time limit replacement problem with imperfect repair is considered. The authors investigated a single-unit system in which, when the unit fails, one estimates the repair time. If the repair can be completed up to a pre-specified time limit $T_{r\,\max}$, the repair is started immediately, otherwise, the spare unit is ordered with a lead-time L. The expected total cost per unit time in the steady state is given by the following formula:

$$C_s(T_r) = \frac{(k_f + c_n)\int_0^{T_r} t\,dG(t) + (k_f L + c_o)\overline{G}(T_r)}{\int_0^{T_r} t\,dG(t) + \frac{1}{\lambda} + \left(L + \frac{1}{\lambda^s} - \frac{1}{\lambda} \right)\overline{G}(T_r)} \tag{15}$$

Where: k_f – penalty cost per unit time when system is in downstate,
T_r – random repair time,
$G(t)$ – p.d.f. for repair time; $\overline{G}(t) = 1 - G(t)$,
λ^s – failure rate of new unit,
λ^n – failure rate of repaired unit.

The solution of the presented model is obtained with the use of the graphical method based on the Lorenz transform.

The main classification of the models regarding the optimisation of inventory and maintenance policy parameters is presented in [11].

3. An example

Consider a repairable system of systems under continuous monitoring, in which there are integrated two independent systems: operational and its supporting system. Both systems have only two states: upstate, when they are operable and can perform its specified functions, and downstate, otherwise.

The system of systems total task is defined as the continuous performing of exploitation process. Moreover, in the presented model, the logistic support functions are narrowed down only to providing the necessary spare parts to the operational system. As a result, the logistic support system is inoperable when there is no capability of supplying the operational processes with necessary spare parts.

The operational system is composed of M identical elements working in a reliability structure, which determines the moments when the system goes to a downstate. Let us also assume that element failures are random in time, and each failure entails a random duration of repair before the element/system is put back into service. Let us also assume that any information about failures in this system is reliable and comes immediately to the logistic system.

In the investigated model, when logistic support system is in upstate, the ability of the system of systems depends only on the following:

- The time, when the operational system is operable, and
- The time of technical system repair.

In this situation, when the supporting system is inoperable due to the lack of spare parts, the system of systems availability also depends on the logistic delayed time, which is necessary to solve logistic problems.

Moreover, if there is a restriction on the system of systems' total task completion time, defined as the time of operational system recovery process, the system of systems remains in upstate if this defined time will be shorter than time resource. Otherwise, the system of systems will fail and remain in downstate until the end of the delivery process.

Consequently, the following additional model assumptions are taken into account to define the system of systems performance process:

- The randomness and independence of all the performed processes,
- Critical inventory level (CIL) used as a stock policy, and
- the individual time redundancy used to model the system of systems performance [11].

To the best of the authors' knowledge, an effective way for achieving the reliable operational systems logistic support is especially based on meeting two targets: reliability/availability and cost constraints [11]:

$$\begin{cases} C_s = \dfrac{C_{sj}}{E[T_j]} \\ A \le A_{\min} \end{cases} \tag{16}$$

Where: C_{sj} – the expected total system of systems costs in a jth procurement cycle,
T_j – the random time of the jth cycle,
A – the system of systems' availability ratio,
A_{min} – the limiting availability ratio of system of systems performance.

More information can be found in [11].

3.2. Simulation model and obtained results

The analytical model of the performance of the presented system of systems with time dependency is investigated in, e.g. [16, 17, 18].

The analytical results of the modelled problem can be obtained only for a small amount of cases, when the operational system is a single-unit system and the performed processes are modelled according to the exponential distributions, *etc*. (see. [11]). Thus, there can be written the following conclusion, that this analytical model is an oversimplified version of the real system behaviour, so the obtained results are not traceable to practical situations.

To overcome this problem, there is proposed a simulation model of time dependent system of systems performance, which has been developed with the use of *GNU Octave* program. The simulation algorithm of the modelled system of systems is given in Fig. 4.

The systems of systems level of availability ratio, the probability of system of systems downtime occurrence, or economic results strongly depend on the operational system reliability structure. That is why the model of the time dependent system of systems performance was created for the three various system reliability structures – series, parallel, and "*k* out of *n*."

The simulation results of the modelled system of systems have been carried out for the input parameters, presented in Table 1.

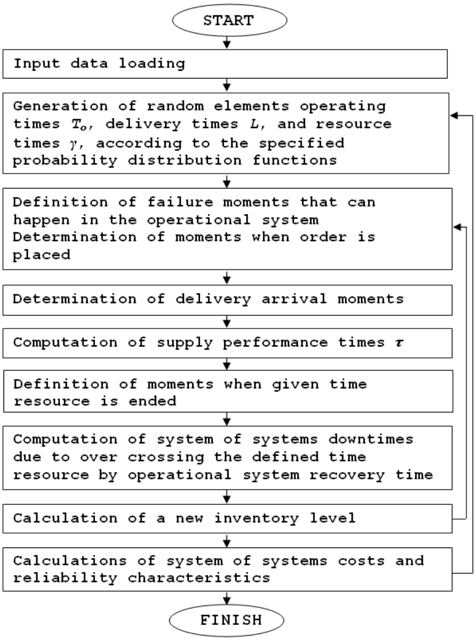

Fig. 4. Simulation algorithm of time dependent system of systems performance [11]

Rys. 4. Algorytm realizacji komputerowej symulacji procesu funkcjonowania nadsystemu [11]

Table 1. Input parameters of modelled system of systems [11]

Tabela 1. Parametry wejściowe modelu nadsystemu [11]

	Initial value	Explanation of denotation
A_o	1	Weibull's shape parameter of single operational element time to failure
$1/B_o$	100	Weibull's scale parameter of single operational element time to failure
A_r	1	Weibull's shape parameter of single operational element replacement time
$1/B_r$	10	Weibull's scale parameter of single operational element replacement time
A_L	1	Weibull's shape parameter of lead-time time
$1/B_L$	1000	Weibull's scale parameter of lead-time time
A_γ	1	Weibull's shape parameter of time resource time
$1/B_\gamma$	10	Weibull's scale parameter of time resource time
k	1	"k" out of "M"
M	5	number of elements working in an operational system
s	5	critical inventory level
Q	30	order quantity
c_w	50	replacement cost of a unit
c_o	50	ordering cost
c_p	50	purchase cost of one unit
c_h	100	inventory unit holding cost per unit time
k_f	1000	penalty cost of system of systems failure occurrence
c_{dw}	1000	cost of system of systems downtime unit

The main reliability and economic results are presented in Fig. 5–10.

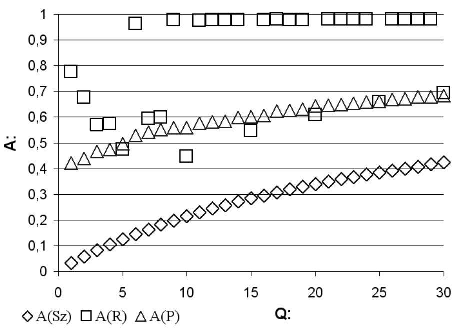

Fig. 5. System of systems availability ratio for various levels of order quantity of spare elements

Rys. 5. Zmienność poziomu wskaźnika gotowości w funkcji zmiany parametru wielkości partii zamówienia

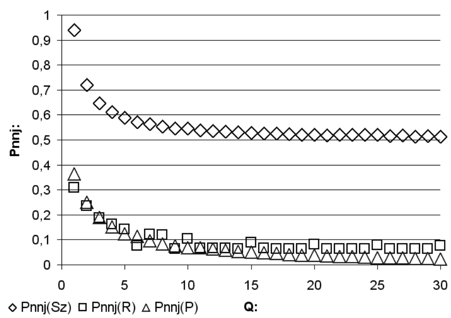

Fig. 6. The system of systems downtime probability for various levels of order quantity of spare elements

Rys. 6. Zmienność poziomu prawdopodobieństwa niezdatności nadsystemu w funkcji zmiany parametru wielkości partii zamówienia

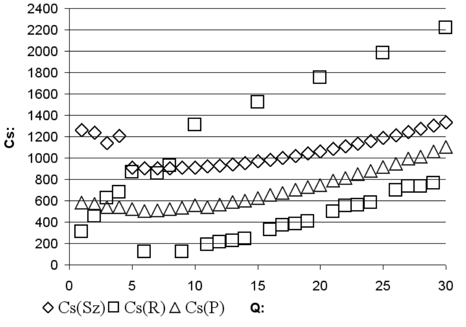

Fig. 7. Expected cost per unit time function for various levels of order quantity of spare elements
Rys. 7. Zmienność oczekiwanych kosztów funkcjonowania nadsystemu w funkcji zmiany parametru poziomu wielkości zamówienia

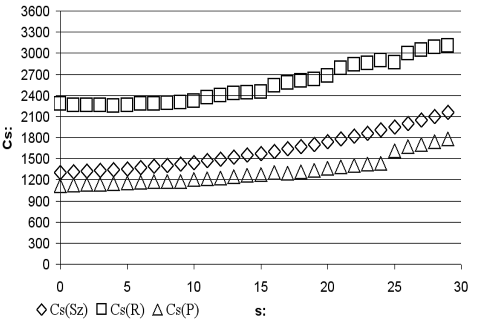

Fig. 8. Expected cost per unit time function for various critical inventory levels
Rys. 8. Zmienność oczekiwanych kosztów funkcjonowania nadsystemu w funkcji zmiany parametru poziomu zapasu alarmowego

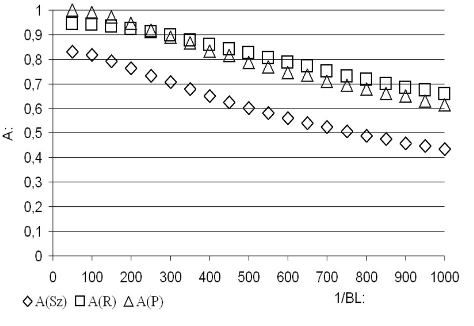

Fig. 9. System of systems availability ratio for various lead-time lengths
Rys. 9. Zmienność poziomu wskaźnika gotowości w funkcji zmiany parametru oczekiwanego czasu realizacji dostaw

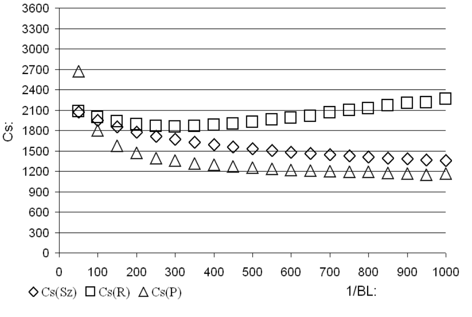

Fig. 10. Expected cost per unit time function for various lead-time lengths
Rys. 10. Zmienność oczekiwanych kosztów funkcjonowania nadsystemu w funkcji zmiany parametru oczekiwanego czasu realizacji dostaw

The ordered and delivered spare parts quantity determines the length of a single procurement cycle (time that elapses between the two consecutive moments when the inventories on-hand drop to a critical level). As a result, the bigger the ordered quantity, the higher mean stock level in the system and rarer deliveries are performed.

The expected costs incurred by the system of systems with a different-structured operational system are mainly determined by the inventory holding costs and the system of systems downtime costs (Fig. 7). When the level of ordered quantity rises, the expected costs function has the local minimum in the case of series and k out of M systems. It is a result of rarer deliveries and downtimes occurrence that arise from inventory lack. On the other hand, the more spare elements are purchased, the higher inventory holding costs are incurred.

The worst solutions for the system of systems with an operational system in parallel occur when the ordered quantity *Q* is a multiple of *M*. If all *M* elements are replaced and there is no spare parts remaining in a logistic system, there is a higher downtime probability and economical consequences, than if there are some elements in a stock. These downtime costs together with the inventory holding costs have the greatest influence on the system of systems economic results.

The same effect can be seen when availability of the system of systems with the operational system in parallel is analysed. The system of systems reaches the lowest availability ratio level when *Q* is a multiple of *M*. For the system of systems with other reliability structures of operational systems, the rarer and bigger deliveries, the higher availability ratio is achieved.

The level of ordered quantity also has an influence on the probability of the system of systems downtime occurrence, which is especially evident for a series structure case (Fig. 6). A lower level of ordered quantity forces frequent deliveries, and as a result, there is higher probability that the possible delays of the delivery cause the system of systems downtime.

The next parameter of the procurement process, which affects the system of systems performance, is the critical inventory level (Fig. 8). The higher critical inventory level, the higher the mean inventory level in a system what incurs higher inventory holding costs. However, the higher *s* level also gives the possibility to reduce the delivery delay consequences, which has a positive impact on the system of systems reliability results.

Moreover, there can also be seen the influence of lead-time length on the system of systems behaviour. The longer lead-time especially affects the reliability characteristic of the system of systems (Fig. 9). On the other hand, the longer the lead-time, the lower inventory holding costs and the higher downtime costs incurred, which is connected with the higher probability of the system of systems downtime occurrence. This relation can be seen in the Fig. 10 as a local minimum of the C_s value.

In order to model the time dependencies between the operational system and its logistic support system, basic relations have to be identified, which result from the system of systems structure, component parameters, or process execution times.

In other words, the presented model can especially support decision processes in the area of supply task performance requirements. It especially gives a convenient tool to decide which supplier can provide the desirable time of supply delivery in order to achieve a defined system of systems operational capability.

On the other side, the developed model can be helpful to assess the reliability requirements of operational system elements in order to provide the continuous system of systems total task performance.

Conclusions

To sum up, all the presented models from the area of supply process parameters optimisation, when a system is maintained according to defined PM, can be divided into two groups:

- Searching for effective optimisation methods for already known models (see e.g. [37, 38]), and
- Searching for system models in which new assumptions are made (e.g. new reliability structure, dependent elements in a system) (see e.g. [23, 24]).

Moreover, literature on modelling relations between logistic and operational systems is scarce. Up to now, the interactions between the operational system and its supporting systems have not been clearly investigated. The research has focused on the evaluation of reliability and economical characteristics for both systems in a separate way.

Moreover, the logistic systems have been evaluating and designing mostly in terms of inventory modelling, supply processes organisation, and transportation processes modelling.

However, the simultaneous setting of all structural parameters (e.g. redundancy, repair shop capacity) and control variables (e.g. spare part inventory levels, maintenance policy parameters, repair job priorities, time resource) is mathematically a hard problem, and cannot be done without many simplified assumptions taken.

References

[1] Pierskalla, W.P. & Voelker, J.A., A survey of maintenance models: the control and surveillance of deteriorating systems. Naval Research Logistics Quarterly 23, (1976) 353-388

[2] Valdez-Flores, C. & Feldman, R., A survey of preventive maintenance models for stochastically deteriorating single-unit systems. Naval Research Logistics Vol. 36, (1989) 419-446

[3] Cho, I.D. & Parlar, M. , A survey of maintenance models for multi-unit systems. European Journal of Operational Research 51, (1991), 1-23

[4] Nakagawa, T., A summary of discrete replacement policies. European Journal of Operational Research 17, (1984) 382-392

[5] Nicolai, R.P. & Dekker, R., Optimal maintenance of multicomponent systems: a review. Economic Institute Report 2006

[6] Nowakowski T. & Werbińska S., Maintenance modelling concepts – state of art. International Journal of Materials and Structural Reliability. vol. 6, nr 2,(2008) s. 229-254

[7] Pham, H. & Wang, H., Imperfect maintenance. European Journal of Operational Research 94, (1996) 425-438

[8] Sherif, Y.S., Reliability analysis: Optimal inspection & maintenance schedules of failing equipment. Microelectronics and Reliability Vol. 22, No. 1, (1982) 59-115

[9] Thomas, L.C., A survey of maintenance and replacement models for maintainability and reliability of multi-item systems. Reliability Engineering 16, (1986) 297-309

[10] Wang, H., A survey of maintenance policies of deteriorating systems. European Journal of Operational Research 139, (2002) 469-489

[11] Werbińska S., Model of logistic support for exploitation system of means of transport. PhD thesis (not publ.). Wroclaw Technical University, Wroclaw, (2008)

[12] Sarjusz-Wolski, Z., Strategy of supply management. Practice of business logistics. (in Polish). A.W. PLACET, Warsaw, (1998)

[13] Guide Jr V.D.R., Srivastava R., Repairable inventory theory: Models and applications, European Journal of Operational Research, 102, (1997)

[14] Kennedy W.J., Patterson J.W. & Fredendall L.D., An overview of recent literature on spare parts inventories, International Journal of Production Economics, 76, (2002)

[15] Crossley W.A.: System of systems: an introduction of Purdue University Schools of Engineering Signature Area. Available on: http://esd.mit.edu/symposium/pdfs/papers/crossley.pdf (29.11.2006)

[16] Werbińska S., Interactions between logistic and operational system – an availability model, Risk, reliability and societal safety. Eds T. Aven, J. E. Vinnem, Leiden: Taylor and Francis, (2007)

[17] Werbińska S., The reliability model of logistic support system with Time Dependency (in Polish). Logistics No. 3, (2007)

[18] Werbińska S., Problems of designing logistic support system with time dependency. SYSTEMS Journal of Transdisciplinary Systems Science, Vol. 12, No. 2, (2007)

[19] Diaz A., Fu M.C., Multi-echelon models for repairable items: a review, Working Paper, University of Maryland, (1995)

[20] Diks E.B., de Kok A.G., Lagodimos A.G., Multi-echelon systems: A service measure perspective, European Journal of Operational Research, 95, (1996)

[21] Sridharan V., A note on ordering policies with cost of observation and lead times, International Journal of Production Economics, 32, (1993)

[22] Thomas L.C., Osaki S., An optimal ordering policy for a spare unit with lead time, European Journal of Operational Research, 2, (1978)

[23] Dohi T., Kaio N., Osaki S., On the optimal ordering policies in maintenance theory – survey and applications, Applied Stochastic Models and Data Analysis, 14, (1998)

[24] Dohi T., Osaki S., Kaio N., Optimal planned maintenance with salvage cost for a two-unit standby redundant system, Microelectronics and Reliability, Vol. 36, No. 10, (1996)

[25] Vujosevic M., Petrovic R., Senborn A., Spare parts inventory planning for a redundant system subject to a phased mission, Engineering Costs and Production Economics, 19, (1990)

[26] Chelbi A., Ait-Kadi D., Spare provisioning strategy for preventively replaced systems subjected to random failure, International Journal of Production Economics, 74, (2001)

[27] Hagmark P.E., Pernu H., Risk evaluation of a spare part stock by stochastic simulation, Safety and reliability for managing risk. Eds C. G. Soares, E. Zio, Leiden: Taylor and Francis, (2006)

[28] Sarker R., Haque A., Optimization of maintenance and spare provisioning policy using simulation, Applied Mathematical Modelling, 24, (2000)

[29] Mehrez A., Stulman A., Age replacement in the presence of inventory constraints, European Journal of Operational Research, 12, (1983)

[30] Vaughan T.S., Failure replacement and preventive maintenance spare parts ordering policy, European Journal of Operational Research, 161, (2005)

[31] Gharbi A., Kenne J-P., Beit M. , Optimal safety stocks and preventive maintenance periods in unreliable manufacturing systems, International Journal of Production Economics, 107, (2007)
[32] Kenne J-P., Gharbi A., Beit M., Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale, European Journal of Operational Research, 178, (2007)
[33] Smith Ch. H., Schaefer M.K., Optimal inventories for repairable redundant systems with aging components, Journal of Operations Management, Vol. 5, No. 3, (1985)
[34] Park Y.T., Park K.S., Optimal stocking for replacement with minimal repair, Microelectronics and Reliability, Vol. 25, No. 1 (1985)
[35] Sheu S-H., Liou Ch-T., Tseng B-Ch., Optimal ordering policies and optimal number of minimal repairs before replacement, Microelectronics and Reliability, Vol. 32, No. 7, (1992)
[36] Dohi T., Ashioka A., Kaio N., Osaki S., The optimal repair-time limit replacement policy with imperfect repair: Lorenz Transform approach, Mathematical and Computer Modelling, 38, (20003)
[37] Catuneanu V.M., Moldovan C., Popentiu Fl., Gheorghiu M. , Optimum system availability and spare allocation, Microelectronics and Reliability, Vol. 28, No. 3. (1988)
[38] Migdalski J. (ed.), Reliability handbook. Mathematical basis. (in Polish) WEMA, Warszawa 1982

Manuscript received by Editorial Board, February11th, 2011

Modele utrzymania systemu technicznego – rola wsparcia logistycznego, przegląd

S t r e s z c z e n i e

W artykule przedstawiono przegląd literatury z zakresu modelowania logistycznego wsparcia funkcjonowania systemu technicznego ze szczególnym uwzględnieniem modeli zaopatrzenia systemów technicznych w części wymienne. Autorzy przedstawili klasyfikację danych modeli, wyróżniając cztery podstawowe grupy: modele doboru optymalnych parametrów strategii sterowania zapasami przy ustalonej strategii obsługi profilaktycznej obiektu, modele doboru optymalnych strategii sterowania zapasami zapewniających maksymalną niezawodność obiektu, modele niezawodności magazynowanych elementów wymiennych oraz modele doboru optymalnej strategii sterowania zapasami obiektu technicznego z wielopoziomowym systemem obsługiwania. W artykule skupiono się na omówieniu pierwszej z wymienionych grup modeli. Następnie w artykule przedstawiono model nadsystemu z rezerwą czasową. Wykorzystano analizę komputerowej symulacji w celu oceny wpływu parametrów procesu zaopatrzenia (wielkości partii zamówienia, poziomu zamawiania, czasu dostawy) na zachowanie nadsystemu.