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Method of describing a catastrophic failure of an element of an aircraft

K e y w o r d s

Crack initiation, fatigue, limit state, fatigue crack, reliability, catastrophic failure, risk.

S ł o w a k l u c z o w e

Inicjacja pęknięcia, zmęczenie, stan graniczny, pęknięcie zmęczeniowe, niezawodność, uszkodzenie katastroficzne, ryzyko.

S u m m a r y

Failures resulting from fatigue processes are a dangerous type of aircraft damages.

This article presents an attempt to determine the probability of the occurrence of catastrophic failures of aircraft elements as a result of fatigue processes including basic stages, i.e. the crack initiation and the crack growth after the initiation in subcritical states.

The possibility to assess the probability of the occurrence of catastrophic failures in the function of the flying time is essential to develop control systems of a technical state of basic aircraft systems. In other words, it is essential for maintaining the required flight safety level. The probability of the catastrophic damage (failure) can be also considered as an element of the risk in the operation of aircraft.

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1. Introduction

During the operation of aircraft, the construction undergoes a degradation process as a result of random load, which leads to failure. Fatigue of construction is the process of degradation. Catastrophic failures are caused by fatigue process [2, 3, 4]. Catastrophic failures are random events in the process of aircraft operation. They are rare but fraught with consequences.

It is assumed that the process of formation of catastrophic fatigue failures is characterised (in some cases) by certain stages. A simple course of fatigue process includes the following three basic stages:

- Crack initiation,
- Crack growth in subcritical state, and
- The destruction of the construction element after the exceeding of the critical crack length.

The formation process of the risk of catastrophic failure (in a particular case) begins with the crack initiation, which leads to the formation of a crack of a particular length. This crack relates to relations describing the crack growth, for example, the Paris formula.

The period in which the process of the crack initiation takes place is the stage that precedes the fundamental process of the crack growth until the critical value is reached. The critical value involves the destruction of the construction. Therefore, the stage of the crack initiation can be treated as the first stage of the destruction of the construction after which there is the second stage including the crack growth until the critical value is reached.

Therefore, it can be assumed that a parallel reliability structure of the destruction of the construction element is formed. The structure includes the crack initiation, then the subcritical crack growth, and the third stage, i.e. the catastrophic destruction of the construction.

2. Determining the probability of the crack initiation as a random process

We assume that the crack initiation in the element is caused by the accumulation of the degradation of an internal structure of the element as a result of the changing load. The changing load leads to accumulation of fatigue symptoms in different parts of the element, for example, various kinds of „obstacles”.

We assume that, among places where effects accumulate, there is one leading place in which the crack initiation occurs as a result of the accumulation of fatigue effects. As an example, near this selected “obstacle” dislocation, accumulation takes place.

Let Ψ be a parameter that is used to measure accumulated destructive symptoms of fatigue of the element structure surrounded by obstacles. Therefore, we can assume that a prognostic parameter for measuring the chance of the crack occurrence (its initiation) is the parameter Ψ. We digitise the prognostic parameter Ψ in the following way: $E_0, E_1, E_2, ..., E_k, ...$ We define these points as states of the process of the increase in fatigue effects before the crack initiation as a result of the action of load. Accumulated fatigue effects in the surrounding of the obstacle favour the crack initiation.

We assume that, in case of each state, there is a specific probability of the crack occurrence (the crack initiation). The probability of the crack initiation increases along with the increase in the state E_i (i = 0, 1, 2, …).

Figure 1 presents the increase in fatigue symptoms in the surrounding of the obstacle as a result of load, which connects with higher and higher state. A factor that forces the change of a state is the probability of the occurrence of the load cycle $\lambda\Delta t$, where λ is the intensity of the occurrence of the load cycle. In each state, there is probability of the crack initiation.

$$q_k(t) = (\mu_0 + k\mu)\Delta t \tag{1}$$

where: μ_0 – the intensity of the crack initiation at the initial moment,

$k\mu$ – the intensity of the crack that depends on the state of accumulated fatigue effects.

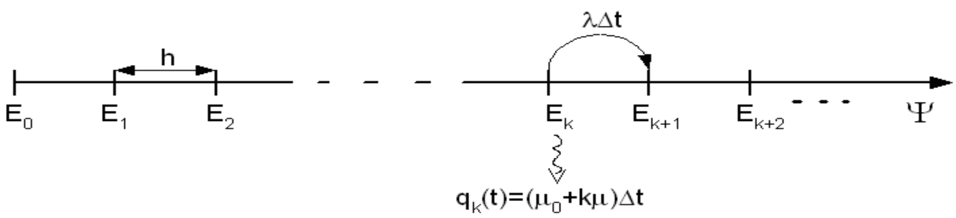

Fig. 1. Diagram of diagnostic parameter digitising: h – mean value of diagnostic parameter increase at the time Δt, $\lambda\Delta t$ – the probability of load cycle occurrence at the time Δt

Rys. 1. Schemat dyskretyzacji parametru diagnostycznego: h – średnia wartość przyrostu parametru diagnostycznego w czasie Δt, $\lambda\Delta t$ – prawdopodobieństwo pojawienia się cyklu obciążenia elementu w czasie Δt

Let $P_k(t)$ denote the probability that, at the moment t, the value of a diagnostic parameter reached the state E_k (where k = 0, 1, 2, …). Having the above assumptions, we can form the following set of equations (with infinite number of equations) [1, 5]:

$$P_0(t+\Delta t) = P_0(t)[1-(\mu_0+\lambda)\Delta t] + 0(\Delta t),$$
$$\vdots \qquad\qquad for\ k = 1,2,... \qquad (2)$$
$$P_k(t+\Delta t) = P_k(t)[1-(\mu_0+k\mu+\lambda)\Delta t] + P_{k-1}(t)\lambda\Delta t + 0(\Delta t)$$

After converting and dividing both sides of $k-$ equation by Δt with the transition to the limit $\Delta t \to 0$, we obtain the following set of equations:

$$P_0^{'}(t) = -(\mu_0+\lambda)P_0(t),$$
$$\vdots \qquad\qquad for\ k = 1,2,... \qquad (3)$$
$$P_k^{'}(t) = -(\mu_0+\lambda+k\mu)P_k(t) + \lambda P_k(t)$$

Initial condition for each of these equations can be written in the following form:

$$P_i(0) = \begin{cases} 1\ for\ i = 0 \\ 0\ for\ i \neq 0 \end{cases} \qquad (4)$$

Using a recursive method, we solve the set of equations (3).

Solution for *k* = 0

$$P_0^{'}(t) = -(\mu_0+\lambda)P_0(t),$$

$$\int_0^t \frac{P_0^{'}(t)}{P_0(t)}dt = -\int_0^t (\mu_0+\lambda)dt.$$

Hence,

$$P_0(t) = C_0\ e^{-(\mu_0+\lambda)t} \qquad (5)$$

For $t = 0,\quad P_0(0) = 1$ hence $C_0 = 1$.

Solution for any *k*

For any *k*, the differential equation has the following form:

$$P_k^{'}(t) = -(\mu_0+k\mu+\lambda)P_k(t) + \lambda P_{k-1}(t) \quad \text{for } k=1,2,\ldots \qquad (6)$$

In this case, we provide the following solution:

$$P_k(t) = C_k(t)\, e^{-(\mu_0+\lambda)t} \tag{7}$$

The derivative of the relation (7) has the following form:

$$P_k'(t) = C_k'(t)e^{-(\mu_0+\lambda)t} + C_k(t)\big(-(\lambda+\mu_0)\big)e^{-(\mu_0+\lambda)t} \tag{8}$$

Substituting the above equation into the relation (6), the following formula was obtained:

$$C_k'(t)e^{-(\mu_0+\lambda)t} - (\lambda+\mu_0)C_k(t)e^{-(\mu_0+\lambda)t} = \\ = -(\mu_0 + k\mu + \lambda)\underbrace{P_k(t)}_{C_k(t)e^{-(\mu_0+\lambda)t}} + \lambda \underbrace{P_{k-1}(t)}_{C_{k-1}(t)e^{-(\mu_0+\lambda)t}}$$

Hence, we obtain the following equation:

$$\begin{aligned} C_k'(t) &= -k\mu\, C_k(t) + \lambda C_{k-1}(t), \\ C_k'(t) + k\mu\, C_k(t) &= \lambda C_{k-1}(t) \end{aligned} \tag{9}$$

The equation (9) for *k*=1 will equal

$$C_1'(t) + \mu\, C_1(t) = \lambda \tag{10}$$

The general notation of the differential equation (10) has the following form:

$$y' + P(x)y = Q(x).$$

The solution of the relation is below:

$$y = e^{-\int_0^t P dx}\left(\int_0^t Q\, e^{\int_0^t P dx} dx\right) \tag{11}$$

Using Formula [11], we can write the solution of equation (10) in the following form:

$$C_1(t) = e^{-\int_0^t \mu\, dt}\left(\int_0^t \lambda\, e^{\int_0^t \mu\, dt} dt\right) = e^{-\mu t}\left(\int_0^t \lambda\, e^{\mu t} dt\right) = e^{-\mu t}\lambda \frac{1}{\mu} e^{\mu t}\Big|_0^t =$$

$$= e^{-\mu t}\,\frac{\lambda}{\mu}(e^{\mu t}-1) = \frac{\lambda}{\mu} - \frac{\lambda}{\mu}\, e^{-\mu t} \tag{11}$$

For $k = 2$, Equation [10] has the following form:

$$C_2^{'}(t) = 2\mu\, C_2(t) = \lambda\left(\frac{\lambda}{\mu} - \frac{\lambda}{\mu}\, e^{-\mu t}\right) \tag{12}$$

The solution of the Equation [12]

$$C_2(t) = e^{-\int_0^t 2\mu t} \int_0^t \lambda\left(\frac{\lambda^2}{\mu} - \frac{\lambda^2}{\mu}\, e^{-\mu t}\right) e^{\int_0^t 2\mu\, dt} dt = e^{-2\mu t}\left(\frac{\lambda^2}{\mu}\,\frac{1}{2\mu} e^{2\mu t} - \frac{\lambda^2}{\mu}\,\frac{1}{\mu} e^{2\mu t}\right)\Big|_0^t =$$

$$= e^{-2\mu t}\left(\frac{\lambda^2}{2\mu^2} e^{2\mu t} - \frac{\lambda^2}{2\mu^2} - \frac{\lambda^2}{\mu^2}\, e^{\mu t} + \frac{\lambda^2}{\mu^2}\right) = e^{-2\mu t}\left(\frac{\lambda^2}{2\mu^2}\, e^{2\mu t} + \frac{2\lambda^2 - \lambda^2}{2\mu^2} - \frac{\lambda^2}{\mu^2}\, e^{\mu t}\right) =$$

$$= \frac{\lambda^2}{2\mu^2} + \frac{\lambda^2}{2\mu^2}\, e^{-2\mu t} - \frac{\lambda^2}{\mu^2} e^{-\mu t} = \frac{\lambda^2}{2\mu^2}\left(1 + e^{-2\mu t}\right) - \frac{\lambda^2}{\mu^2}\, e^{-\mu t} \tag{13}$$

The equation describing the function was converted to the form that suggests the form of this function in a general case

$$C_2(t) = \frac{\lambda^2}{2\mu^2} + \frac{\lambda^2}{2\mu^2}\, e^{-2\mu t} - \frac{\lambda^2}{\mu^2} e^{-\mu t} \qquad | \cdot 2,$$

$$2C_2(t) = \frac{\lambda^2}{\mu^2} - \frac{2\lambda^2}{\mu^2}\, e^{-\mu t} + \frac{\lambda^2}{\mu^2} e^{-2\mu t}.$$

Hence,

$$C_2(t) = \left(\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t} \right)^2 \frac{1}{2} \tag{14}$$

The form of the equation (14) enables us to provide the notion of the function in a general form. This relation has the following form:

$$C_k(t) = \frac{1}{k!} \left(\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t} \right)^k \tag{15}$$

Using (15), we can write the solution of the equation (6). These solutions have the following form:

$$P_k(t) = \frac{1}{k!} \left(\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t} \right)^k e^{-(\mu_0 + \lambda)t} \qquad \mathrm{k} = 1, 2, ... \tag{16}$$

Using Relations (5) and (16), we can determine the reliability of the element (non-initiation of the crack). Hence:

$$R_1(t) = \sum_{k=0}^{\infty} P_k(t),$$

$$R_1(t) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t} \right)^k e^{-(\mu_0 + \lambda)t} \tag{17}$$

The following equality occurs:

$$\sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t} \right)^k = e^{\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t}} \tag{18}$$

Using the relation (18), the formula for the reliability has the following form:

$$R_1(t) = e^{\frac{\lambda}{\mu} - \frac{\lambda}{\mu} e^{-\mu t}} e^{-(\mu_0 + \lambda)t}$$

Hence, $$R_1(t) = e^{\frac{\lambda}{\mu}(1-e^{-\mu t})-(\mu_0+\lambda)t} \tag{19}$$

Based on the above relation, the probability of the crack initiation for the flying time t will equal

$$Q_1(t) = 1 - e^{\frac{\lambda}{\mu}(1-e^{-\mu t})-(\mu_0+\lambda)t} \tag{20}$$

3. Determining the relation for the crack growth after the occurrence of the crack initiation in the construction element

1) We assume the following [5, 6]:
   - After initiation, a small crack l_0 occurs in the construction element;
   - A technical state of the element is determined by one parameter in the form of the crack length. The current value of diagnostic parameter is marked with l;
   - The change of the crack length can occur only during the operation of a device;
   - In the analysed case, the Paris formula has the following form:

$$\frac{dl}{d N_z} = C M_k^m (\sigma_{\max})^m \pi^{\frac{m}{2}} l^{\frac{m}{2}} \tag{21}$$

where: c, m – material constants;
N_z – the variable meaning the number of cycles in the assumed load spectrum;
M_k – the coefficient of finiteness of dimensions of the element in the crack location;
$\sigma_{\max}$ – max. load that is determined by the relation (3).

2) It is assumed that a destructive factor is the load of the element in the form of the assumed load spectrum. We assume that this load spectrum enables determination of the following:
   - The total number of load cycles N_c during one flight (the standard cycle);
   - In the assumed spectrum, max. threshold loads are $\sigma_{\max}^1, \sigma_{\max}^2, \ldots, \sigma_{\max}^L$ (we assume that there is L-thresholds in the load spectrum);

– The number of repetitions of determined load threshold values equals n_i, where

$$N_c = \sum_{l=1}^{L} n_i \tag{22}$$

3) Max. values of loads for the assumed thresholds are determined in the following way:

$$\sigma^i_{\max} = \overline{\sigma}^i_{sr} + \overline{\sigma}^i_a \tag{23}$$

where: $\sigma^i_{\max}$ – max. value of load for *i*-threshold;

$\overline{\sigma}^i_{sr}$ – mean value of load for *i*-threshold;

$$\overline{\sigma}^i_{sr} = \frac{\sigma^i_{\max} - \sigma^i_{\min}}{2}$$

$\overline{\sigma}^i_a$ – the amplitude of cyclic load for *i*-threshold.

4) Values of threshold load $\sigma^1_{\max}, \sigma^2_{\max}, \ldots, \sigma^L_{\max}$ correspond to the following frequencies of their occurrence:

$$\frac{n_i}{N_c} = P_1; \quad \frac{n_2}{N_c} = P_2, \ldots, \quad \frac{n_L}{N_c} = P_L,$$

where: $P_1 + P_2 + \ldots P_L = 1.$

Based on the above assumptions, we will attempt to determine the form of the density function of the crack length that depends on the time of the operation of an aircraft (flying time).

The relation (21) can be represented in the form of the function of the flying time of an aircraft. For this purpose, we assume the following:

$$N_z = \lambda t \tag{24}$$

where: λ – the intensity of the occurrence of load cycles in the assumed spectrum.

t – the flying time of an aircraft.

In the assumed case

$$\lambda = \frac{1}{\Delta t}$$

where: Δt – mean time of fatigue cycle in the assumed spectrum.

We can assume a working formula for determining in the following form:

$$\Delta t = \frac{T}{N_c}$$

where: T – flight duration time of the standard cycle.

The relation (21) in the function of the flying time has the following form:

$$\frac{dl}{dt} = \lambda C M_k^m (\sigma_{\max})^m \pi^{\frac{m}{2}} l^{\frac{m}{2}} \tag{25}$$

The form of the solution of the equation (25) depends on the value of the index of the power *m*. In the considered case, we assume *m*=2. Hence, the equation (25) has the following form:

$$\frac{dl}{dt} = \lambda C M_k^2 (\sigma_{\max})^2 \pi l \tag{26}$$

The crack growth for the increase of the flying time Δt is:

$$\Delta l = \lambda C M_k^2 (\sigma_{\max})^2 \pi l \Delta t \tag{27}$$

Using the previous findings, we can determine the relation for the density function of the crack length in the function of the flying time of an aircraft. Let $U_{l,t}$ denote the probability that, at the moment *t* (for the flying time = *t*), the crack length will *l*. For the assumed notation, the dynamics of the crack length growth was described by the following differential equation:

$$U_{l,t+\Delta t} = \sum_{l=1}^{L} P_i U_{l-\Delta l_1}, t \tag{28}$$

where: $\Delta l_1 = CM_k^2(\sigma_{\max}^i)^2 \pi l \underbrace{\lambda \Delta t}_{1}; \qquad i = 1,2,...,L$ (29)

P_i – the probability of the occurrence of, provided that $P_1+P_2+\ldots+P_L=1$.

The Equation (8) in the functional notation has the following form:

$$U(l,t+\Delta t) = \sum_{l=1}^{L} P_i U(l-\Delta l_i, t) \tag{30}$$

We convert the equation (10) to a partial differential equation. We assume the following approximations:

$$u(l,t+\Delta t) \cong u(l,t) + \frac{\partial u(l,t)}{\partial t}\Delta t$$

$$u(l-\Delta l_i, t) = u(l,t) - \frac{\partial u(l,t)}{\partial l}\Delta l_i + \frac{1}{2}\frac{\partial^2 u(l,t)}{\partial l^2}(\Delta l_i)^2 \tag{31}$$

Substituting (31) into (30), we obtain the following:

$$u(l,t) + \frac{\partial u(l,t)}{\partial t}\Delta t = \sum_{i=1}^{L} P_i \left\{ u(l,t) - \frac{\partial u(l,t)}{\partial l}\Delta l_i + \frac{1}{2}\frac{\partial^2 u(l,t)}{\partial l^2}(\Delta l_i)^2 \right\}$$

$$\frac{\partial u(l,t)}{\partial t}\Delta t = -\frac{\partial u(l,t)}{\partial l}\sum_{i=1}^{L} P_i \Delta l_i + \frac{1}{2}\frac{\partial^2 (l,t)}{\partial l^2}\sum_{i=1}^{L} (\Delta l_i)^2 P_i$$

Hence

$$\frac{\partial u(l,t)}{\partial t} = -\underbrace{\frac{1}{\Delta t}\sum_{i=1}^{L} P_i l_i}_{\alpha(t)} \frac{\partial u(l,t)}{\partial l} + \frac{1}{2}\underbrace{\frac{1}{\Delta t}\sum_{i=1}^{L} P_i (\Delta l_i)^2}_{\beta(t)} \frac{\partial^2 u(l,t)}{\partial l^2} \tag{32}$$

where: $\alpha(t)$ – mean crack length growth per unit of time;

$\beta(t)$ – mean square of the crack length growth per unit of time.

The transformation of the coefficient $\alpha(t)$ of Equation (32):

$$\alpha(t) = \frac{1}{\Delta t}\sum_{i=1}^{L} \Delta l_i = \frac{1}{\Delta t}\sum_{i=1}^{L} CM_k^2 P_i(\sigma_{\max}^i)^2 \pi l \lambda \Delta t =$$

$$= \lambda CM_k^2 \pi l \underbrace{[P_1(\sigma_{\max}^1)^2 + P_2(\sigma_{\max}^2)^2 + ... + P_L(\sigma_{\max}^L)^2]}_{E[\sigma_{\max}^2]} =$$

$$= \lambda CM_k^2 \pi E[\sigma_{\max}^2] l \qquad (33)$$

where: $E[\sigma_{\max}^2]$ – the second moment of load of the construction element.

For the purpose of determining the relation for the crack length l from deterministic perspective, the following relation was used:

$$\frac{dl}{dt} = \lambda CM_k^2 \pi E[\sigma_{\max}^2] l.$$

Hence

$$\int_{L_o}^{l} \frac{dl}{l} = \int_0^t \lambda CM_k^2 \pi E[\sigma_{\max}^2] dt,$$

Therefore

$$l = l_o \, e^{\lambda CM_k^2 E[\sigma_{\max}^2] \pi t},$$

We will denote

$$CM_k^2 \pi = C_1$$

$$C_1 E[\sigma_{\max}^2] = \overline{C}_1 \qquad (34)$$

The relation for the coefficient $\alpha(t)$ has the following form:

$$\alpha(t) = \lambda \overline{C}_1 l_0 \, e^{\lambda \overline{C}_1 t} \qquad (35)$$

Acting in a similar way, we can determine the relation for the value of the coefficient $\beta(t)$. After transformations, the equation (32) has the following form:

$$\frac{\partial u(l,t)}{\partial t} = -\alpha(t)\frac{\partial u(l,t)}{\partial t} + \frac{1}{2}\beta(t)\frac{\partial^2 u(l,t)}{\partial l^2} \tag{36}$$

The special solution of the equation (36) is the density function of the crack length in the following form:

$$u(l,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(l-B(t))^2}{2A(t)}} \tag{37}$$

where: $B(t)$ – mean value of the crack growth for the flying time t;
$A(t)$ – the variance of the crack length for the flying time t.

For the material constant m=2, coefficients $A(t)$ and $B(t)$ are the solution of integrals:

$$B(t) = \int_0^t \alpha(t)\,dt = l_0(e^{\lambda \overline{C}_1 t} - 1) \tag{38}$$

$$A(t) = \int_0^t \beta(t)\,dt = \frac{1}{2} l_0^2\, \overline{C}_1\, \omega (e^{2\lambda \overline{C}_1 t} - 1) \tag{39}$$

where:

$$\omega = \frac{E[\sigma_{\max}^4]}{(E[\sigma_{\max}^2])^2}.$$

Using the previous findings, the reliability of the construction element is as follows:

$$R_2(t) \cong \int_{-\infty}^{l_{kr}} u(l,t)dl \tag{40}$$

where: $$u(l,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(l-B(t))^2}{2A(t)}}$$

Considering the Relations (38) and (39), we obtain the following form of the integrand in Relation (40):

$$u(l,t)=\frac{1}{\sqrt{2\pi\frac{1}{2}l_0^2\,\overline{C}_1\,\omega(e^{2\lambda\overline{C}_1 t}-1)}}\,e^{-\frac{(l-l_o(e^{\lambda\overline{C}_1 t}-1))^2}{l_0^2\,\overline{C}_1\,\omega(e^{2\lambda\overline{C}_1 t}-1)}} \tag{41}$$

We standardise the random variable l. As a result of standardisation, we obtain the new random variable „z". Its mean value equals 0 and its variance equals 1.

$$z=\frac{l-B(t)}{\sqrt{A(t)}}$$

After standardising the random variable, the Formula (23) has the following form:

$$R_2(t)\cong\int_{-\infty}^{\frac{l_{kr}-B(t)}{\sqrt{A(t)}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}dz \tag{42}$$

4. Final relations for the assumed stages of the crack development

The reliability of the element including the stages of the crack development is as follows:

$$R(t)=R_1(t)+\left(1-R_1(t)R_2(t)\right) \tag{43}$$

The unreliability, i.e. a specific risk of catastrophic failure is as follows:

$$Q(t)=Q_1(t)\cdot Q_2(t) \tag{44}$$

The relations (43) and (44) can be written in the following form:

$$R(t)=R_1(t)-(1-R_1(t))\int_{-\infty}^{l_d<l_{kr}}u(l,t)\,da \tag{45}$$

$$Q(t)=(1-R_1(t))\int_{l_{kr}}^{\infty}u(l,t)\,da \tag{46}$$

It can be presented that the stages of the crack growth form a parallel reliability structure. Failure of a parallel structure occurs when its all elements are damaged. Hence, it can be written as follows:

$$Q(t) = (1 - R_1(t)) \int_{l_{kr}}^{\infty} u(l,t)\, da,$$

$$R(t) = 1 - (1 - R_1(t)) \int_{l_{kr}}^{\infty} u(l,t)\, da.$$

Hence, we obtain the following relation (43):

$$R(t) = R_1(t) + (1 - R_1(t)) \int_{-\infty}^{l_{kr}} u(l,t)\, da.$$

Considering only the probability, the risk of the catastrophic failure of the construction element, including the crack, will be determined by Relation (46).

The value of the possibility of failures of the construction can be used to develop a control system of a technical state of an aircraft in the function of the flying time.

References

[1] Gerebach J.B., Kordoński Ch.B.: Modele niezawodnościowe obiektów technicznych. Wydawnictwo Naukowo-Techniczne, Warszawa 1968.

[2] Kałmucki W.S.: Prognozirowanije resursow detali maszin i elementów konstrukcji. Kiszinier, 1989.

[3] Smirnov N.N., Ickovicz A.A.: Obsłuzivanije i remont aviacjonnej techniki po sostojaniju. Transport 1980.

[4] Tomaszek H., Wróblewski M.: Podstawy oceny efektywności eksploatacji systemów uzbrojenia lotniczego. Dom Wydawniczy – Bellona, Warszawa 2001.

[5] Tomaszek H., Żurek J., Jasztal M.: Prognozowanie uszkodzeń zagrażających bezpieczeństwu lotów statków powietrznych. Warszawa 2008.

[6] Tomaszek H., Klimaszewski S., Zieja M.: Zarys probabilistycznej metody wyznaczania trwałości zmęczeniowej elementów konstrukcji z wykorzystaniem wzoru Parisa i funkcji gęstości czasu przekroczenia stanu granicznego. 35th International Scientific Congress on Powertrian and Transport Means – European KONES 2009, Zakopane 2009.

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Metoda opisu uszkodzenia katastroficznego elementu konstrukcji statku powietrznego

S t r e s z c z e n i e

Niebezpiecznym rodzajem uszkodzeń statków powietrznego są awarie konstrukcji na tle procesów zmęczeniowych.

W artykule podjęto próbę określenia prawdopodobieństwa powstawania uszkodzeń katastroficznych elementów konstrukcji w wyniku działania procesów zmęczeniowych, uwzględniając podstawowe etapy, tj. inicjacji pęknięcia elementu konstrukcji i rozwoju pęknięcia po inicjacji w stanach podkrytycznych.

Możliwość szacowania prawdopodobieństwa pojawiania się uszkodzeń katastroficznych w funkcji nalotu statku jest niezbędna dla opracowania systemów kontroli stanu technicznego podstawowych układów statku powietrznego dla zachowania wymaganego poziomu bezpieczeństwa lotów. Prawdopodobieństwo uszkodzenia katastroficznego (awarii) może być również przyjęte jako element składowy pojęcia ryzyka w eksploatacji statków powietrznych.