SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
2 (162) 2010

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Aircraft flight safety with the risk of failure during performance of an aviation task

Key words

Reliability, safety, risk, failure, safety system, event, probability.

Słowa kluczowe

Niezawodność, bezpieczeństwo, ryzyko, awaria, układ zabezpieczający, zdarzenie, prawdopodobieństwo.

Summary

This article presents the outline of a method for the assessment of aircraft flight safety with the risk of failure. Despite efforts, appliance failures can occur. Appliance failures result in dangerous situations during flight. Cases of failures contribute to actions that have initiated the incorporation of backup systems into operations. These systems are aircraft units designed to

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prevent dangerous situations during flight. Moreover, they enable saving either an aircraft from damage or the crew in case of military aircraft. Backup systems include the following events:
– remaining in a state of operational readiness;
– taking over the function of a basic system after its damage;
– enabling landing of an aircraft or saving a pilot's life.

The article describes the above mentioned events and presents formulas for determining the probability of these events and formulas for the assessment of aircraft flight safety with the risk of aircraft failure.

1. Introduction

Flight safety is one of the most important undertakings in military and civil aviation. Despite efforts of technical services, appliance failures may occur. These failures cause dangerous situations during flight and force safety systems to take over some functions. These systems are integral units of aircraft. Safety systems are designed to prevent dangerous situations. They enable saving either an aircraft or the crew (a pilot) in case of military aircraft.

Safety systems require a special treatment during aircraft operation. This comes down to the following:
- maintaining them in a suitable operational readiness in case of basic unit failure, and
- taking over basic unit functions at the right time that guarantees aircraft efficiency.

The effectiveness of safety systems depends mainly on the following:
1) their technical state at the moment of the need to use them (i.e. operational readiness),
2) their operational reliability during their usage under conditions provided by a manufacturer, and
3) security of opportunity to save an aircraft, i.e. landing.

The performance of a task by a safety system (or safety systems) concerns the following events [3]:

A – An event that a safety system (or safety systems) is mounted on an aircraft, where there is no damage at the moment of the need to use it, i.e. it is in its operational readiness.

B – An event that a safety system will operate without damage during failure and it will take over basic system functions at the right time, i.e. there will be a reliable takeover of functions of a damaged basic system by a safety system.

C – An event that safety systems with aircraft systems in working order will, and this enable a safe landing, i.e. saving an aircraft from destruction.

Based on the above issues, it shall be stressed that safety systems with aircraft systems in working order will perform an aviation task if all the above mentioned events occur and an aircraft is saved.

A measure of a pilot's effective behaviour is the probability of a task performance after the occurrence of failure. This probability can be determined in the following way:

$$P_s(\tau) = P(A) \cdot P(B/A) \cdot P(C/A \cap B), \tag{1}$$

where:

- $P_S(\tau)$ – the probability of a pilot's effective performance at a given time τ resulting from the scale of failure;
- $P(A)$ – the probability of the occurrence of the event *A* – safety systems will be ready to take over operation in case of danger;
- $P(B/A)$ – the conditional probability of the occurrence of the event *B* on the condition of the occurrence of the event A. The computational formula has the following form [1]:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}, \tag{2}$$

- $P(C/A \cap B)$ – the conditional probability of the occurrence of the event *C* (the landing of an aircraft) on the condition of the occurrence of the event *A* and *B*; The computational formula has the following form [1]:

$$P(C/A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)}. \tag{3}$$

Substituting the formulas (2) and (3) into the formula (1), we obtain

$$P_s(\tau) = P(A) \cdot \frac{P(A \cap B)}{P(A)} \cdot \frac{P(A \cap B \cap C)}{P(A \cap B)} = P(A \cap B \cap C). \tag{4}$$

Aircraft safety depends on aircraft reliability until the occurrence of failure and the effectiveness of saving an aircraft after failure. The effectiveness of saving an aircraft is determined by the formula (4), and including aircraft reliability, it can be presented in the following way:

$$P_{BS} = R(t) + \left(1 - R(t)\right)P_s(\tau), \tag{5}$$

where:

P_{BS} – the probability of aircraft safety with failure;

$R(t)$ – the reliability of an aircraft till the occurrence of failure (*t* means the time of aircraft flight);

$(1 - R(t)) = Q(t)$ – the probability of the occurrence of failure till the time *t*, i.e. in the range of flight $(0,t)$;

$P_s(\tau)$ – the probability of saving an aircraft after failure.

Based on the above data, the determination of aircraft safety comes down to the determination of the above mentioned probabilities. For the purpose of simplifying the notation, the term "a safety system" will mean both a safety system and safety systems in the further part of this article.

2. Determining the probability of the operational readiness of a safety system – *P(A)*

The operational readiness of a safety system is subject to a suitable control during aircraft operation via the use of appropriate diagnostic procedures. A safety system undergoes repair in case of deviation from requirements. Therefore, we can distinguish (a) the operational readiness of a safety system when all parameters do not diverge from requirements, and (b) the state of unreadiness when conditions are not met and the safety system is exposed to risk.

A diagram of the maintenance of the readiness state of a safety system is presented in Fig. 1.

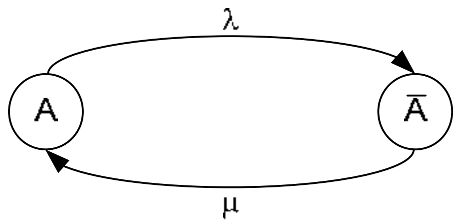

Fig. 1. Diagram of the maintenance of the readiness state of a safety system during aircraft operation: A – the readiness state of a safety system, $\bar{A}$ – the unreadiness state of a safety system, λ-intensity of loss of the readiness state of a safety system, μ-intensity of the restoring of the readiness state

Rys. 1. Schemat utrzymania stanu gotowości układu zabezpieczającego w procesie eksploatacji statków: A – stan gotowości układu zabezpieczającego, $\bar{A}$ – stan niegotowości układu zabezpieczającego, λ – intensywność utraty stanu gotowości układu zabezpieczającego, μ – intensywność przywracania stanu gotowości

Let $P(A,t)$ denote the probability of a safety system staying in the state "A," and $P(\bar{A},t) = 1 - P(A,t)$ the probability of a safety system staying in the state "$\bar{A}$." The time *t* is the time of a safety system staying in operation time. The probability $P(A,t)$ is the unknown quantity of the probability $P(A)$. The implemented notation aims at highlighting the variability of this probability in the function of time.

$$P(A,t) \equiv P(A).$$

The following equation of state is true [4]:

$$P(A,t+\Delta t) = [1-\lambda\Delta t]P(A,t) + \mu\Delta t P(\bar{A},t) + O(\Delta t), \quad (6)$$

where:

$\lambda\Delta t \leq 1$ – the probability of the loss of the readiness state at the time interval Δt;

$[1-\lambda\Delta t]$ – the probability of the lack of the loss of the readiness state of a safety system at the time interval Δt;

$\mu\Delta t$ – the probability of the restoring of the readiness state at the time interval Δt;

$O(\Delta t)$ – a small quantity of higher order.

After dividing both sides of the equation (6) by Δt and ordering the notion and after reaching the limit 0 by Δt, we obtain the following differential equation:

$$P'(A,t) = -\lambda P(A,t) + \mu P(\bar{A},t). \quad (7)$$

Substituting in the equation (7) $P(\bar{A},t) = 1 - P(A,t)$, we obtain the following equation:

$$P'(A,t) + (\lambda+\mu)P(A,t) = \mu. \quad (8)$$

The solution of the equation (8) is the probability of a safety system staying in the readiness state. It has the following form:

$$P(A,t) \equiv P(A) = \frac{\mu + \lambda e^{-(\lambda+\mu)t}}{\mu+\lambda}. \quad (9)$$

When $t \to \infty$, we obtain a stationary value of the probability of a safety system staying in the readiness state $P_{st}(A)$

$$P_{st}(A) = \frac{\mu}{\mu+\lambda} = K_g, \tag{10}$$

where: K_g - the readiness coefficient determined as the probability of a safety system staying in the readiness state.

It can be seen that the stationary value of the probability $P(A)$, i.e. $P_{st}(A)$, is the readiness coefficient K_g which is known from the theory of reliability.

3. Determining the probability of the event *P(B/A)*

For the purpose of determining the conditional probability *P(B/A)*, i.e. the occurrence of the event *B* under the condition of the occurrence of the event *A*, we will use deliberations from the Renewal Theory when renewal time is negligible. In our case, the probability *P(B/A)* will be determined by the reliability of a device at a particular time interval (in this case, after the occurrence of failure).

This characteristic is significant for aeronautical devices that, in operational state, perform their functions only in a limited time (in this case, after the occurrence of failure).

Therefore, we will determine the probability of a failure-free operation of a safety system at a finite time interval $(t, t+\tau)$, where τ is the time of flight duration after failure. We will denote this probability by the following:

$$R_t(\tau) = P(B/A). \tag{11}$$

For the purpose of determining the searched probability, a set of independent events is implemented [5]:

$$A_0 = \{t + \tau < T_1\},$$
$$A_n = \{t_n < t < t + \tau < t_n + T_{n+1}\} \quad n = 1, 2, \ldots, \tag{12}$$

where:

$n = 1, 2, \ldots$ – subsequent damages during the time of use of a safety,

T_i – random variables of duration of being in operational state (where $i=1, 2, \ldots$),

t_n – moments of damages and at the same time repairs of a safety system (*n=1, 2, …*) determined on the axis of time.

The event A_0 means that there was no damage of a safety system until the moment *t* and at the time interval $(t, t+\tau)$.

The event A_n means that there were *n* damages until the moment *t*, and there were no damages at the time interval $(t, t+\tau)$.

The probability $R_t(\tau)$ is the probability of the occurrence of the event that is determined in the following way:

$$B = \bigcup_{n=0}^{\infty} A_n, \tag{13}$$

that is as follows:

$$R_t(\tau) = P(B) = \sum_{n=0}^{\infty} P(A_n). \tag{14}$$

In Paper [2], it was showed that we obtain the following integral equation from the relation (14)

$$R_t(\tau) = 1 - F(t+\tau) + \int_0^t [1 - F(t+\tau-x)]h(x)dx, \tag{15}$$

where:

$F(t)$ – the distribution function of the time of correct operation of a safety system,

$h(x)$ – the renewal density function.

The formula (15) is seldom used in practice, because we usually are interested in distant moments of time when the renewal process becomes stationary; thus, the probability $R_t(\tau)$ is no longer dependent on *t*. Therefore, in the equation (15), we use the transition to the limit $t \to \infty$.

In this situation, the component $1-F(t+\tau)$ of the formula (15) approaches 0 as *t* increases. For the purpose of finding the limit of the integral the following formula is used:

$$\lim_{t\to\infty} \int_0^t [1 - F(t+\tau-x)]h(x)dx. \tag{16}$$

We will use the renewal Theorem [2]. This theorem has the following content: if the time τ of the operation of an element has a continuous distribution, and $Q(t)$ is a non-increasing monotone function and the integrable function is in the range *(0, ∞)*, then:

$$\lim_{t\to\infty}\int_0^t Q(t-\tau)\,dH(\tau)=\frac{1}{T_0}\int_0^t Q(x)\,dx, \tag{17}$$

where:

T_0 – the expected value of the operation time of an "element,"

$H(\tau)$ – the renewal function.

In our case, $Q(x)$ has the following form:

$$Q(x)=1-F(x+\tau). \tag{18}$$

Considering the above, from the formula (15), after transition *t*→∞, we obtain the following:

$$R(\tau)=\lim_{t\to\infty}R_t(\tau)=\frac{1}{\Theta}\int_0^\infty[1-F(x+\tau)]\,dx. \tag{19}$$

Hence:

$$R(\tau)=\frac{1}{\Theta}\int_\tau^\infty[1-F(\tau)]\,dt, \tag{20}$$

where: [– the mean value of time till the damage of a safety system.

For constant intensity of damages of a safety system $\bar{\lambda}$, the relation (20) has the following form:

$$R(\tau)=\frac{1}{\Theta}\int_\tau^\infty[1-F(t)]\,dt=\frac{1}{\Theta}\int_\tau^\infty\left[1-\left(1-e^{-\bar{\lambda}t}\right)\right]dt=$$

$$=\frac{1}{\Theta}\int_\tau^\infty e^{-\bar{\lambda}t}\,dt=\frac{1}{\Theta}\left(-\frac{1}{\bar{\lambda}}e^{-\bar{\lambda}\tau}\right)\Bigg|_\tau^\infty=e^{-\bar{\lambda}\tau}.$$

Therefore:

$$R(\tau)=e^{-\bar{\lambda}\tau}. \tag{21}$$

Using the relation (21) to assess the searched probability of the event *B* (i.e. that a safety system will perform a task), under the condition of the occurrence of the event *A* (i.e. that a safety system was in working order at the moment when it was needed), the final formula has the following form:

$$P(B/A) = R(\tau) = e^{-\overline{\lambda}t}. \tag{22}$$

4. Determining the probability of the event *C* under the condition of the occurrence of the event *A* and *B*

If the event *A* and *B* occurred, then conditions for the event *C* are created, i.e. the landing of an aircraft.

The event *C* means that all systems and devices that guarantee the landing of an aircraft will operate without failures. For the purpose of determining this probability, we will use the intensity of damage [5]:

$$\chi(x) = \lim_{\Delta x \to 0} \frac{P(x < X \leq x + \Delta x / x < X)}{\Delta x}, \tag{23}$$

where:

- X – the random variable of time until the damage during landing (during the event *C*),
- x – the current value of the time of the course of landing,
- $P(x < X \leq x + \Delta x / x < X)$ – the conditional probability of the damage at the time interval $x < X \leq x + \Delta x$ under the condition that the random variable X is greater than x,
- Δx – the increase of time during an aircraft landing.

The conditional probability can be presented in the following way:

$$P(x < X \leq x + \Delta x / X > x) = \frac{P(x < X \leq x + \Delta x)}{P(X > x)}. \tag{24}$$

Substituting (24) into (23), we obtain the following:

$$\chi(x) = \lim_{\Delta x \to 0} \frac{P(x < X \leq x + \Delta x)}{\Delta x P(X > x)}. \tag{25}$$

In the relation (25), after transition to the limit $\Delta x \rightarrow 0$, we obtain:

$$\chi(x) = \frac{f(x)}{R(x)} = \frac{-R'(x)}{R(x)}. \tag{26}$$

Hence, we obtain the following differential equation:

$$R'(x) + \chi(x)R(x) = 0, \tag{27}$$

where: $R(x)$ - the probability of the performance of the event C at the time interval *(0, x)*.

Assuming that the performance of the event C is $x = x_k$ and that the intensity of the damage during landing is the constant χ^*. Then

$$P(C/A \cap B) = R_{x_k} = e^{-\chi^* x_k}. \tag{28}$$

5. Final remarks

Using the obtained partial relations for the assumed events, we can provide combined relations for the safety of the flight of an aircraft with failure.

The effectiveness of saving an aircraft with a crew is determined with the formula (4). Hence, we obtain the following:

$$P_s(\tau) = \underbrace{K_g}_{P(A)} \cdot \underbrace{e^{-\bar{\lambda}\tau}}_{P(B/A)} \cdot \underbrace{e^{-\chi^* x_k}}_{P(C/A \cap B)}. \tag{29}$$

We obtain the following relation for the safety of the flight of an aircraft with failure:

$$P_{BS} = \bar{R}(t+\tau) = R(t) + (1 - R(t))P_s(\tau), \tag{30}$$

where:

$P_s(\tau)$ – determined by the relation (29),

$R(t)$ – the reliability of the flight of an aircraft till the occurrence of failure.

The above-presented outline of the assessment of the safety of an aircraft with a crew requires further analysis aimed at the improvement of the obtained relations.

If an aircraft cannot be saved, it is possible to save a pilot's life with the use of a safety system in the form of an ejector seat. Similar logic patterns can be used to assess the chances of saving a pilot's life [3].

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Manuscript received by Editorial Board, May 17th 2010

Bezpieczeństwo lotów statków powietrznych z ryzykiem awarii w czasie wykonywania zadania lotniczego

S t r e s z c z e n i e

W niniejszym artykule przestawiono zarys metody oceny bezpieczeństwa lotu statku powietrznego z ryzykiem awarii. Pomimo starań zdarzają się awarie sprzętu, które są przyczyną niebezpiecznych sytuacji w locie. Przypadki awarii sprzętu przyczyniają się do podjęcia działań mających na celu włączenie do pracy układów pełniących rolę układów rezerwowych. Układy te są zespołami statku powietrznego przeznaczonymi do przeciwdziałania niebezpiecznym sytuacjom w locie. Ponadto umożliwiają one bądź to ratowanie statku powietrznego przed zniszczeniem, bądź tylko załogi w przypadku wojskowych statków powietrznych. Z układami zabezpieczającymi wiążą się następujące zdarzenia:

– pozostawanie w stanie gotowości do użycia;
– przejęcie funkcji układu podstawowego po jego uszkodzeniu;
– umożliwienie lądowania statku powietrznego lub tylko ratowanie życia pilota.

W artykule określono te zdarzenia i przedstawiono wzory do wyznaczenia ich prawdopodobieństw. Mając określone zależności na prawdopodobieństwo tych zdarzeń, podano wzory na szacowanie bezpieczeństwa lotu z ryzykiem awarii statku.