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GRZEGORZ SZALA*

The modification of the generalised two-parametric fatigue characteristic based on haigh diagram conception

K e y w o r d s

Fatigue life, two-parametric characteristics, S355J0 steel.

S ł o w a k l u c z o w e

Trwałość zmęczeniowa, dwuparametryczne charakterystyki zmęczeniowe, stal S355JO.

S u m m a r y

The experimental verification of the generalised two-parametric fatigue characteristics has shown good compatibility between the calculation and the fatigue life examination results with use of constant amplitude load and variable asymmetry coefficients, according to the model based on the Haigh diagram conception. The level of compatibility depended on the durability range, and it decreased considerably for the low durability values (high-tension values).

The analysis of the experimental verification results indicated that the significant factor having an influence on the level of compatibility between the calculation and examination results is the material stress sensitivity coefficient that is present in the mathematical model. In this work, the modification of the generalised two-parametric fatigue characteristics based on the Haigh diagram conception, based on example of steel S355JO examination, is presented.

* University of Technology and Life Sciences in Bydgoszcz Prof. Kaliskiego Street 7, 85-789 Bydgoszcz, Poland, tel.: +48 52 340 82 95, fax: +48 52 340 82 71; e-mail: gszala@utp.edu.pl

Nomenclature:

A	–	elongation in %,
C	–	constant in formula describing the S-N curve for the fluctuating stress ($R = 0$),
C_0	–	constant in formula describing the S-N curve for the alternating stress ($R = -1$),
N	–	the number of cycles general designation (fatigue life),
N_0	–	the number of cycles of the fatigue life corresponding to fatigue limit,
$R = S_{min}/ S_{max}$	–	the stress ratio,
R_e	–	material plasticity limit in MPa,
R_f	–	general fatigue limit designation in MPa,
R_m	–	tensile strength in MPa,
R_0	–	fatigue limit for fluctuating stress ($R = 0$) for the number of cycle N_0 in MPa,
R_0^N	–	fatigue limit for the sinusoidal fluctuating stress ($R = 0$) for the number of cycles N in MPa,
R_{-1}	–	fatigue limit for the alternating stress ($R = -1$) for the number of cycles N_0 in MPa,
R_{-1}^N	–	fatigue limit for the sinusoidal alternating stress ($R = -1$) for the number of cycles N in MPa,
S	–	general stress designation in the specimen in MPa,
$S_a = 0,5(S_{max} - S_{min})$	–	stress amplitude in the sinusoidal cycle in MPa,
$S_m = 0,5(S_{max} + S_{min})$	–	the mean stress in the sinusoidal cycle in MPa,
$S_{max} = S_m + S_a$	–	the maximum stress in the sinusoidal cycle in MPa,
$S_{min} = S_m - S_a$	–	the minimum stress in the sinusoidal cycle in MPa,
Z	–	contraction in %,
m	–	exponent in the formula describing the S-N curve for the fluctuating stress ($R = 0$),
m_0	–	exponent in the formula describing the S-N curve for the alternating stress ($R = -1$),
ψ	–	material stress sensitivity coefficient for $N = N_0$,
ψ_N	–	material stress sensitivity coefficient for $N \neq N_0$.

1. Introduction

In work [1] the need to determinate the two-parametric fatigue characteristic in the calculations of fatigue life of the structural components, which, in the operation, conditions, have been subjected to random loads of a wide spectrum has been widely substantiated. Additionally, experimental verification has been made of the Heywood characteristics [2] and five models.

The models are marked with Roman numerals: I, II, III, IV, and V. From the experimental verification carried out on the specimens made of S355JO steel, it turned out that Model I has a lot of merits and that the compatibility between the calculation results of fatigue life according to this model and the examination results, apart from the low durability range ($N = 10^2$ - 10^4), is satisfactory, which corresponded to the high levels of the variable stresses.

The similar conclusions can be formed regarding examinations described the work [3] on the specimens made of D16CzATW aluminium air alloy. From the analysis of the verification results, it appeared that the material stress sensitivity coefficient ψ_N is the factor that has the significant impact on the compatibility between the calculation results and the examination results.

The aim of this work is to modify the generalised two-parametric fatigue characteristic based on the Haigh diagram conception, which means assuming a suitable relation between the value of the material stress sensitivity coefficient ψ_N and its fatigue life. The modified characteristics have been experimentally verified on the specimens made of S355JO steel.

2. Formulation of the problem

In work [1], the description of two-parametric fatigue characteristic $N(S_a, S_m)$ is given – Model I in the following form:

$$N = \frac{N_0 R_{-1}^{m_0}}{\left(S_a + \psi_N S_m\right)^{m_0}} \quad -\infty < R \le 0 \tag{1}$$

and

$$N = N_0 \left[\frac{R_{-1}(R_m + S_a - S_m)}{S_a R_m (1+\psi_N)} \right]^{m_0} \quad 0 < \mathrm{R} \le 1.0 \tag{2}$$

or, in the form which is more convenient to draw a contour line diagram (the contour line corresponds with the condition of the constant durability N for variables S_a and S_m):

$$\frac{S_a}{R_m} = -\psi_N \frac{S_m}{R_m} + \frac{R_{-1}}{R_m}\left(\frac{N_0}{N}\right)^{\frac{1}{m_0}} \quad -\infty < R \le 0 \tag{3}$$

and

$$\frac{S_a}{R_m} = \frac{R_{-1}}{R_m\left(\frac{N}{N_0}\right)^{\frac{1}{m_0}}\left(1+\psi_N\right) - R_{-1}}\left(1 - \frac{S_m}{R_m}\right) \quad 0 < \mathrm{R} \le 1.0 \tag{4}$$

In this article, the impact of the ψ_N coefficient on the fatigue calculations according to the formulas given above are the subject of the analysis.

The $\psi_N = \psi$ coefficient in various publications, e.g. [4], and it is dependent on the material and on the type of variable load. For example, for bending in extreme cases: $\psi = 0.07 - 0.23$, for axial load: $\psi = 0.05 - 0.19$, and for torsion: $\psi = 0 - 0.14$. The values in the low range apply to low-strength steel (e.g. steel 10), whereas the values in the high range apply to high-strength steel (e.g. thermally improved steel 36HNM).

For the high cycle fatigue (HCF), Formula [5] enables one to calculate the sensitivity coefficient of a material for $N \neq N_0$ ($\psi_N \neq \psi$), and the following form has been derived:

$$\psi_N = 2C_0^{\frac{1}{m_0}} C^{-\frac{1}{m}} N^{(\frac{1}{m}-\frac{1}{m_0})} - 1 \tag{5}$$

The application of the given relation to Formulas (1) to (4) causes the difference in the fatigue calculation results and the examination results, especially for the number of cycles N in the range 10^2 - 10^4, which has been pointed out in work [6]. Therefore, a more beneficial solution is the application of the experimental relation described by the following Formula in work [5], which has already been mentioned:

$$\psi_N = N^k \tag{6}$$

where k is the index exponent dependent on the material and variable load type.

The modification of Model I of the two-parametric fatigue characteristic means replacing the analytic solution to the stress sensitivity coefficient of a material ψ_N according to Formula (5) with the experimental relation Formula (6)

3. The experimental verification of modified Model I of the two-parametric fatigue characteristic $N(S_a, S_m)$

The modified two-parametric fatigue characteristic are marked in the following text as an IM model.

3.1. The calculation and examination results for steel S355JO

The static properties of steel S355JO are in Table 1, and the cyclic properties are in Table 2.

Table 1. The static strength properties of steel S355JO
Tabela 1. Statyczne własności wytrzymałościowe stali S355J0

	The static properties of steel S355JO				
	R_e	R_m	E	A_5	Z
	MPa	MPa	MPa	%	%
Average value	499.9	678.0	208159	17.2	59.8
Standard deviation	8.4	7.1	1306	0.99	0.9

Table 2. The cyclic mechanical properties of steel S355JO
Tabela 2. Cykliczne własności mechaniczne stali S355J0

Load type	equation form	Exponent	Absolute term	Fatigue limit	
				R_f	N_0
Alternating (R=-1)	$S_a^{m_0} \cdot N = C_0$	$m_0 = 12.33$	$C_0 = 1.156 \cdot 10^{36}$	$R_{-1} = 274$	10^6
Fluctuating (R=0)	$S_{max}^{m_0} \cdot N = C$	m = 15.92	$C = 6.163 \cdot 10^{48}$	$R_0 = 480$	10^6

From the data contained in work [5], the index exponent k in the Formula (6) is equal 0.1586.

The static and cyclic properties given above and the value of the index exponent k enables one to make fatigue calculations of the two-parametric fatigue characteristic according to Formulas 3, 4 and 6. These formulas for the analysed steel and data contained in Tables 1 and 2, after necessary transformation, assume the following form:

– Formula 1:

$$N = 10^{36} \frac{1{,}14147}{\left[S_a \left(1 - \psi_N \frac{R+1}{R-1} \right) \right]^{12{,}33}} \tag{1a}$$

– Formula 2:

$$N = 1{,}14147 \cdot 10^{36} \left[\frac{678 + S_a \dfrac{2R}{1-R}}{678 S_a (1 + \psi_N)} \right]^{12{,}33} \tag{2a}$$

– Formula 3:

$$S_a = \frac{840}{N^{0{,}081} \left(1 + \psi_N \dfrac{R+1}{R-1} \right)} \tag{3a}$$

– Formula 4:

$$S_a = \frac{185772}{221{,}14 (1 + \psi_N) + 274 \dfrac{2R}{1-R}} \tag{4a}$$

The results of the fatigue calculations, according to formulas given above for various stress values are depicted in the Table 3.

Column 2 depicts the values of the stress ratio coefficient, column 3 – N durability in cycles, for which the amplitude value S_a corresponding to fatigue strength of the specimen R_{-1}^N has been calculated. Column 4 shows the values of the material stress sensitivity coefficient calculated with use of Formula (6). Data from columns 2 through 4 provide the fatigue calculation for Model I (column 5) and for the model after the modification IM (column 6) possible. In order to compare the calculated values S_{ac} with the experimental data S_{aex}, the data from the examination derived from work [6] have been presented in column 7 of Table 3.

4. Analysis of the results of calculations and their experimental verification

The data contained in Table 3 can provide a graph of the two-parametric fatigue characteristics in the form of the contour line diagram. In Fig. 1, the following diagrams are depicted:

a – characteristics diagram according to Model I (column 5, Table 3),

b – characteristics diagram according to the modified IM model (column 6, Table 3),

c – diagram according to the experimental data (column 7, Table 3).

From the comparison of the diagrams, it turns out that the calculations, both for Model I and the modified IM model, result in higher values than the experimental ones. The only exceptions are the data for the stress sensitivity coefficient R = -1.0, for which the compatibility is total, which results from the assumptions accepted by the model design.

Table 3. The results of the fatigue calculations according to Model I and IM as well as experimental data for S355JO steel

Tabela 3. Wyniki obliczeń zmęczeniowych według modelu I i IM oraz dane doświadczalne dla stali S355JO

Lp.	R	N	Ψ_N	Calculations S_a acc. to model		Experimental data S_{aex}
				I	IM	
1	2	3	4	5	6	7
1	-3.0	10^4	0.23	448.4	449.0	350.0
2		10^5	0.162	361.8	359.4	300.0
3		10^6	0.115	292.4	290.7	265.0
4		10^7	0.08	236.7	236.5	250.0
5	-2.0	10^2	0.465	----------	---------	---------
6		10^3	0.35	----------	---------	---------
7		10^4	0.23	430.3	431.3	385.9
8		10^5	0.162	350.6	349.1	336.7
9		10^6	0.115	285.9	285.1	293.8

Lp.	R	N	Ψ_N	Calculations S_a acc. to model		Experimental data S_{aex}
				I	IM	
10		10^7	0.08	233.5	233.3	256.4
11	-1.25	10^2	0.465	600.4	609.8	559.2
12		10^3	0.35	495.0	449.2	475.0
13		10^4	0.23	408.3	399.1	403.5
14		10^5	0.162	336.8	336.4	343.1
15		10^6	0.115	277.9	277.5	291.1
16		10^7	0.08	229.3	229.0	247.6
17	-1.0	10^2	0.465	578.3	578.3	578.7
18		10^3	0.35	479.8	480.3	480.2
19		10^4	0.23	398.1	398.5	398.4
20		10^5	0.162	330.3	330.6	330.5
21		10^6	0.115	274.0	274.3	274.2
22		10^7	0.08	227.3	227.6	227.5
23	-0.5	10^2	0.465	500.7	500.8	473.6
24		10^3	0.35	439.2	429.7	419.4
25		10^4	0.23	370.3	369.8	371.4
26		10^5	0.162	312.1	313.4	328.9
27		10^6	0.115	263.0	264.0	291.2
28		10^7	0.08	221.5	221.1	257.9
29	0.0	10^2	0.465	434.1	394.7	433.5
30		10^3	0.35	375.6	355.4	375.5
31		10^4	0.23	325.1	323.6	324.9
32		10^5	0.162	281.3	284.2	281.2
33		10^6	0.115	243.4	245.7	243.3
34		10^7	0.08	210.6	210.5	210.6
35	0.5	10^2	0.465	289.6	182.4	175.0
36		10^3	0.35	261.9	173.5	165.0
37		10^4	0.23	237.8	165.1	151.0
38		10^5	0.162	216.9	154.6	149.0
39		10^6	0.115	199.0	142.5	136.0
40		10^7	0.08	183.6	130.0	125.0

Designation: R – the stress ratio $R = S_{min}/S_{max} = (S_m - S_a)/(S_m + S_a)$
Ψ_N – material stress sensitivity coefficient

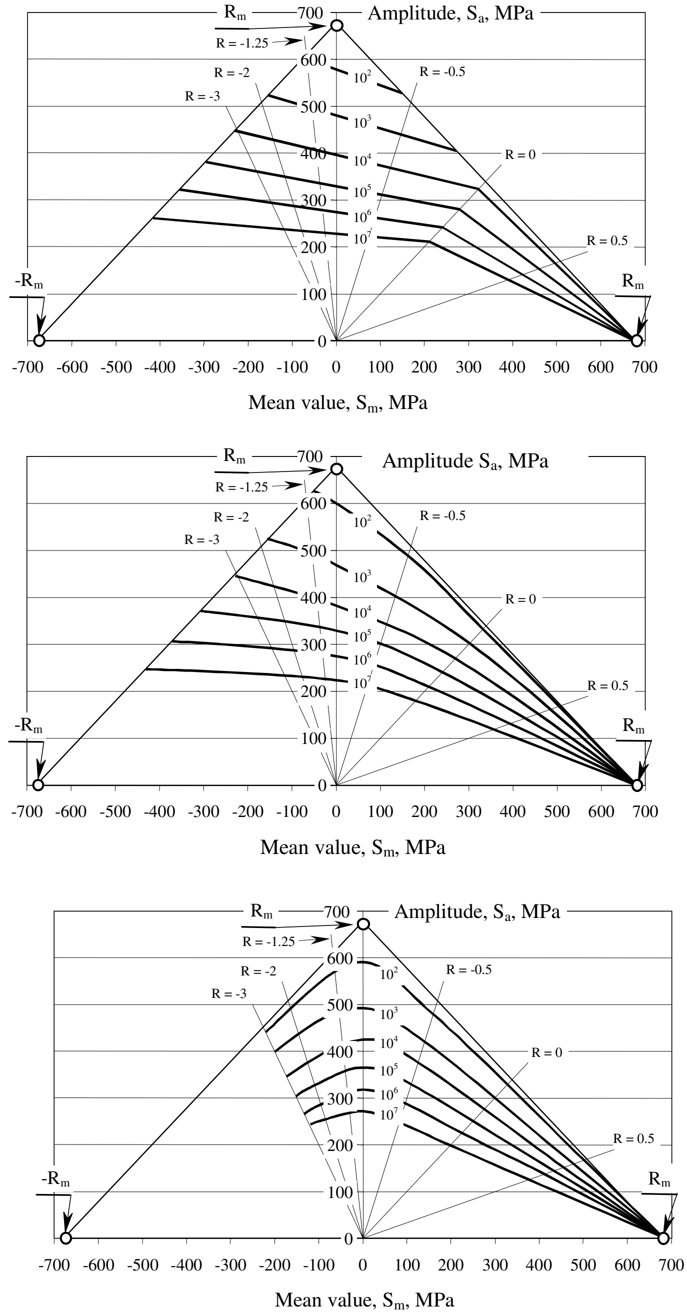
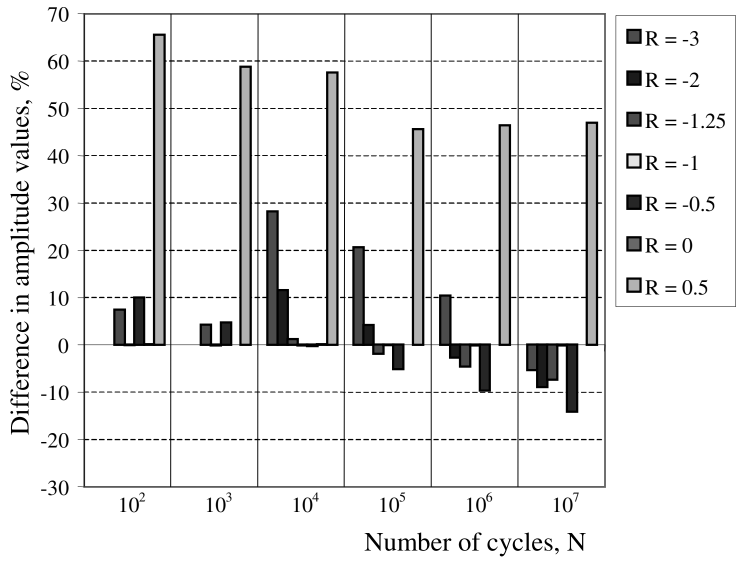


Fig. 1. Contour line diagrams of the two-parametric fatigue characteristics for the steel S355JO: a – according to Model I, b – according to the modified IM model, c – diagram determined on the base of experimental data

Rys. 1. Wykresy warstwicowe dwuparametrycznych charakterystyk zmęczeniowych dla stali S355JO: a – według modelu I, b – według zmodyfikowanego modelu IM, c – wykres wyznaczony na podstawie danych doświadczalnych



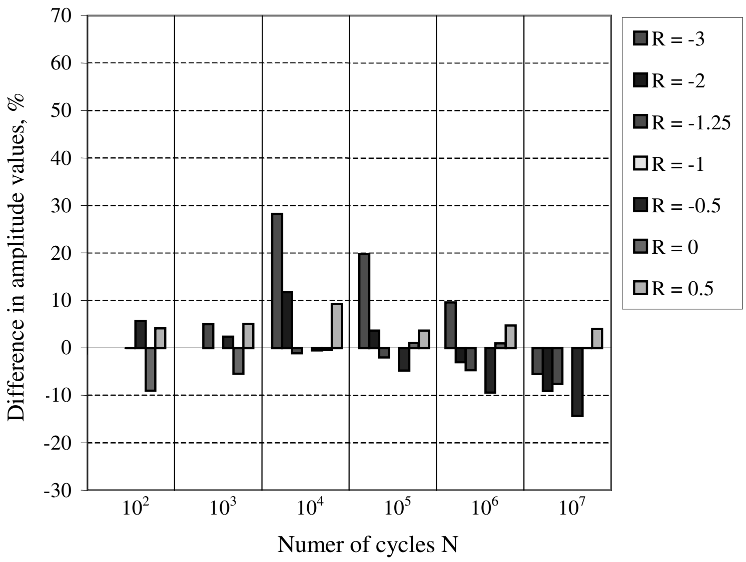


Fig. 2. Relative difference of the fatigue strength calculated and determined experimentally:
a – for Model I, b – for IM model
Rys. 2. Względna różnica wytrzymałości zmęczeniowej obliczonej i wyznaczonej doświadczalnie:
a – dla modelu I, b – dla modelu IM

Please notice that the shape of the diagram is determined using the experimental examination for the cycles with a negative average value (on the side of compressive stress). It is generally assumed that the negative values of the average stress S_m, increase the S_a amplitude values.

The methods to increase the fatigue durability of structural components with preload and technological treatment, introducing compressive stress in the cracking area are based on this assumption. The discussed observation requires further research supplemented with metallographic examination.

When the results of the calculations from Model I and the modified IM model are compared, their high compatibility in the range of the variability of the coefficient R from (-0.5) to (-3.0), and significant differences in the range of variability of the coefficient R from 0 to 1.0 are observed. The results of the calculations from the modified IM model are closer to the experimental data in this range.

The quantitative evaluation of the differences between the calculation and experimental examinations results has been carried out by introducing a relative difference measure calculated according to the following formula:

$$\delta = \frac{S_{ac} - S_{aex}}{S_{aex}} \cdot 100\% \tag{7}$$

The illustration of relative differences calculated according to Formula (7) is depicted in the Fig. 2. From the diagram (Fig. 2), it turns out that the maximal relative difference values δ are 65 % and apply to R= 0.5 and strength range N from 10^2 to 10^3 apply to Model I (Fig. 2a) and for 28 % R= -3 from….and N 10^4 – model IM (Fig. 2b).

From the data it turns out that especially in the range of the coefficient R variability from 0 to 1,0, as it was given above, the compatibility of the model IM calculation results with the experimental data is higher than in the Model I.

5. Conclusions

- The quantitative relative differences between the calculation and experimental examination results for different values of the material stress sensitivity coefficient R (-3; -2; -1.25; -1; -0.5; 0; 0.5) and different strength values N (10^2, 10^3, 10^4, 10^5, 10^6, 10^7) show that the results closer to experimental ones are received in calculations with use of the IM (modified) model. The significant improvement of the compatibility between the calculation and experimental examination results is observed for the values of R in the range 0 through 1.0. In the remaining ranges, the relative differences in the stress calculated according to comparable models are small.
- The application of the IM model in the fatigue calculation is more significantly beneficial, because the material stress coefficient value is calculated from empirical Formula (7). Therefore, it is not necessary to know

the parameters of the S-N curve for the cycle asymmetry coefficient R = 0 (fluctuating stress), which is necessary in the basic Model I, because the value of the material stress sensitivity coefficient is being calculated from Formula (5). Data about the S-N curve for various materials and for coefficient R = -1 (alternating stress) are widely available in publications, but data for the S-N curve for coefficient R = 0, are rare.

– The two-parametric fatigue characteristics have significant merits in the calculation of the service fatigue life in cases of broadband random service stress. Using these conditions, developed stress spectra are characterised by considerable dispersion of amplitude S_{ai} and average S_{mi} values of the sinusoidal cycles. In this situation, it is significant to evaluate the impact of the analysed differences between the models of the two-parametric fatigue characteristics on the calculated fatigue life. There are some cases in which small differences in the fatigue diagrams caused considerable differences in calculated fatigue life. The analysis of this problem is a subject for further research and calculations, which will be presented in the next publications.

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Modyfikacja uogólnionej dwuparametrycznej charakterystyki zmęczeniowej opartej na koncepcji wykresu zmęczeniowego Haigha

S t r e s z c z e n i e

Weryfikacja doświadczalna uogólnionych dwuparametrycznych charakterystyk zmęczeniowych wykazała dobrą zgodność wyników obliczeń z wynikami badań trwałości zmęczeniowej w warunkach obciążenia stałoamplitudowego i zmiennych współczynników asymetrii według modelu opartego na koncepcji wykresu zmęczeniowego Haigha. Poziom zgodności zależny był od zakresu trwałości i istotnie obniżał się dla niskich trwałości (wysokich wartości naprężeń).

Analiza wyników weryfikacji doświadczalnej wskazywała na to, że istotnym czynnikiem wpływającym na poziom zgodności wyników obliczeń i badań ma występujący w modelu matematycznym współczynnik wrażliwości materiału na asymetrię cyklu. W tej pracy przedstawiona zostanie modyfikacja uogólnionej dwuparametrycznej charakterystyki zmęczeniowej opartej na koncepcji wykresu zmęczeniowego Haigha na przykładzie badań stali S355JO.