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Self-heating effect in laminate plates during harmonic forced loading

K e y w o r d s

Self-heating, polymer laminates, diagnostics of laminates, model-based diagnostics.

S ł o w a k l u c z o w e

Samorozgrzewanie, laminaty polimerowe, diagnostyka laminatów, diagnostyka wsparta modelowo.

S u m m a r y

Laminate structures on a polymer base are widely used in many responsible applications. Therefore, behaviour of these structures must be predictable in different physical conditions and working loads. Behaviour of polymers can be described by an elastic rheological model only in static loading and specific thermophysical conditions, while, in the case of harmonic loading, it must be described using a viscoelastic model. Out-of-phase oscillations between stress and strain amplitudes cause energy dissipation, which introduce heating processes. An important phenomenon in polymers is self-heating. There are two approaches for the interpretation of the phenomenon: macromechanical, which base on energy dissipation, and micromechanical, which is based on the friction of broken polymer chains. The increase of self-heating temperature is dangerous in during exploitation in the case of polymers, because polymers possess a low glass transition temperature and a low heat transfer coefficient. The paper deals with three cases of the geometry of the structure: rectangular cantilever plate, circular clamped plate, and ring plate

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clamped on internal edge. These models are often used in many engineering solutions. The high-accuracy dependencies for laminate rigidity homogenisation are also presented. A solution to the heat generation problem in steady state is also shown. Furthermore, the possibility of obtaining self-heating temperature in non-steady state based on an approximate model is presented. Identification and evaluation of area of the laminates where the self-heating effect appeared allow one to specify the degradation degree evaluation and the residual life prediction of the laminate.

1. Introduction

Laminate structures on a polymer base are used more frequently in many responsible elements such as turbine blades, helicopter propellers, rotors, the bottoms of containers and many others. Therefore, the behaviour of the laminate must be predictable during all phases of exploitation and in different physical conditions and working loads. One important phenomena occurring during exploitation of laminates is the self-heating process. It is initialised during cyclic loading as an effect of dissipative energy. Polymers in a glassy state (such as thermoplasts) can be described by an elastic rheological model only in static loads, small strains, and specific thermophysical conditions. When time-varying loading is applied, polymers must be described by a viscoelastic rheological model. Therefore, oscillations of stress and strain amplitudes are out-of-phase (Fig.1) and cause energy dissipation in the form of heat. This phenomenon can create dangerous states in the exploited system or part, because most polymers are characterised by a low glass-transition temperature and a low heat transfer coefficient. In such conditions of operation, the internal temperature increases to the glass-transition temperature (Fig. 2), and this can cause catastrophic consequences [1]. Therefore, the parameters of dangerous states and the character of property changes of the laminate must be investigated and applied both in design and maintenance phases.

Research on the phenomena of self-heating in polymers was initiated by Ratner and Korobov [2, 3] in late 1960s. In these works, the authors experimentally investigated the character of time-temperature curves and proposed basic dependencies between heat generation and changing phase angle δ between stress and strain amplitudes. They postulated that the breakdown of polymer is characterised by the critical temperature, when δ reaches the maximum value [2] and applied it in Zhurkov's micromechanical fatigue model [3].

The main research on self-heating was made by Karnaukhov's scientific group from S. P. Timoshenko Institute of Mechanics of National Academy of Sciences of Ukraine [4–6]. In their research, several problems of coupled thermoviscoelasticity were investigated. In [5] the resulting constitutive equations were investigated in complete and approximate formulation of thermomechanical viscoelasticity and were presented for different geometrical models.

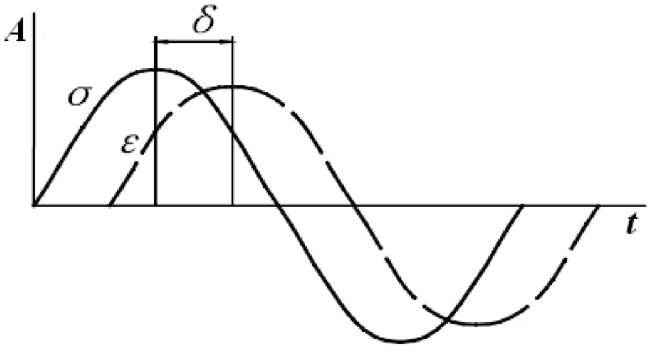

Fig.1. Out-of-phase stress-strain amplitudes of a viscoelastic material
Rys. 1. Niewspółfazowe amplitudy naprężeń odkształceń dla materiału lekkosprężystego

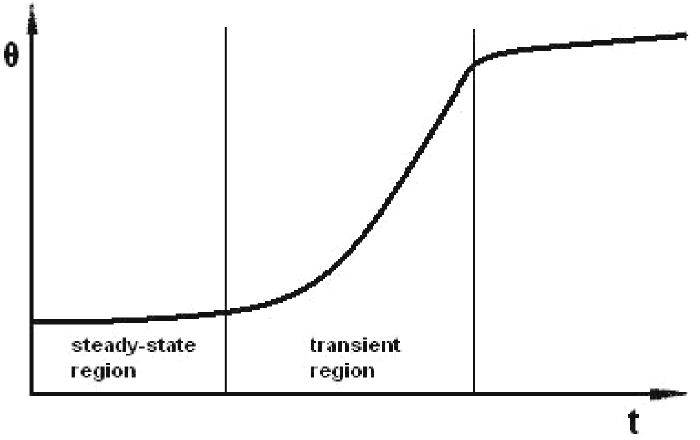

Fig.2. Self-heating temperature increasing during cyclic deformations [3]
Rys. 2. Wzrost temperatury samorozgrzania podczas cyklicznych odkształceń

The research on the mentioned phenomena was continued by Molinari's group from Paul Verlaine University [7, 8]. In [8], the dynamic thermomechanical response of a viscoelastic beam subjected to pure bending was described. The storage and loss moduli, according to the approximate formulation [5], as functions of frequency and temperature in steady states were assumed.

In this paper, the problem of self-heating was investigated on other types of geometry: a cantilever rectangular plate forced uniformly on the free edge and clamped circular plate with axial loading. As the research shows [1, 9, 10], there is a necessity to investigate these structures for predicting the degree of degradation of laminates. The geometrical structures under the author's consideration have important meaning in applied engineering solutions; therefore, their behaviour was investigated.

2. Problem formulation

Let us consider a cantilever rectangular transversal isotropic multilayer thin plate with k layers subjected to Kirchhoff's plate theory with length l, width b and thickness h. The plate is uniformly loaded by a harmonic force $P(t)$ on the free edge. The model can be simplified to a one-dimensional one, because, in case of pure bending, $u(x,0,t)=u(x,b,t)$ is constant, where t is time variable. The mechanical boundary and initial conditions can be presented by the following:

$$u(0,t)=0\,,\ \frac{\partial u(0,t)}{\partial y}=0\,;\qquad D\frac{\partial^3 u(L,t)}{\partial y^3}=-P(t)\,; \tag{1}$$

where D is substitutive flexural rigidity of laminate, and P is:

$$P(t)=P_0\sin\omega t. \tag{2}$$

For determining rigidity of the laminate plate according to the assumed simplification, the homogenisation can be used [1]. In case of isotropic materials, the flexural rigidity D can be expressed as follows:

$$D=\frac{Eh^3}{12\left(1-\nu^2\right)}, \tag{3}$$

where E is Young's flexural modulus and ν is Poisson's ratio.

In the transversal isotropy case, the flexural rigidity (3) becomes a matrix:

$$[D]=\begin{bmatrix} D_{11} & D_{12} & 0 \\ & D_{22} & 0 \\ sym. & & D_{33} \end{bmatrix},\qquad \left\{D_{ij}\right\}_k=\frac{1}{3}\sum_{k=1}^{n}\left[Q_{ij}\right]_k\left(z_k^3-z_{k-1}^3\right), \tag{4}$$

where Q_{ij} denote elasticity matrix coefficients, and z is the distance from the k-th lamina to the mid-plane of plate. For describing ply orientations different then 0^0, the cosine matrix must be taken into consideration. After several transformations, the substitutive flexural rigidity can be obtained.

$$D=\frac{1}{6}\sum_{k=1}^{n}\left(D_{11k}+D_{22k}+\frac{1}{6}D_{33k}\right). \tag{5}$$

Considering (4), and introducing the flexibility matrix and cosine matrix, we obtain the following formula based on material parameters:

$$D=\frac{1}{36}\sum_{k=1}^{n}\frac{z_k^3-z_{k-1}^3}{1-\nu_{12}\nu_{21}}\left[\cos^2\phi_k\sin^2\phi_k\left(E_1+E_2\nu_{21}(22+\nu_{21})-\frac{48G_{12}}{1-\nu_{12}\nu_{21}}\right)+\right.$$
$$\left.\cos^4\phi_k\left(E_1+E_2+\frac{G_{12}}{1-\nu_{12}\nu_{21}}\right)+\sin^4\phi_k\left(E_1+E_2-\frac{G_{12}}{1-\nu_{12}\nu_{21}}\right)\right]. \tag{6}$$

3. Viscoelastic behaviour of laminate

According to the viscoelastic behaviour of the harmonically loaded laminate plate, Boltzmann-Volterra equation (7) can be used for describing the behaviour of the material [11]:

$$E\varepsilon(t)=\sigma(t)+\int_0^t\Pi(t-\tau)d\tau, \tag{7}$$

where $\Pi(t-\tau)$ is a relaxation kernel. Let us consider the stress tensor:

$$\sigma_{ij}=s_{ij}-\frac{1}{3}\sigma_{ij}\delta_{ij}, \tag{8}$$

where s_{ij} is the stress deviator and δ_{ij} is the Kronecker's delta. In the investigated case, the subscripts $i=j=1$ under the assumption of pure bending, and (8) can be simplified to

$$\sigma_{11}=\frac{3}{2}s_{11}. \tag{9}$$

The deviatoric stress s_{11} and temperature dependence can be presented as (compare with [8]) while taking into consideration (6) and (7).

$$s_{11}=2D_e(\theta)\varepsilon_{11}+2\int_0^t D(t-\tau,\theta)\frac{d\varepsilon_{11}}{d\tau}d\tau. \tag{10}$$

The equilibrium modulus D_e depends only on temperature θ and does not depend on time, because, using the generalised Maxwell model of viscoelastic material, the time dependence of rigidity (in our case) can be presented as

$$D(t)=D_e+\sum_{i=1}^{I}D_i\exp\left(-\frac{t}{\tau}\right), \tag{11}$$

where I is the number of Maxwell elements. According to (11), $D(0)=\sum_{i=0}^{I} D_i$, $D_e = D$. The under-integral part $D(t-\tau,\theta)$ denotes the resolvent kernel in comparison with (7).

Taking into consideration (2), the strain in time dependency is expressed as

$$\varepsilon_{11}(u,t)=\frac{2u}{h}\varepsilon_0 \sin \omega t . \tag{12}$$

In cyclic loading, the temperature and strain will be oscillating equivalently. According to small evolution of temperature in one cycle, we can use the averaged value per cycle:

$$\theta_a = f\int_t^{t+T}\theta(t)dt, \tag{13}$$

where $f=\omega/2\pi$ denotes frequency and T denotes cycle period. Following the assumptions mentioned above, the deviatoric stress can be expressed as

$$s_{11}=\frac{4u}{h}\left[D_e(\theta_a)+\varepsilon_0\hat{D}(\omega,\theta_a)\right], \tag{14}$$

where

$$\hat{D}(\omega,\theta_a)=\int_0^{\infty} D(\vartheta,\theta_a)\exp(-i\omega\vartheta)d\vartheta, \quad \vartheta=t-\tau \tag{15}$$

is a complex rigidity, which decomposes in the form:

$$\hat{D}(\omega,\theta_a)=D'(\omega,\theta_a)+iD''(\omega,\theta_a), \tag{16}$$

$$\begin{aligned} D'(\omega,\theta_a)&=\omega\int_0^{\infty} D(t,\theta_a)\sin(\omega t)dt, \\ D''(\omega,\theta_a)&=\omega\int_0^{\infty} D(t,\theta_a)\cos(\omega t)dt, \end{aligned} \tag{17}$$

In Fig. 2, the complex rigidity and its components were presented on a hysteresis loop. The expression $(17)_1$ is an elastic energy component, and we will call it “stored rigidity,” and $(17)_2$ is the viscous energy component, which is dissipated as heat, and we will call it “loss rigidity.” Finally, after substitution (17) to (14), we obtain

$$s_{11} = \frac{4u}{h}\left[D_e(\theta_a) + \varepsilon_0\left(D'(\omega,\theta_a)\sin\omega t + D''(\omega,\theta_a)\cos\omega t\right)\right]. \tag{18}$$

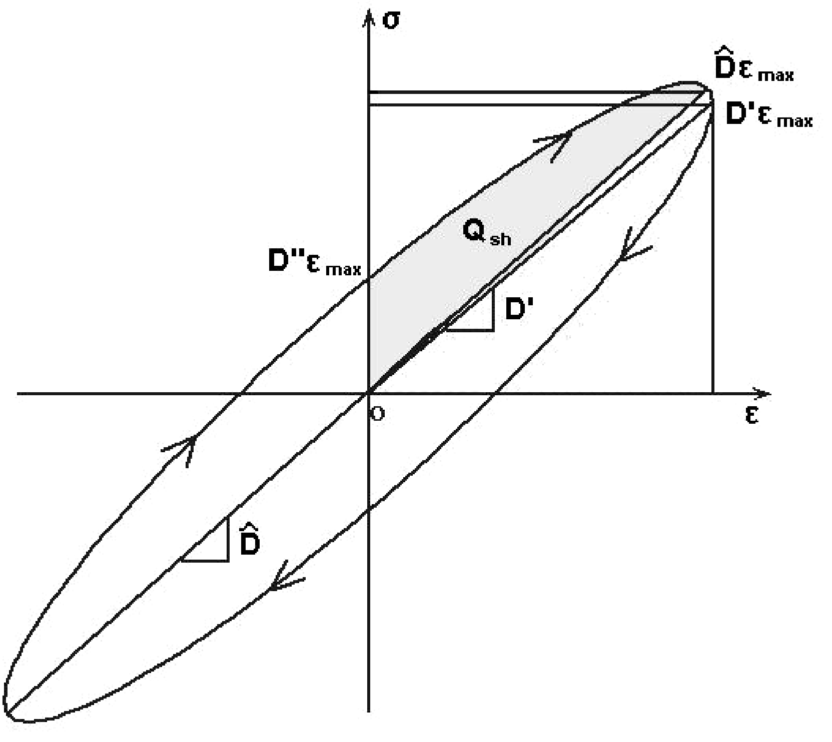

Fig.3. Typical hysteresis loop with complex rigidity, its components and dissipated energy
Rys. 3. Typowa pętla histerezy ze sztywnością zespoloną, jej składowymi i energią dyssypowaną

4. Heat transfer and temperature distribution

Heat transfer due to thermal processes in the laminate can be described by a linear heat equation with the assumption of homogeneity of the material (see Section 2) and thermal isotropy:

$$\rho c \frac{\partial \theta}{\partial t} - \lambda \frac{\partial^2 \theta}{\partial u^2} = Q_{sh}, \tag{19}$$

where ρ is the material density, c is the specific heat capacity, λ is the thermal conductivity, and Q_{sh} is an energy dissipated per unit time and unit volume. Here we assumed that the thermomechanical properties in (19) do not depend on temperature.

Let us consider the energy dissipation from a mechanical point of view. We can write the next expression according to the area of a hysteresis loop:

$$Q_{sh} = f \int_0^T \sigma_{11} \dot{\varepsilon}_{11} dt . \tag{20}$$

The heat conduction with surroundings, i.e. thermal boundary conditions, can be presented by

$$\begin{gathered} \theta = \theta_r \quad \text{for} \quad u = 0, \\ \lambda \frac{\partial \theta}{\partial u} = -\alpha(\theta_a - \theta_0) \quad \text{for} \quad u = \pm \frac{h}{2}, \end{gathered} \tag{21}$$

where α is the heat transfer coefficient, θ_0 and θ_r are an external temperature and reference temperature, respectively, and h is the thickness of the plate.

By substituting (9) and (18) to (20), the cycle-averaged dissipated energy calculates as

$$Q_{sh} = 6\omega\varepsilon_0^2 u^2 h^{-2} D''(\omega, \theta_a). \tag{22}$$

According to the experimental research presented in [2], the temperature evolution of dissipated energy can be divided into three regions (see Fig. 2). The first one corresponds to the steady state, when the balance between heat from energy dissipation and heat conducted to surroundings takes place. The second one is for the non-steady state, when the heat from dissipated energy is growing (when stiffness is decreasing). In the last region, the temperature has stabilised and demonstrates the linear behaviour. As it can be noticed, the non-stationary temperature evolution is determined by the loss of rigidity, i.e. the heat equation (19) can be reduced to a steady state form for the first and third regions. Taking into consideration the self-heating energy (22), the heat transfer equation for the presented problem was constructed:

$$\lambda \frac{d^2\theta}{du^2} + 6\omega\varepsilon_0^2 u^2 h^{-2} D''(\omega, \theta_a) = 0 . \tag{23}$$

Taking into account the boundary conditions (21), the governing equation (23) was solved as follows:

$$\theta(u) = \frac{3}{2} D''(\omega, \theta_a) \frac{\omega\varepsilon_0^2 u^3 (2u - h)(1 - \alpha\lambda^{-1})}{h^2(\lambda - \alpha)} + \frac{2u\theta_r}{h(\lambda - \alpha)} \left(1 + \alpha - \alpha\frac{\theta_0}{\theta_r}\right) + \theta_r . \tag{24}$$

The non-stationary temperature evolution (second region) occurs when the rigidity loss increases. The critical point of temperature evolution is the value of a glass-transition temperature θ_g. We seek the state where $\theta_a \leq \theta_g$. Based on FEM model [12], the temperature in non-steady state can be obtained using the following dependence:

$$\theta(t', \omega) = \theta_g - f \exp(-gt'), \tag{25}$$

where $f = 34.04$ and $g = 2.3987 \cdot 10^{-5}$ are approximation constants, t` is the time variable. The rigidity loss function in the non-steady state can be determined by substitution (25) to (24).

5. Other geometry cases

Similarly for the investigated phenomena for cantilever rectangular plate, we can obtain solutions by analogy for circular plates.

Let us consider a circular transversal isotropic multilayer thin plate with k layers clamped on the edge, according to Kirchhoff-Love's plate theory, with radius R and thickness h loaded by concentrated harmonic force $P(t)$ in the centre. According to the axi-symmetrical problem, boundary conditions can be presented as follows:

$$u(R) = 0; \quad \frac{\partial u(R)}{\partial y} = 0; \quad D\frac{\partial^3 u(0)}{\partial y^3} = -P(t); \tag{26}$$

where $P(t)$ is defined by (2).

In the same way, we can present boundary conditions for a ring plate with thickness h clamped on the internal edge with radius R_1 and loaded by force $P(t)$ distributed on the external edge with radius R_2:

$$u(R_1) = 0; \quad \frac{\partial u(R_1)}{\partial y} = 0; \quad D\frac{\partial^3 u(R_2)}{\partial y^3} = -P(t). \tag{27}$$

The investigated phenomenon is applicable to other one-dimensional systems or systems, which can be reduced to one-dimensional systems.

6. Conclusions and remarks

The superposition method was used for obtaining the dissipation energy and self-heating temperature in plate laminate structures in steady and non-steady states in cross-section and axial loading. The new homogenisation method of laminate stiffness, while taking into consideration variable laminas orientations, was proposed, which gives high-accuracy results (near 0.5% relative error). The viscoelastic behaviour of the laminate was determined by the complex rigidity, where the initial rigidity was determined by laminate's homogenised rigidity and used for obtaining self-heating energy. The steady state expression for temperature in the coupled problem was obtained and non-steady state was determined by the approximate solution of the numerical model. To verify the obtained solution, it is necessary to investigate the complex rigidity variability experimentally in different temperatures and loading frequencies. Also, it is shown that the proposed solution can be applied for other geometry structures, e.g. circular clamped plate or ring plate in case of axi-symmetrical loading, and generally it can be applied for one-dimensional systems and systems reduced to one-dimension.

The most interesting for the presented phenomena is the coupled problem solution in the horizontal section of laminate, which can be obtained by solving the two-dimensional heat transfer represented by Poisson's partial differential equation. The next problem is to take into consideration the non-ideal contact of the laminas in the coupled problem.

The obtained dependencies can be applied in residual life evaluation as well and used in the model-based diagnostics for polymer laminates. In addition, the understanding and describing the self-heating effect allows one to specify diagnostics and monitoring methods for laminates..

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Efekt samorozgrzewania w płytach laminatowych podczas harmonicznych obciążeń wymuszonych

S t r e s z c z e n i e

Struktury laminatowe o osnowie polimerowej są szeroko stosowane w wielu odpowiedzialnych aplikacjach. Dlatego zachowanie tych struktur powinno być przewidywalne w różnych warunkach fizycznych i warunkach obciążeń. Polimery mogą być opisane sprężystym modelem reologicznym tylko przy statycznym obciążaniu i odpowiednich warunkach termofizycznych. W przypadku obciążeń harmonicznych polimery powinny być opisywane modelem lepkosprężystym. Niewspółfazowość oscylacji pomiędzy amplitudami naprężeń i odkształceń powoduje dyssypację energii, co jest przyczyną generowania ciepła. Ważnym zjawiskiem w polimerach jest samorozgrzanie, które wynika z dyssypacji energii w ujęciu makromechanicznym lub z tarcia pomiędzy zerwanymi łańcuchami polimerowymi w ujęciu mikromechanicznym. Wzrost temperatury samorozgrzania jest niebezpieczny ze względów eksploatacyjnych w przypadku polimerów, gdyż charakteryzują się one niską temperaturą zeszklenia oraz niskim współczynnikiem przewodności cieplnej. Przy wzroście temperatury własności materiałowe stuktury maleją aż do jej zniszczenia. W pracy rozpatrzono trzy przypadki geometrii struktur: prostokątna płyta jednostronnie utwierdzona, okrągła płyta utwierdzona na brzegu oraz płyta pierścieniowa utwierdzona na wewnętrznym promieniu. Takie modele często używa się w wielu rozwiązaniach inżynierskich. W rozważaniach zaprezentowano zależności dla homogenizacji sztywności laminatu o wysokiej dokładności. Zostało przedstawione rozwiązanie zagadnienia generowania ciepła w stanie ustalonym, a także przedstawiono możliwość otrzymania temperatury samorozgrzania w stanie nieustalonym na podstawie modelu aproksymacyjnego. Identyfikacja i oszacowanie obszaru laminatu objętego efektem samorozgrzania umożliwia sprecyzowanie oceny stopnia degradacji i predykcji wytrzymałości resztkowej laminatu.