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Semi-Markov process for a pair of elements

Key words

Semi-Markov process, pair of elements, stochastically uniform utilization, steady-state availability.

Słowa kluczowe

Proces semi-Markowa, para elementów, równomierne stochastycznie użytkowanie, graniczny współczynnik gotowości.

Summary

In the paper a problem of determination of basic reliability parameters and characteristics for a system of pair of elements is considered. Different methods of operation of the system are discussed, however one method was chosen for further analysis as the most convenient one from practical point of view. It was presumed that the system operates following semi-Markov scheme. Basing on that presumption reliability characteristics were constructed and the steady-state availability of the system as well. Because the system consisted of two elements only, Authors indicated that it will be convenient to consider a system of two identical series systems operating in parallel with stochastically equal utilization.

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Introduction

One of the elementary systems that was the point of interest of theoreticians several times over the years is the pair of elements (e.g. Gnyedenko 1964, 1969, Gnyedenko et al. 1965, Kopociński 1973). The system consists of two identical elements (e_1, e_2) being in parallel in the reliability sense (Fig. 1).

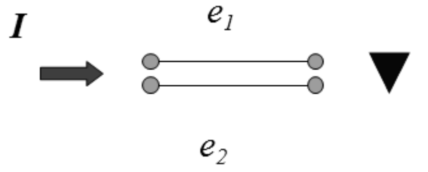

Fig. 1. Pair of elements system
Rys. 1. Para elementów

One element executes its duties and the second one is a reserve. Each unit can be in two of its own states: work and repair and in one state being a result of the system existence – standstill/reserve.

Generally, there are three problems associated with an operation of this system, namely:

(a) An intensity of failures of the spare element,
(b) The selection of a method of the system utilisation, and
(c) A manner of the system modelling and calculation.

Problem (a) was discussed at the very beginning of the problem creation. The general case was formulated assuming the additionally possibility of a failure for an element in the reserve. It means that the reserve is of a warm type. If the intensity of failures for an element being a spare is identical as for element in work, it means we have a hot reserve. The reserve is a cold one if the intensity of failures equals zero or the intensity is negligible.

Some interesting results were presented in cited papers in connection with different types of the reserve; however, they were mainly in a shape of the Laplace transforms. However, it appears that the most important result, especially from a practical point of view, is that one assuming a cold type reserve and is constant for the intensity of element failure. Notice that it is the simplest case from theoretical point of view, because the process of changes of states for the system is a Markov type.

Analysing real pairs of elements operating in different systems, reliability engineers have discovered that a vital problem to consider is a manner of the system utilisation, since rule three different ways of the system operation were taken into account (e.g. Czaplicki and Lutyński 1987)[1]:

[1] This problem was recalled in Czaplicki's monograph of 2010 with more detailed discussion.

(1) **A symmetric pair.** One element works; a second one is in a reserve – a cold one. When a failure occurs in the working element, the second element commences its duties without delay. The first element is in a repair state. When the repair is finished the first one becomes the reserve. This situation exists until the moment when failure occurs in the second element. The situation is then reversed. A failure of the system occurs when a failure occurs in the working element during the repair of the other.

(2) **A pair in order.** One element executes its duties; a second one is in reserve – a cold one. When failure occurs in the working element, the second element commences its duties. The first element is in a repair state. When the repair is finished – a renewal occurs – this repaired element restarts its duties again. The second element becomes a reserve, a backup. Failure of the system is the same as in point (1). This system is a hierarchical one.

(3) **Elements half loaded**. Let us assume that a stream of mineral is delivered to the system. Instead of fully loading one element, both elements carry half of the load. The idea of this solution is that a half-loaded element should have a higher reliability, perhaps with a mean work time significantly longer. Recently carried in-field research in underground coal mines allows stating that is better when the belt conveyor is not fully loaded, but the speed of the belt should be higher to obtain required output. This finding comes across by way of the system utilisation. Obviously, when one element is in failure, the second one takes the full load. Failure of the system is the same as in (1) and (2).

An important question can be formulated here. Which solution is the best one? Some more in-depth questions may be the following: What kind of changes in the system parameters can be observed after the application of the reserve? What is the reliability of this system?

Let us discuss these modes of system operation taking into account the experience gained from mining practice.

The main idea of the last proposition (3) is that half-loaded elements will have higher reliability. This higher reliability will pay for almost double element utilisation, and additionally, will earn a profit.

Research investigations in this regard have shown that this increase in reliability is usually small and the operational cost is almost doubled compared to the solution with a cold reserve. In some special cases, the profit due to application of this type of utilization of the system can be significant; however, this method generally is not recommended.

Utilisation of the system ‘a pair in order’ generates at least two problems. One element is in reserve, and it does not work for the majority of the time. If a belt conveyor for example (or other mechanical device) is in a standstill state for a longer time some troublesome processes are observed. Re-starting generates problems. The intensity of failures during this operation is significantly higher than during regular transportation. This means that problems occur when they should not. A second worrying property is connected with the fact that, after a longer period of time of the system's operation, one conveyor may become worn out, the second one that still almost new becomes old but in a different sense. These two elements turn out to be different in the sense of their properties. They are not identical from a reliability point of view. Generally, this solution is impractical if the elements are mechanical ones because of the ageing process. If such a system consists of electronic items, these annoying phenomena are rather not observed. But we are not analysing electronic systems here. For these reasons this way of system utilisation is also not recommended.

The third solution – a symmetric pair – looks most practical at first glance. Elements wear out at the same intensity and during a longer period of time, and the total work time will be approximately the same for both elements. However, for such pieces of equipment as belt conveyors, this method of utilisation is unsuitable because conveyors are 'too reliable'. Failures occur rarely and the element being held in reserve is frequently in this state for a long time. If this happens, several annoying phenomena can be observed (greater belt sag between idlers, local belt deformations, etc.). Generally, it is a well-known fact that it is not good to keep a mechanical system in a standstill state for a longer period of time. For these reasons a fourth solution, a fourth method (4) of the system utilisation is the best one – to switch an element being in reserve to work, not waiting for a failure to occur. If this action is repeated periodically with appropriate frequency, and the reliability of both elements will be the same and failure problems connected with re-starting are eliminated to great extent. In some cases, the intensity of the failures of elements is slightly reduced. It is worth noting that this method of system utilisation is equivalent to a symmetric pair in a reliability sense. Therefore, such a solution will be taken into further considerations.

Method of modelling

Having some idea about the behaviour of the element in reserve and knowing which method of utilisation of the system should be applied, we can now consider a method of modelling and analysis of the system operation.

Starting from the early seventies of the previous century, a Markov process was employed as a rule. The only reason for this situation was quite obvious – it was the only theoretical model ready to use in those days. To clarify the situation, experimental research on many machines and mechanical devices gathered data allowed researchers to verify statistical hypothesis stating that the empirical distribution of work time between two neighbouring failures can be satisfactory described by exponential distribution. For times of repair, the situation was different. In some cases, exponential distribution was appropriate to describe empirical data, but in many cases it was not. For these reasons, application of Markov processes was rational in some instances but in some other instances it was not.

Normally, before the selection of a model, conditions for the selection of a model were tested in order to check whether a given model could be used. An important issue was the verification of a stipulation that times of states are independent. And in the majority of cases, this condition was fulfilled.

Therefore, besides a Markov process, a semi-Markov process should be considered when at least one probability distribution is not exponential.

The theory of semi-Markov processes was introduced by Levy (1954) and Smith (1955). Takács (1954, 1955) considered similar processes. The foundations of the theory of semi-Markov processes were mainly laid by Pyke (1961a, b), Pyke and Schaufele (1964), Çinclar (1969) and Korolyuk and Turbin (1976). Recently, several new publications were issued such as Bousfiha et al. (1996, 1997), Limnios and Oprişan (2001), and Harlamov (2008).

Let us then consider the application of a semi-Markov system to find the basic reliability characteristics of system analysis.

Semi-Markov approach

Consider an exploitation repertoire for the process of changes of states for the system analysed. Each element can be in three states, namely work (W), repair (R), and standstill in reserve (S). Therefore, the set of theoretically possible states consists of $2^3 = 8$ elements; however, the system can technically be in five states only. They are as follows:

$$\{ s_1, \ldots, s_5 \} = \{ WS, WR, SW, RW, RR \}.$$

An exploitation graph is shown in Fig. 2.

Knowing both probability distributions, that is the probability distribution $F(t)$ of work time and the probability distribution $G(t)$ of repair time, we are able to calculate the following passage probabilities:

$$p_{43} = 1 - p_{45} = \int_0^\infty [1 - Q_{45}(t)] q_{43}(t) dt$$

$$p_{21} = 1 - p_{25} = \int_0^\infty [1 - Q_{25}(t)] q_{21}(t) dt$$

$$p_{52} = 1 - p_{54} = \int_0^\infty [1 - Q_{54}(t)] q_{52}(t) dt .$$

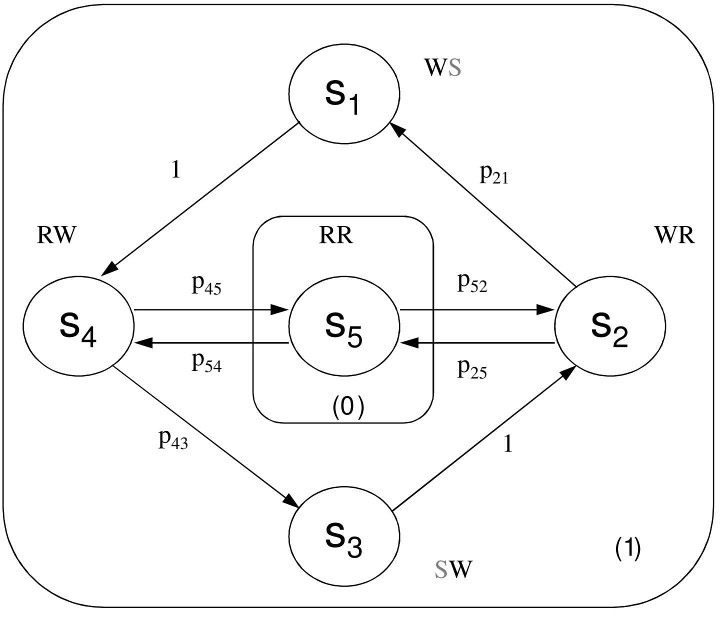

Fig. 2. Exploitation graph for a symmetric pair
Rys. 2. Graf eksploatacyjny dla pary elementów

The semi-Markov kernel ○(t), where the matrix of transitions between states is given by the following equation:

$$
\circ(t) = \begin{pmatrix} 0 & 0 & 0 & \hat{Q}_{14}(t) & 0 \\ \hat{Q}_{21}(t) & 0 & 0 & 0 & \hat{Q}_{25}(t) \\ 0 & \hat{Q}_{32}(t) & 0 & 0 & 0 \\ 0 & 0 & \hat{Q}_{43}(t) & 0 & \hat{Q}_{45}(t) \\ 0 & \hat{Q}_{52}(t) & 0 & \hat{Q}_{54}(t) & 0 \end{pmatrix}.
$$

To define the components of the above matrix, we have

$$
\hat{Q}_{14}(t) = Q_{14}(t) = F(t)
$$
$$
\hat{Q}_{21}(t) = p_{21}Q_{21}(t) = p_{21}G(t)
$$
$$
\hat{Q}_{25}(t) = p_{25}Q_{25}(t) = p_{25}F(t)
$$
$$
\hat{Q}_{32}(t) = Q_{21}(t) = F(t)
$$

$$\hat{Q}_{43}(t) = p_{43}Q_{43}(t) = p_{43}G(t)$$
$$\hat{Q}_{45}(t) = p_{45}Q_{45}(t) = p_{45}F(t)$$
$$\hat{Q}_{52}(t) = p_{52}Q_{52}(t) = p_{52}G(t)$$
$$\hat{Q}_{54}(t) = p_{54}Q_{54}(t) = p_{54}G(t).$$

To determine the initial distribution of states we assume

$$\boldsymbol{\alpha} = (1 \quad 0 \quad 0 \quad 0 \quad 0) \qquad \boldsymbol{\alpha}_1 = (\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4)\, \boldsymbol{\alpha}_0 = (\alpha_5) \qquad \boldsymbol{\alpha} = (\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_0)$$

This means that we assume that the system is in a good state from the very beginning.

The matrix of the imbedded Markov chain can be presented as follows:

We have two states: work $1 \equiv (s_1\ s_2\ s_3\ s_4)$ repair $0 \equiv (s_5)$.

The matrix:

00

$$P = \begin{pmatrix} \mathbf{P}_{11} & \mathbf{P}_{10} \\ \mathbf{P}_{01} & \mathbf{P}_{00} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ p_{21} & 0 & 0 & 0 & p_{25} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & p_{43} & 0 & p_{45} \\ 0 & p_{52} & 0 & p_{54} & 0 \end{pmatrix}$$

The ergodic probability distribution

$$\boldsymbol{\Pi} = (\Pi_1 \quad \Pi_2 \quad \Pi_3 \quad \Pi_4 \quad \Pi_5)$$

can be calculated by solving the following matrix equation

$$\boldsymbol{\Pi}\ P = \boldsymbol{\Pi},$$

having in mind that the sum of these probabilities is closed to unity, $\sum_{i=1}^{5} \Pi_i = 1$.

Now we can calculate the expected values of times for all states as follows:

$$m_1 = m_{14} = \int_0^\infty x dQ_{14}(x) dx = \int_0^\infty x f(x) dx$$

$$m_2 = p_{21}m_{21} + p_{25}m_{25} = p_{21}\int_0^\infty xdQ_{21}(x)dx + p_{25}\int_0^\infty xdQ_{25}(x)dx =$$

$$p_{21}\int_0^\infty xg(x)dx + p_{25}\int_0^\infty xf(x)dx$$

$$m_3 = m_{32} = \int_0^\infty xdQ_{32}(x)dx = \int_0^\infty xf(x)dx$$

$$m_4 = p_{43}m_{43} + p_{45}m_{45} = p_{43}\int_0^\infty xdQ_{43}(x)dx + p_{45}\int_0^\infty xdQ_{45}(x)dx =$$

$$= p_{43}\int_0^\infty xg(x)dx + p_{45}\int_0^\infty xf(x)dx$$

$$m_5 = p_{52}m_{52} + p_{54}m_{54} = p_{52}\int_0^\infty xdQ_{52}(x)dx + p_{54}\int_0^\infty xdQ_{54}(x)dx =$$

$$= p_{52}\int_0^\infty xg(x)dx + p_{54}\int_0^\infty xg(x)dx = (p_{52} + p_{54})\int_0^\infty xg(x)dx .$$

Matrixes of these mean values can be determined as

$$\boldsymbol{m} = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 & m_5 \end{pmatrix} = (\boldsymbol{m}_1 \ \boldsymbol{m}_0)$$

$$\boldsymbol{m}_1 = \begin{pmatrix} m_1 & m_2 & m_3 & m_4 \end{pmatrix} \qquad \boldsymbol{m}_0 = (m_5).$$

The ergodic probability distribution for the semi-Markov process can be calculated from the following equations:

$$\rho_i = \frac{\Pi_i m_i}{M} \quad i = 1, 2, \ldots, 5; \qquad M = \sum_{i=1}^{5} m_i \Pi_i$$

The steady-state availability of the pair of elements is given by

$$A = \sum_{i=1}^{4} \rho_i = \frac{1}{M}\sum_{i=1}^{4} \Pi_i m_i .$$

Final remarks

Having the steady-state availability assessed, we are able to study the rationale of the application of a spare element in a general case, i.e. when the process of the changes of states for a pair of elements can be satisfactorily described by a semi-Markov process. Here we may repeat comprehensive considerations that were presented in Czaplicki's monograph of 2010 (Example 7.4).

An interesting problem to analyse is a case when a pair of elements is a multi-unit system that is two duplicate systems of *n* identical units connected in a series. We consider the rationale of the construction of a second system that will serve as a reserve. When the process of changes of states of this system can be described by Markov process, the problem is simple, and the formulas given in the cited monograph can be applied to evaluate the steady-state availability. In a case when the process of changes of states must be a semi-Markov one, the problem is more complicated. However, evaluation of series system for a semi-Markov case was recently presented in Czaplicki's paper of 2011.

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Semimarkowski proces zmiany stanów pary elementów

S t r e s z c z e n i e

W pracy omówione zostało zagadnienie niezawodności systemu składającego się z pary identycznych elementów, w którym jeden element pracuje, a drugi stanowi rezerwę. Różne metody użytkowania tej pary zostały wzięte pod uwagę i jedna metoda wybrana z racji jej najkorzystniejszych właściwości dla praktyki inżynierskiej. W pracy przedyskutowano przypadek, w którym proces eksploatacji pary identyfikuje się jako proces typu semimarkowskiego. Podstawowe charakterystyki procesu zostały skonstruowane, a także zdefiniowano współczynnik gotowości systemu. Autorzy wskazali na dalszy kierunek analizy, w którym rozważa się system skonstruowany nie z dwóch tylko elementów, lecz z dwóch identycznych systemów szeregowych tworzących parę symetryczną.