Model of the Vibration Signal of the Vehicle Drive System as a Diagnostic Tool

Key words
Vehicle drive system, vibration signal, diagnostic model.

Summary
Propositions of modelling vibrations signals of a vehicle drive system in such a way that it will become an efficient tool of diagnostic mechanical defects are presented in the paper. In order to fulfil such requirements the model must be easily identifiable. The base model and its measures are established for a new engine to which the control measurements, performed at definite time intervals of vehicle operation, are compared. This base model must be actualised after each engine overhaul and as the casual wear progresses, since the vibration characteristics are – in such cases – changing. A complicated structural model of the vehicle drive system was given up for the mathematical description at a high abstraction stage. Since diagnostics is carried out during the normal engine exploitation, the defect signals must be really ‘strong’. Depending on the place of recording, the vibration signal can be either cyclostationary or transient. Three methods of modelling were proposed in the dependence of the tested element and frequency range: the harmonic series, the parametric model, and the wavelet model. These methods allow the on-board
diagnostics of the gearbox, main gearbox, differential gear, and triple joints. They make possible engine fault detection, such as the exhaust valve burning-out, valve clearance change, and the head gasket defect. These defects are, in general, not detected by the diagnostic system of the modern OBD engines, since they are masked by the electronic system of the engine control. The diagnostic method was verified on the example of the vibration signals recorded for the drive system of a Fiat Punto automobile.

Introduction

The connection between the defects of vehicle drive system and the vibroacoustic characteristics was proved several times [1-3]. However, diagnostics on the basis of experimentally found symptoms is costly, since it requires gathering broad databases and is true for only one investigated object. A much better method is performing the diagnostics on the model basis [4-6], however, under the condition that it will be identifiable fast.

During the modelling process of the drive system, the problem of selecting the abstraction (complexity) degree of the model occurs. Of course, it is possible to design one global model of the vehicle drive system [7] and expanding it, but in such case the complication degree will not allow to use it for the on-line (on-board) diagnostics.

Instead of the structural model, which is too complicated, the model of the vibroacoustic signal propagation – on a very high abstraction level – is proposed in the presented hereby study. The necessary condition of the proper model operation is then its accurate formal identification [8]. This model is created in the adaptive way, which means, firstly, the accelerated model identification, and, secondly, adjusting the model to the given type of the drive system and its working time [9]. Designing such a model requires performing a series of measurements in the selected points of the drive system as well as other additional synchronising signals during a normal vehicle maintenance, which is during operation at a steady speed and without abrupt load changes. The base model takes into account the influence of responses of all elements of the drive system.

Investigations and results discussion

The results of experimental investigations determine the functionality of building the proposed ‘block’ model of the drive system and its diagnostic usefulness. We will now analyse the results of the active experiments performed on the real object, which was the drive system of a Fiat Punto automobile. Its technical data are listed in Table 1.
Table 1. Technical specifications of the vehicle driving line
Tabela 1. Dane techniczne samochodowego układu napędowego

<table>
<thead>
<tr>
<th>Engine type</th>
<th>FIRE 1.2 MPI, gasoline, 4-stroke, 8-valve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder diameter</td>
<td>70.8 mm</td>
</tr>
<tr>
<td>Piston stroke</td>
<td>78.9 mm</td>
</tr>
<tr>
<td>Displacement</td>
<td>1242 cm³</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>9.8</td>
</tr>
<tr>
<td>Compression pressure</td>
<td>1.15 Mpa</td>
</tr>
<tr>
<td>Maximum power</td>
<td>54 kW at 6000 rpm</td>
</tr>
<tr>
<td>Maximum torque</td>
<td>106 Nm at 4000 rpm</td>
</tr>
<tr>
<td>Gearbox</td>
<td>5-gear</td>
</tr>
<tr>
<td>Clutch</td>
<td>Single-plate, dry</td>
</tr>
</tbody>
</table>

The experiments were performed during road tests at constant rotational speeds and loads with a warm engine (temperature of the cooling liquid: 95°C). The following signals were recorded:
- Accelerations of the engine head vibrations at the 1st cylinder, in the vertical and horizontal directions;
- Accelerations of the engine head vibrations at the 4th cylinder, in the vertical direction;
- Accelerations of the gearbox housing vibrations;
- Accelerations of the clutch shell vibrations;
- Accelerations of the main gear housing vibrations;
- Accelerations of the triple joint housing vibrations;
- Acoustic pressure in the engine compartment;
- Voltage from the sensor of the crankshaft position;
- Voltage from the sensor of the throttle position; and,
- Voltage signal of the 1st cylinder ignition coil.

Accelerations of the engine head vibrations were processed by means of the Bruel & Kjaer DeltaShear sensors, type 4393 of the frequency range: 0.1–16500 Hz, resonance frequency 55 kHz and a work temperature from -74 to 250°C, fixed by a screw joint. The remaining signals of the drive system accelerations of vibrations, due to the narrower frequency range of the investigated signal, were processed by means of the Bruel & Kjaer sensors type IEPE No. 4514. Signals were recorded by means of the portable recording device of the data, Bruel & Kjaer PULSE type 3560E with the sampling frequency of 65536 Hz.

Signals of approximately 1–minute duration were recorded while driving on the highway at constant rotational speeds of 2000, 3000 and 4000 rpm, without abrupt changes of an engine load. Small fluctuations of rotational speeds were eliminated during further analysis by means of synchronisation and interpolation operations. Maintaining a constant rotational speed of the engine is essential, since this parameter strongly influences the vibration amplitude. The load influence is much less significant [59]. The crankshaft position signal and the coil signal allowed the identification of engine work cycles, ignition moments, and timing gear phases.
Transient processes, being responses for valves closing, are dominating in the vibration signal recorded during the work cycle. In order to be able to draw conclusions on the engine on-line state – under various maintenance conditions – on the grounds of vibrations, the influence of engine work parameters on the vibration signal should be investigated. The performed investigations indicate that the most important is the crankshaft rotational speed. This speed increase is accompanied by the increase of the vibration signal amplitude, especially components being the responses for valves closing (Fig. 1).

Fig. 1. Instantaneous values of accelerations of the engine head vibrations in the vertical direction, at a speed of 3000 rpm
CEV – closing of the exhaust valve; CIV – closing of the inlet valve
Rys.1. Chwilowe wartości przyspieszeń drgań głowicy silnika w kierunku pionowym, przy prędkości 3000 obr/min
CEV – zamykanie zaworu wylotowego; CIV – zamykanie zaworu dolotowego

Signals of vibration accelerations of the remaining elements of the vehicle drive system are of a quite different character. Those are periodical signals, and for the constant rotational speed also stationary and ergodic with elements of random noise.

The spectrum of the vibration signal generated by the gearbox is presented in Fig. 2. Its main components are trains of harmonics of the crankshaft rotational speed. This is the broad-band spectrum and the generation of so many harmonics is related, among others, to the unbalancing and misalignments of shafts – changing during the drive – as well as to transferring of vibrations caused by the cyclic operations of engine valves. The successive harmonics corresponding to operations of next cylinders are visible in the band of lower frequencies (for the 4-cylinder engine this is the half of the crankshaft frequency). The next excitation source constitutes the tooth-passing of the gearbox and the main gearbox and also their harmonics. In the range of low frequencies (up to the crankshaft rotational frequency \( f_0 \)) the component of the half-shaft and the components related to operations of individual cylinders are seen.
Above analysis of the vibration signal shows that the drive system can be reduced to two blocks joined by the clutch, which vibrations are recorded for the needs of diagnostics (Fig. 3).

The vibroacoustic signal spectrum of the investigated system observed on the engine is described by the formula:

\[ Y_e(j\omega) = X_{ee} + H_{ee}(X_{cc} + H_{cg}X_{g-w}) + \Psi \]  

(1)

where:
- \(X_{ee}(j\omega)\) – engine excitation spectrum,
- \(X_{cc}(j\omega)\) – clutch excitation spectrum,
- \(X_{g-w}(j\omega)\) – spectrum of excitation of the remaining part of the system (from the gearbox to the automobile wheel),
- \(H_{ee}(j\omega)\) – engine – clutch transfer function,
- \(H_{cg}(j\omega)\) – clutch – gearbox transfer function,
- \(\Psi(j\omega)\) – noise spectrum.

This description concerns the linearised model, where all components formed due to non-linear influences are contained in the noise \(\Psi\). The
acceleration spectra of the engine and gearbox vibrations in Fig. 2 were divided into components generated by individual elements of the drive system. 4-cylinder internal combustion engine generates several harmonics related to the cyclic operations of a frequency: \( k \frac{f_{w}}{2} \), which is a multiple of the half of the crank shaft rotations frequency \( f_{w} \). The vibration spectrum is then very broad; whereas, components related to the operation of toothed gears, shafts, half axles, etc. can be observed on the gearbox. Some of these components are apparently divided due to their mutual super positioning, e.g. harmonic components of the crank shaft rotational speed and components of the gearbox toothed wheels (and their multiples).

**Modelling of vibration signal**

During the lifetime of the drive system, its vibration characteristics are changing, the spectrum becomes smeared, new components are added, etc. To be a universal model, it has to be able to adapt itself for various drive systems being in various states of wear. First of all, the base model remembered for the new system must exist. Such a model should automatically update itself after each repair or change of parts. Finally, it must be determined for the selected engine rotational speeds, since the vibration characteristics are strongly dependent on this parameter.

On the grounds of the adaptive base model for the system in good technical condition, the basic measures of the vibration signal – constituting the reference in the diagnostic process – are determined.

**Train of harmonics**

The vibration signal can be considered as polyharmonic and analysed by means of the Fourier transform, after filtering off the high-frequency components. In this situation, the signal reconstruction is performed as the sum of components of the largest amplitudes. Thus, signals of vibrations of shafts, gears, wheels, etc., can be used for diagnostic purposes.

Let us consider the simple model of a vibration signal of the gearbox for good and worn teeth (Fig. 4).

The basic dominant aspects of the vibroacoustic signal are the tooth mesh frequency and its multiples. As the result of the tooth cooperation, the coefficient of tooth stiffness is changing with time, which causes the formation of side bands around the tooth mesh frequency and its harmonics (the second group of spectrum components). When teeth are wearing out and the inter-teeth clearance is increasing, the amplitude of these components also increases [10, 11].
The third group of the vibroacoustic signal components of the gear constitutes the harmonics of the basic frequency (shaft rotational speed). Numerous observations [12-14] indicate that, as the gear is wearing out, all production and assembling deviations are increasing these very components.

The gearbox in the automobile works under dynamic conditions of misalignment and unbalancing; therefore, when the toothed gear is wearing out, it is increasing the occurrence of the harmonic rotational speed series of the crankshaft as expected.

The mathematical model of the gearbox is highly nonlinear. Wearing out of the gearbox causes qualitative and quantitative changes in the time waveform, the spectrum of vibrations, and acoustic pressure signals. Two various approaches to the gearbox vibroacoustic signal are possible. Either we can look for nonlinear model coefficients or we can write the linear model introducing into it, in an evident way, components related to wear and defects.

As known from investigations, the dominant elements of the signal are the rotational speed of the crankshaft and its harmonics. The gear mesh frequency and its harmonics are also integral crankshaft multiplies. The simplest signal model can be written in the time domain.

\[ x(t) = \sum_{i=1}^{n} A_i \sin\left(2\pi f_w t - \phi_i\right) \]  

where:
- \( i \) – number of the harmonics of the shaft
- \( n \) – number of the considered harmonics (in this case \( n=70 \))
- \( f_w \) – frequency of crank shaft rotations
- \( A_i \) – amplitude of the \( i^{th} \) component
- \( \phi_i \) – phase shift of the \( i^{th} \) component

or in the frequency domain:
In order to perform diagnostics of the gearbox state, the broader frequency range should be compared. The envelopes of the vibration spectra of the gearbox in good technical condition and the one with the worn out teeth of the 5th gear are compared in Fig. 5.

\[ X(f) = \sum_{i=1}^{n} A_i (if_n) \]  

(3)

Auto Regressive model

Generating the base spectrum and its comparison with the signal spectrum is the easiest method; however, it is less suitable for diagnostics of valve defects or head gasket faults. Additionally, if we do not have the recorded waveforms long enough to obtain the sufficient resolution of the spectrum, the time waveforms seem more suitable. They require, of course, a synchronisation or synchronous averaging.

One of the methods of the model determination is the parametric identification [15, 16].

The Auto Regressive (AR) model is characterised by the lack of the input signal. In this case, the vibration response is treated as the result of the influence of a certain non-measurable disturbance \( \varepsilon(k) \), expressed by the following equation:

\[ y(k) = -a_1 y(k - 1) - ... - a_n y(k - n) + \varepsilon(k) = -\sum_{i=1}^{n} a_i y(k - i) + \varepsilon(k) \]  

(4)
Number $n$ of time instants is the order of the model.

The AR model is used mainly in case of a non-measurable disturbance $\epsilon(k)$ of properties close to a white noise. The drawback of the AR model is the large number of coefficients necessary for obtaining a sufficiently accurate description of the output signal.

Before the identification the signal must be properly prepared by:
- removal of constant components and trends,
- filtration in order to select the characteristic frequency range,
- determination of the autocorrelation function or synchronous averaging, and
- decimation.

The methodology of proceedings can be presented for the defective differential gear (Fig. 6).

Thus, let us perform the parametric identification for the time-waveform of the gearbox vibration signal, in the low-frequency range, of the drive system in good technical condition. This signal should be prepared by filtering out high-frequencies (higher than three-times the crankshaft frequency) and by synchronous averaging. Then the autocorrelation function must be determined. 10-times decimation was also applied, since the signal was over-sampled and the model identification would require too many, rather useless, coefficients.

The model quality assessment is done based on the following indices: Akaike’s Information Criterion AIC, Final Prediction Error FPE, Fit [9]. On the bases of the model assessment indices, it can be assumed that the auto-regression model of the 4th order is satisfactory. The comparison of the recorded vibration signal with the AR model output of 4 coefficients $a$ is presented in Fig. 7.
The comparison of the residual signals of the base model low-frequency vibration signal of the gearbox with the signals recorded for the defected differential gear and triple joint and, are presented in Fig. 8.

Since wavelet daughters are medium-pass filters the wavelet transform is a filtering operation, the wavelet transform constitutes the proper tool for the description of non-stationary signals [17–20]. The methodology of proceedings can be presented based on the exhaust valve defect in the Fiat engine (Fig. 9).
Defects of valves and a head gasket cause the natural intensification of vibrations and entering into the resonance zone. Regardless of the response character (damping random vibrations or damping resonant vibrations) the time-frequency analysis, especially the wavelet analysis, provides good results.

The wavelet transform enables the linear signal decomposition by means of the arbitrary base function characterised by the finished and short interval that assumes non-zero values. If $\psi(t)$ is the mother wavelet, then the daughter wavelet has the following form [21]:

$$\psi_{a,b}(t) = \frac{\psi(t-b)}{a}$$

where:

- $a$ – scale coefficient
- $b$ – shifting.

By changing parameters $a$ and $b$, the wavelet family can be formed. The continuous wavelet transform of function $x(t)$ is determined by the following equation:

$$W(a,b) = \frac{1}{\sqrt{a}} \int x(t) \psi_{a,b}^*(t) dt$$

where ‘*’ means a convolution operation.

The results of the wavelet decomposition of the engine vibration signal, by means of the Daubechies 4 wavelet (db4) [22], are presented in Fig. 10.
Figure 10a presents the result of the wavelet vibration analysis for the good engine. The only meaningful vibrations constitute the response of the system for closing the inlet valve. The analogous analysis for the defective valve is seen in Fig.10b. Within the higher frequencies range (lower scales), several transient components can be noticed. Quite a few components occur for the pseudo-frequency of 21 kHz.

The inverse discrete wavelet transform allows for the signal reconstruction and is described by the following equation:

$$ f(t) = \sum_{m,n} (f, \psi_{m,n}) \psi_{m,n} = \sum_{m,n} d_m[n] \psi_{m,n} $$

(7)

where:

- \(d_m[n] = (f, \psi_{m,n})\) - wavelet coefficients,
- \(\psi_{m,n}\) – wavelets of the frequency scale coefficients \(m\) and displacement in time \(n\).

On the grounds of wavelet coefficients as a scale coefficient and displacement in time (or a shaft rotation angle) function, the signal reconstruction can be performed. Thus, the model identification can be reduced to the detection of coefficients \(d_m[n]\). Since, in case of random signals, there can be many wavelet coefficients, their compression can be performed taking into account only ‘the most energetic’. The results of this compression are shown in Fig.11. After the compression of 64 wavelet coefficients describing the signal, only 10 of the highest energy were left.
The wavelet model describes the time-frequency structure of the vibration signal. For the comparison of the base model and recorded signal, the envelopes of signals are more applicable (Fig.12).

![Figure 11](image1.png)

**Fig. 11.** Model identification using wavelet reconstruction of the engine vibration response signal on closing the exhaust valve

![Figure 12](image2.png)

**Fig. 12.** Envelopes of the reconstructed signal for the engine work cycle with the good exhaust valve (base model) and with the defective valve

**Conclusions**

The diagnostic method of mechanical defects of the vehicle drive system based on comparing the current vibration signal with the reference model was proposed. The model is described at a very high abstraction level, close to the ‘black box’ type. It can be applied for the diagnostics of the selected mechanical defects of subassemblies, however, only under the condition of the proper
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identification of the parameters. Signals of vibrations measured in two places of the drive system, on the engine head and on the gearbox housing, were used in the designing of the base model.

The model is adaptive, which means that the diagnostic program obtains the base model after each change of the drive system element. Later on, the program compares the recorded and properly processed signal with this model.

The model was designed and verified based on the series of examinations performed on a real object, which was the drive system of the automobile Fiat Punto. The fact that this automobile was not new confirmed the applicability of the method, since, as time goes by, the vibration characteristics of the drive system are changing and the defects occur years later. This model was also adapted for the drive systems of Ford Fiesta and Renault Thalia with internal combustion engines of a capacity 1.4 cm$^3$ of low mileage. Regardless of the differences in the level of generated vibrations, the character of vibration waveforms is similar for various engines. In addition, changes in vibration characteristics generated by the examined defects are of an analogous character.

The proposed model is open for the diagnostics of other defects for which the vibroacoustic signal is sensitive. This only requires further studies and program modifications. Obtaining the model as well as the diagnostic procedure can be performed during normal vehicle maintenance and operation. It requires the application of the signal processor and memory systems. The proposed system can constitute a supplement to the existing OBD systems.

References

Streszczenie