SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
2 (162) 2010

ANDRZEJ KATUNIN*

Identification of multiple cracks in composite beams using discrete wavelet transform

K e y w o r d s

Faults identification, polymeric laminates, discrete wavelet transform.

S ł o w a k l u c z o w e

Identyfikacja uszkodzeń, laminaty polimerowe, dyskretna transformata falkowa.

S u m m a r y

A method for the identification of multiple cracks based on Discrete Wavelet Transform is presented. The analysis is provided on beams made of polymeric laminate. The estimation of the crack locations is based on the evaluation of natural modal shapes of pre-cracked beams. The modal shapes were estimated experimentally using laser Doppler vibrometry. The dynamic response of multi-cracked beams is processed using Discrete Wavelet Transform and detail coefficients are considered for the crack identification. Next, the methods of detail coefficients denoising are discussed. The principles of the selection of the appropriate wavelet are investigated. The proposed method indicates effectiveness in multiple crack identification and could be applied in industrial solutions of structural health monitoring as well.

1. Introduction

The development of the novel applications of polymeric composites determines the necessity of their diagnosing during exploitation. During

* Department of Fundamentals of Machinery Design, Faculty of Mechanical Engineering, Silesian University of Technology, Konarskiego 18A, 44-100 Gliwice, Poland, e-mail: andrzej.katunin@polsl.pl

workloads, such structures were subjected to several fault initiations. The most typical faults that occurred in composites are cracks. The cracks may occur in different conditions, such as intensive loading, impacts, mechanical and thermal fatigue, etc. Considering the importance of some applications of composite structures, e.g. aircraft elements, diagnosing must be effective, non-destructive, and possibly low-cost and simple. The timely detection and identification of cracks may prevent breakdowns and makes possible appropriate repairs. In practice, one can observe the simultaneous initiation of multiple cracks. When multiple cracks appear in the structure, their dynamic response becomes more complex, taking into consideration their positions. Considering structural properties and loading conditions, cracks may propagate in a different manner. Therefore, the identification method must allow for the identifying of all presented cracks.

One of the most widespread groups of non-destructive methods in technical diagnostics is based on the modal analysis of the structures and specific techniques of signal processing. In the diagnostics of the structural health, many techniques based on measured signal transforms could be used. Some publications [1, 2] on crack detection in composite beams were based on Fast Fourier Transform (FFT); however, such analysis allows only the detection of the crack. Other techniques, which were used for crack detection, contain Short-Time Fourier Transform, kurtosis analysis, cepstrum-based techniques, etc. In the case of an early phase of the damage, the analysis of natural frequencies or signal point features variability often is not effective, and it is impossible to localise the crack and to estimate its dimensions. In this case, the changes in natural frequencies and modes are not directly detectable and require specific processing.

To fulfil the above-mentioned criteria, it is advisable to use multi-resolution techniques, e.g. Wavelet Transform (WT). This technique has some advantages, like the possibility of the application of different wavelets and the simultaneous scaling and shifting of them for approximation purposes. Many researchers use WT for crack identification. The problem of faults detection and isolation in composite beams in the early damage phase using Continuous Wavelet Transform (CWT) was studied and discussed [3]. This problem was also investigated by Douka *et al.* for beams with single [4] and double [5] cracks. They used symlets in CWT for identifying crack locations and the estimation of their relative depth. Research on the damage detection was provided by the research group of Ostachowicz. The authors presented several approaches in damage identification. They used experimental measurements and finite element models for identify cracks based on the deflection profiles [6]. In [7], the Discrete Wavelet Transform (DWT) was used for the crack identification. The authors compared the effectiveness of the identification using different wavelets. DWT on modal shape data used for crack identification allows one to separate the signal into the approximation coefficients and details coefficient parts. Then, the details coefficients part is analysed to identify the damage. This procedure is quite quick and simple, which is an unquestionable advantage of DWT. However, the usage of wavelets in DWT is limited to compactly-supported orthonormal wavelets. Some studies concentrated on theoretical modelling of cracked or multi-cracked beam-like structure response, mainly using Galerkin formulation. The analytical model for Timoshenko beams proposed in [8] allows crack detection and localisation. Another valuable work [9] presents the finite element formulation for multi-cracked shafts.

In solving the problem of damage detection and identification using WT, it is important to select the appropriate wavelet. In previous works [10, 11], the problem was studied and several wavelets were compared. It was noticed that B-spline wavelets applied for damage detection give the best results in comparison with other wavelets both using CWT and DWT. The present paper deals with the method of multiple crack detection and identification in polymeric layered composite beams. In Section 2, the algorithm of single-level DWT will be presented for decomposition and reconstruction operations. In the next section, the parameters of the experimental study and the measurement equipment will be shown. Obtained measurements will be analysed for crack detection and an estimation of their depth. The selection of the appropriate wavelet for the analysis and parameters for detail coefficients filtering will be discussed. The filtering threshold was determined in terms of measurement noise and disturbances that result from vanishing moments of the applied wavelet.

2. The algorithm of Discrete Wavelet Transform and its inversion

The effectiveness of DWT is due to the dyadic bases of scales and translation positions, which makes possible the multi-resolution analysis. The algorithm was proposed by Mallat in [12]. The process of DWT is based on the decomposition of the signal $f(x)$ into the summation of wavelet bases at different dyadic scales. The decomposition of the signal is the filtering operation with the use of two filters, where the first one is the wavelet scaling function $\varphi(x)$ (low-pass filter) and the second one is the wavelet function $\psi(x)$ (high-pass filter). The signal $f(x)$ can be presented as the following relation:

$$f(x) = \sum_{n=-\infty}^{n=\infty} f_n^0 \varphi(x-n) \tag{1}$$

The operation of the decomposition considering (1) and the orthogonality of φ can be presented as follows:

$$f_n^{(j)} = \sum_k h_{2n-k} f_k^{(j-1)} , \quad d_n^{(j)} = \sum_k g_{2n-k} f_k^{(j-1)} \tag{2}$$

where j is the level of the decomposition and h and g are related to the low-pass filter and high-pass filter, respectively. After this operation, one can obtain the approximation coefficients f and the detail coefficients d as a result of the high-pass and low-pass filtering, respectively. Note that, after this operation, the resolution of the filtered signal is reduced twice.

The operation of the discrete signal reconstruction is based on Inverse Discrete Wavelet Transform (IDWT). Here the high-pass filter is applied to approximation coefficients, and the low-pass filter is applied to the detail coefficients. Then the convolution operation is used for the filtered coefficients. For the n-th level of reconstruction IDWT can be presented as follows:

$$f_n^{(j-1)} = \sum_k h_{k-2n} a_k^{(j)} + \sum_k g_{k-2n} d_k^{(j)} \quad (3)$$

The effectiveness of above-presented algorithms is strongly dependent on the analysing wavelet, which will be investigated and discussed later.

3. Experimental setup and measurements

3.1. Specimens and measurement equipment

The specimens were manufactured from glass-fibre reinforced epoxy in the form of unidirectional preimpregnated fibres. The specimens were prepared to achieve transversal isotropic properties. Layers orientation and characteristic material properties of the laminate may be found in [13]. The dimensions of specimen are as follows: length L = 0.25 m, width W = 0.025 m, and thickness H = 0.005 m. In the experiment, four specimens were considered, where the first one was undamaged and others were artificially pre-cracked. The cracks were created at various distances and with various depths. The positions and depth dimensions of the cracks are presented in Fig. 1. Considering the clamp of the specimen and the excitation clamp on the other side of the specimen, an additional dimension was defined as the effective length L_{eff} = 0.215 m on which the measurements are provided.

We provide the measurements using scanning laser vibrometer Polytec PSV-400 and the second point laser vibrometer Polytec PDV-100 was used as a measurement reference. The specimens were excited by the electrodynamic modal shaker TIRA TV-51120 by the random noise signal, which was amplified by the power amplifier TIRA BAA 500. The experimental stand was illustrated in Fig. 2. On the measurement surface of the specimens, the reflecting tape was attached to provide the satisfactory focus and power of the laser beam. Then, using PSV software, we defined 39 measurement points on L_{eff} with the constant interval of ~0.0055 m between them. The frequency bandwidth was set in the range of 0 to 2 kHz with the resolution of 0.625 Hz and the sampling frequency equals 5.12 kHz. During the measurements, the velocity of vibrations in each defined point was measured.

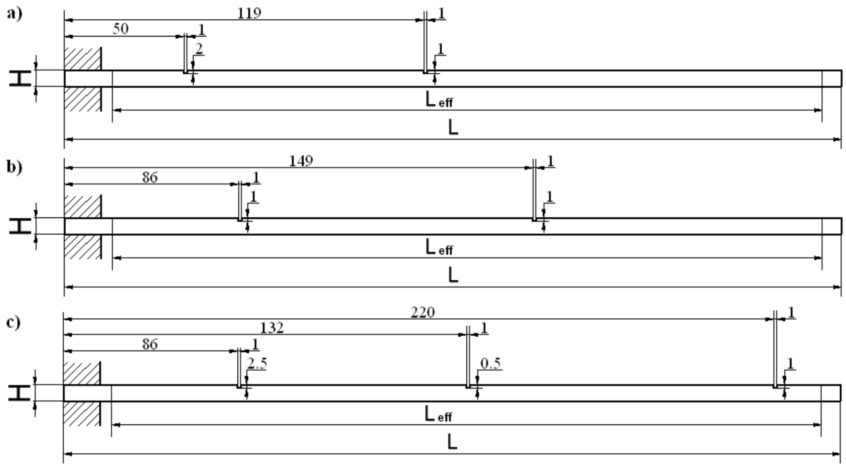

Fig. 1. Crack positions and depths in the investigated specimens
Rys. 1. Położenia i głębokości pęknięć w badanych próbkach

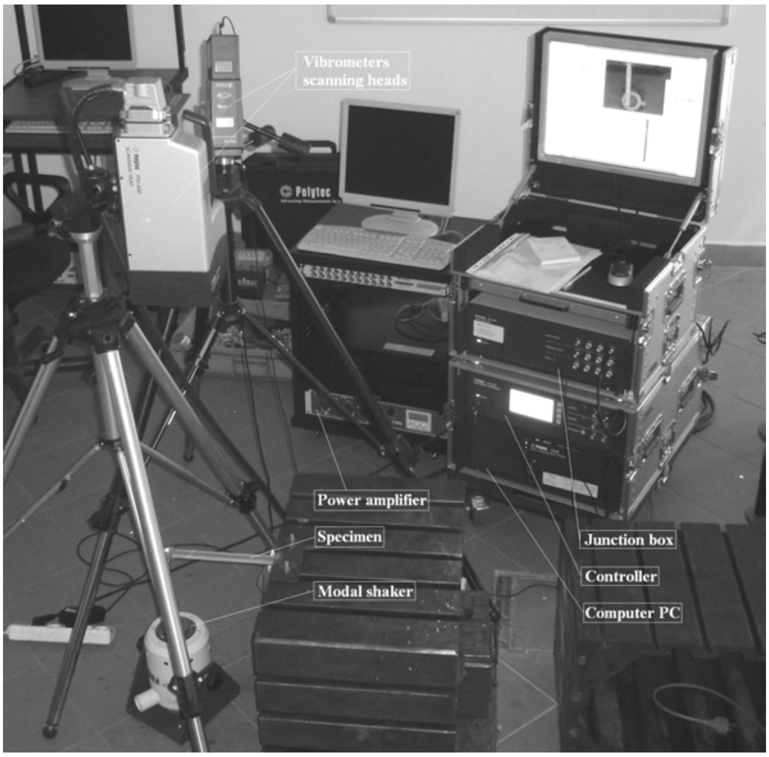

Fig. 2. Experimental setup
Rys. 2. Stanowisko eksperymentalne

3.2. Measurement data preprocessing

As a result of measurements, we obtain the Frequency Response Functions (FRF) for each specimen. Then, we select peaks on the FRF (see Fig. 3) for determining the natural frequencies and modal shapes of the investigated beams. It was necessary to separate only bending frequencies for further analysis. The first three bending, natural frequencies for all specimens are tabulated in Table 1.

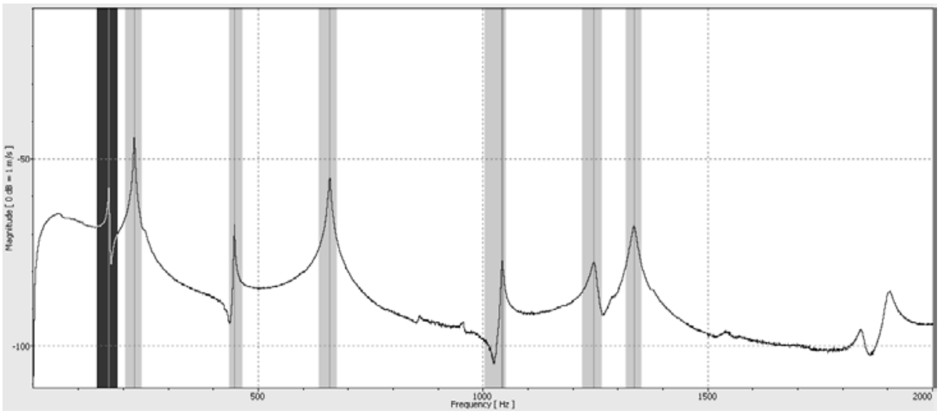

Fig. 3. Exemplary FRF and peaks selection
Rys. 3. Przykładowa funkcja odpowiedzi częstotliwościowej z zaznaczeniem pików

Table 1. Natural frequencies of the investigated specimens
Tabela 1. Częstotliwości własne drgań badanych próbek

Frequency number	Frequency, Hz			
	Undamaged	Case a	Case b	Case c
1	224.375	252.5	237.5	232.5
2	659.375	742.5	708.125	656.25
3	1336.25	1467.5	1415	1346.25

While analysing the obtained frequency values, it is possible to detect the crack presence. The evaluation of obtained values could be provided, e.g. using Modal Assurance Criterion (MAC), which shows good results in such problems [14]. However, for crack identification, methods that are more specific should be used. It will be showed in the next section using DWT.

4. Cracks identification

Signals of velocity values in defined points achieved from the experiment for selected cases were exported to the MATLAB® environment.

4.1. Selection of the analysing wavelet

Obtained signals for cracked beams contain some singularities, this provides information about crack location and depth. However, these singularities were not visible in signal realisations. Therefore, we use DWT to decompose the signals to approximation coefficients, which illustrates the smooth curve, and the detail coefficients, containing useful information about singularities, and this can be used for cracks identification.

The selection of the appropriate wavelet for such problems depends on several factors. The analysing wavelet must be orthogonal. The second criterion concerns some kind of compromise between the length of the effective support and the number of vanishing moments. It is necessary to select the wavelet with maximum possible number of vanishing moments and the shortest possible effective support, which assures stability and higher coefficients in damaged regions. Douka *et al.* [4, 5] used the fourth order symmetrical wavelet (*sym4*) (same as the authors of [16]) and in their further work [15], they select another two wavelets besides *sym4*: coiflet 2 (*coif2*) and biorthogonal 6.8 wavelet (*bior6.8*). The authors of [17] used biorthogonal 5.5 wavelet (*bior5.5*) for this procedure. In the previous author's works [11], the effectiveness of the B-spline wavelets in comparison with other compactly supported orthogonal wavelets was demonstrated using the Degree of Scalogram Density. In [11], the six-order B-spline wavelet was used.

An additional parameter, which may influence crack detection effectiveness, is the number of measuring points. The authors of [15] and [16] used for the analysis 1001 and 601 measuring points, respectively. In present work, we used only 40 measuring points, which implies the necessity of applying the wavelet with shorter effective support and a sufficiently large number of vanishing moments. After some testing, the quadratic B-spline wavelet (*bsp3*) was chosen for the further analysis throughout the present work. The above-presented wavelets has four (*sym4, coif2*) or five (*bior5.5*, *bior6.8*) vanishing moments, but their effective supports are sufficiently large: *sym4* – 7, *coif2* and *bior5.5* – 11, *bior6.8* – 13. The chosen wavelet has 4 vanishing moments and the effective support length of 5.

4.2. Determination of the threshold for details coefficients de-noising

Considering measurement noise and disturbances induced by the wavelet, it is necessary to de-noise detail coefficients obtained from the DWT procedure for a clear detection and location of the cracks. The most used methods for de-noising are the soft- and hard-thresholding. Soft-thresholding is proposed considering its nice mathematical properties. The general procedure of de-noising consists of three steps: firstly, we decompose the signal, then we threshold detail coefficients, and finally we reconstruct the signal after the thresholding. In other words, only the detail coefficients greater than threshold are considered. Authors of [4] used the thresholding for de-noising the detail coefficients; however, they assumed some value of threshold without explanation about its determination. Zhong and Oyadiji [15] proposed the most effective method of the thresholding yielding minimax performance multiplied by the small factor proportional to logarithmized signal length. This method could be slightly improved using a combination of the above-discussed rule and the rule based on the Stein's Unbiased Risk Estimate (SURE). Such a combination is useful when the signal-to-noise ratio is very small, and should be used when there is no opportunity to achieve the response of the undamaged structure. However, this method is based on the stochastic formulation.

In the situation when we could determine the detail coefficients for the undamaged structure, it is suitable to determine the threshold using the peak values of these coefficients. The thresholds were determined both based on statistical method and based on experimental results. Obtained thresholds are tabulated in Table 2.

Table 2. De-noising thresholds for detail coefficients
Tabela 2. Progi filtracji szumu dla współczynników detali

Case	Mode number	Method	
		Statistical (log+SURE)	Experimental
a	1	$2.5042\cdot10^{-6}$	$2.2680\cdot10^{-6}$
b	1	$2.4809\cdot10^{-6}$	
c	1	$2.5625\cdot10^{-6}$	
a	2	$2.3811\cdot10^{-6}$	$2.2293\cdot10^{-6}$
b	2	$2.4685\cdot10^{-6}$	
c	2	$2.2414\cdot10^{-6}$	
a	3	$2.5448\cdot10^{-6}$	$2.3951\cdot10^{-6}$
b	3	$2.5631\cdot10^{-6}$	
c	3	$2.5415\cdot10^{-6}$	

It is noticed that thresholds determined theoretically are slightly higher than experimentally determined. However, de-noising of detail coefficients, using both methods gives almost identical results.

4.3. Cracks detection and localisation

The procedure of cracks detection and localisation is based on DWT and IDWT. Here, the following algorithm was used: measured velocity signal was decomposed on a single-level to approximation coefficients and details coefficients; then, de-noising of obtained detail coefficients was performed using soft thresholding, and finally the reconstruction was applied only for de-noised detail coefficients. The scheme of the above-presented procedure is presented in Fig. 4. Blocks 'g' and 'h' denote low-pass and high-pass filters, respectively.

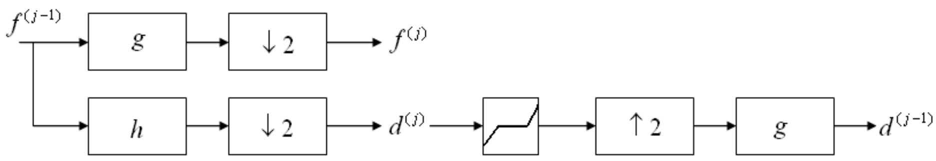

Fig. 4. Scheme of signal processing for crack detection and localisation
Rys. 4. Schemat przetwarzania sygnału dla detekcji i lokalizacji pęknięcia

After using this procedure, the evaluation of crack position was performed. In the obtained local disturbances, the peak value indicates the crack presence. Results of the analysis are shown in Fig. 5.

It should be noted that the peak values do not reflect the crack locations in some cases (e.g. Fig. 5, Case a, Mode 2). In this case, the effect of the sign of detail coefficients appeared. In the some region, where detail coefficients are negative, we could observe a mirror image position of the true crack location [18]. Following this, the true crack position could be determined as a difference of the L_{eff} and pseudo-crack position. In the presented results, the above-mentioned effect could be observed for Case b Mode 2 and Case c Mode 3.

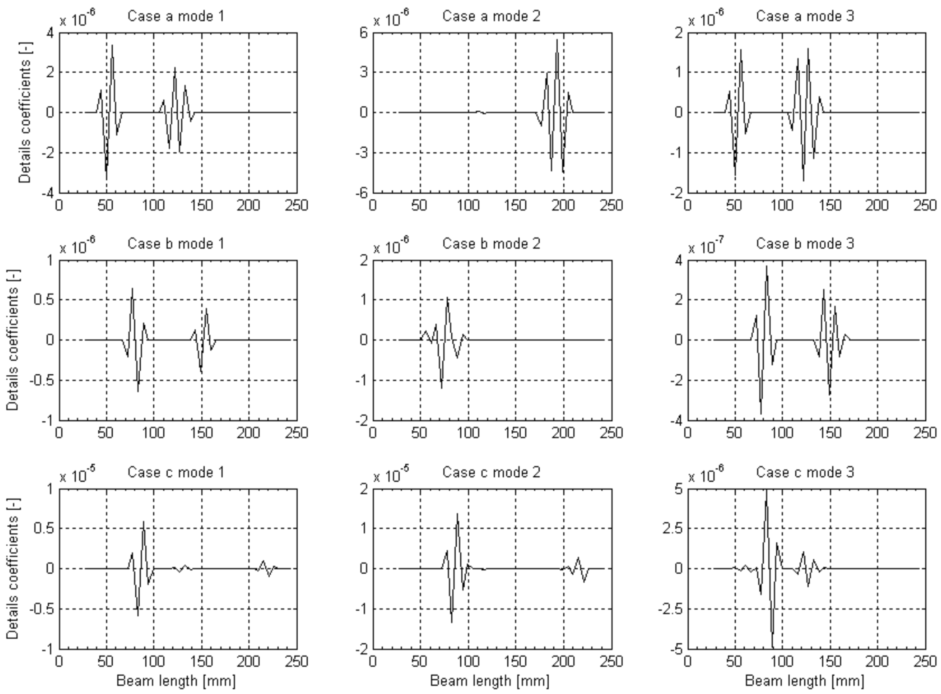

Fig. 5. Results of cracks detection and localization procedure
Rys. 5. Wyniki procedury detekcji i lokalizacji pęknięć

4.4. Discussion on the estimation of cracks depth

The problem of the crack depth estimation using wavelet transform has been discussed by many authors, e.g. [4, 16]. Douka *et al.* in [4, 5] proposed the crack depth estimation using the relative differences observed in the detail coefficients set. They established the intensity factor based on the modal model of a cracked beam. Zhong and Oyadiji in [16] used a similar method and also discussed the influence of crack depth on the sign of detail coefficients. Both of above-presented methods are based upon the comparative analysis of detail coefficients of cracked beams with various crack depths. Such an approach allows the estimation of cracks depth only as a relative one and does not give the depth value explicitly.

The problem of crack depth estimation becomes very complicated, taking into consideration following effects. First, the relation between the crack position and the actual modal shape must be considered. Let us analyse the results in Fig. 5 for Case a. For the first mode, it could be observed that first absolute local peak value is greater than the second one, which confirm the actual state (see Fig. 1). However, for the second mode, we obtain different result: the detail coefficients in the first crack location have great magnitude, but the second one is almost undetectable. This is reasoned by the mode shape: the second crack is located very close to the modal node; thus, the magnitude of the detail coefficients in this location is quite low. For the third mode, we obtain almost identical magnitudes of local peaks of detail coefficients, but originally their depths differ by a factor of two. The influence of modal shape is clearly visible for Case b Mode 1. The beam contains two cracks with different locations but with the same crack depth; however, local peaks of detail coefficients on Fig. 5 show recognisable differences.

Summarising, at the present time, there is no effective method for the explicit estimation of crack depth. The crack depth could be evaluated using the mixed theoretical-experimental approach, but this method will be limited to simple geometrical cases due to the complexity of theoretical modelling of real structural components.

5. Conclusions

The paper considers the problem of multiple crack identification in polymeric laminate beams using DWT. The modal responses of damaged beams were obtained using laser Doppler vibrometer for first three modal shapes. Measured signal realisations, as expected, contain two types of noise: an experimental and wavelet disturbance. Therefore, it was necessary to investigate two subsidiary problems: the selection of the appropriate wavelet for effective crack recognition and simultaneously minimising local disturbances; and the selection of the appropriate method for detail coefficients de-noising.

Comparative analyses and discussions on these problems were performed. Using optimal wavelet and de-noising procedures, the signal processing was carried out using a modified method. After signal decomposition, one considers only details coefficients, which were de-noised and reconstructed. The proposed algorithm gives good results, i.e. we detect and localise all of the cracks in each investigated specimen. Several useful aspects, like the effect of the sign of detail coefficients and the influence of the modal shape on detail coefficient magnitudes, were also discussed. The proposed method has a few limitations. First of all, it can be used only for the transverse crack identification during bending excitation, and the accuracy of the method is strongly influenced by the type of applied wavelet, the sampling distance, and the number of measuring points. However, the method could be applied in a large group of industrial applications, which fulfil the above criteria.

References

[1] Ghoneam S.M., Dynamic analysis of open cracked laminated composite beams, Composite Structures, 32, pp. 3–11, 1995.

[2] Abd El-Hamid Amada A., An investigation into the eigen-nature of cracked composite beams, Composite Structures, 38, pp. 45–55, 1997.

[3] Katunin A., Moczulski W., Faults detection in layered composite structures using wavelet transform, Diagnostyka 1(53), pp. 27–32, 2010.

[4] Douka E., Loutridis S., Trochidis A., Crack identification in beams using wavelet analysis, International Journal of Solids and Structures, 40, pp. 3557–3569, 2003.

[5] Loutridis S., Douka E., Trochidis A., Crack identification in double-cracked beams using wavelet analysis, Journal of Sound and Vibration, 277, pp. 1025–1039, 2004.

[6] Grabowska J., Palacz M., Krawczuk M., Damage identification by wavelet analysis, Mechanical Systems and Signal Processing, 22, pp. 1623–1635, 2008.

[7] Rucka M., Wilde K., Crack identification using wavelets on experimental static deflection profiles, Engineering Structures, 28, pp. 279–288, 2006.

[8] Tian J., Li Z., Su X., Crack detection in beams by wavelet analysis of transient flexural waves, Journal of Sound and Vibration 261, pp. 715–727, 2003.

[9] Sekhar A.S., Multiple cracks effects and identification, Mechanical Systems and Signal Processing, 22, pp. 845–878, 2008.

[10] Katunin A., Korczak A., The possibility of application of B-spline family wavelets in diagnostic signal processing, Acta Mechanica et Automatica, 3(4), pp. 43–48, 2009.

[11] Katunin A., Construction of high-order B-spline wavelets and their decomposition relations for faults detection and localization in composite beams, Acta Mechanica et Automatica, 2010, submitted.

[12] Mallat S., A theory of multiresolution signal decomposition: the wavelet representation, IEEE Trans. Patt. Anal. Mach. Intel., 11, pp. 674–693, 1989.

[13] Katunin A., Moczulski W., The conception of a methodology of degradation degree evaluation in laminates, Maintenance and Reliability, 1(41), pp. 33–38, 2009.

[14] Deraemaeker A., Preumont A., Vibration based damage detection using large array sensors and spatial filters, Mechanical Systems and Signal Processing, 20, pp. 1615–1630, 2006.

[15] Douka E., Loutridis S., Trochidis A., Crack identification in plates using wavelet analysis, Journal of Sound and Vibration, 270, pp. 279–295, 2004.

[16] Zhong S., Oyadiji S.O., Crack detection in simply supported beams using stationary wavelet transform of modal data, Structural Control and Health Monitoring, 2010, early view.

[17] Rucka M., Wilde K., Application of continuous wavelet transform in vibration based damage detection method for beams and plates, Journal of Sound and Vibration, 297, pp. 536–550, 2006.

[18] Zhong S., Oyadiji S.O., Crack detection in simply supported beams without baseline modal parameters by stationary wavelet transform, Mechanical Systems and Signal Processing, 21, pp. 1853–1884, 2006.

Manuscript received by Editorial Board, July 27th 2010

Identyfikacja wielopołożeniowych pęknięć w belkach kompozytowych z zastosowaniem dyskretnej transformacji falkowej

S t r e s z c z e n i e

W pracy przedstawiono metodę identyfikacji pęknięć wielopołożeniowych opartą o dyskrestną transformację falkową. Analiza została przeprowadzona na belkach z laminatu polimerowego. Wyznaczenie lokalizacji pęknięć polegało na ocenie postaci własnych drgań belek z pęknięciami. Postacie własne były otrzymane eksperymentalnie za pomocą dopplerowskiej wibrometrii laserowej. Odpowiedź dynamiczna belek z pęknięciami była przetworzona z zastosowaniem dyskretnej transformacji falkowej, a następnie rozpatrzono współczynniki detali w celu identyfikacji pęknięć. Dalej omówiono metody usuwania szumu ze współczynników detali. Zbadano kryteria wyboru odpowiednich falek. Zaproponowana metoda wykazała efektywność w identyfikacji pęknięć wielopołożeniowych i może być z powodzeniem zastosowana w rozwiązaniach przemysłowych kontroli stanu struktur.