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Semi-Markov model of the availability of the means of municipal transport system

K e y w o r d s

Transport system, municipal transport, semi-Markov model, technological object availability.

S ł o w a k l u c z o w e

System transportowy, komunikacja miejska, model semi-Markowa, gotowość obiektu technicznego.

S u m m a r y

The article presents a method of designing the availability of technological objects used in complex operational systems with the assistance of a theory pertaining to semi-Markov processes. The complete consideration was presented based on the chosen authentic system of transport means operation – municipal bus transport system in a chosen urban complex. Direct realisation of transport goals of the municipal transport system is undertaken by a utilisation subsystem comprised of elementary subsystems of the operator – transport means (driver – bus) type, the availability of which significantly influences the possibility of appropriate realisation of these goals.

In order to build a model of the availability of transport means, significant operational states of the operational process were designed and a division and reduction of the number of states was done taking into consideration the criterion availability for operation. Based on this, both event-based and mathematical models of the process of transport means operation were built, assuming that the mathematical model of the process of transport means operation constitutes a homogenous

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semi-Markov process. Then, for the operational data obtained from the testing of authentic operational process, limit values of the availability factor for the delineated levels of transport means availability in municipal transport system were defined.

This article presents a way to define the availability of a single technological unit – transport means, based on the constructed semi-Markov model of operational process used in a utilisation subsystem of the tested transport means system. The presented model for designing the availability of transport means is part of a decision-making model of controlling transport system availability created within the framework of a larger research project.

1. Introduction

Technological object (element or system) availability is the object's feature, which is characteristic from the point of view of the possibility of timely obtaining or maintaining the state of efficiency (facilitating the realisation of goals) [6, 8, 10, 11]. The term 'availability' pertains to the systems characterised by the necessity of quick reaction in emergency situations; such systems include the military, police, ambulance service, fire department, as well as transport systems. In such systems, whenever there is a goal in an emergency situation, an individual or a team of individuals with their assigned technological objects attempts its *immediate* realisation.

A unique type of transport system is the municipal bus transport system with the main goal of passenger transport. The frequency of rides and a complex number of persons on board determine the size of the transport goals, in an assigned period of time, on a defined route. In systems of this type, the direct implementation of passenger transport remains the responsibility of the utilisation subsystem comprised of elementary subsystems of the operator – transport means (driver – bus) type. It is on the availability of these subsystems that the possibility of appropriate implementation of transport goals depends. Availability of transport means remains at an appropriate level, as a result of the repair processes implemented in the utilisation subsystem by technological support units and in the efficiency implementation in repair posts at the bus depot. The above results in the availability of transport system being dependent on the possibility of correct control of the process of operation is realised both in the utilisation and efficiency implementation subsystems.

This article presents a way of defining availability of a single technological object – means of transport, based on the built semi-Markov model of operational process realised in the utilisation subsystem of the tested municipal transport system. The model of determining the availability of transport means presented here is a part of a decision-making model of controlling transport system availability created within a larger research project. In the following stages, a model for determining utilisation subsystem availability comprised of N number of transport means combined with an appropriate structure will be created, together with a model of defining and the evaluating of the availability of an efficiency implementation subsystem.

2. Event-based operation process model

An event-based model of an operation and maintenance process was built on the basis of the analysis of the operation and maintenance states space and the operation and maintenance events regarding the technical objects being operated and maintained within a real transport system under analysis. Each of the operated and maintained technical object may, at any moment *t*, stay in only one of the distinguished states, forming a finite set of operation and maintenance states of an object. The following significant operation and maintenance states have been distinguished in the analysed operation and maintenance process model:

- S_1 – stopover at bus depot parking space,
- S_2 – repair at bus depot parking space,
- S_3 – carrying out of the transport goal,
- S_4 – fuel intake between transport peak hours,
- S_5 – repair by technical support unit without losing a trip,
- S_6 – repair by technical support unit with losing a trip,
- S_7 – awaiting the start of task realisation after technical support repair,
- S_8 – emergency exit,
- S_9 – technical object repair at the efficiency implementation subsystem posts.

Afterwards, the possible transitions between the distinguished operation and maintenance states were determined. It was the basis for making a graph of the operation and maintenance state changes, as presented in the Fig. 1.

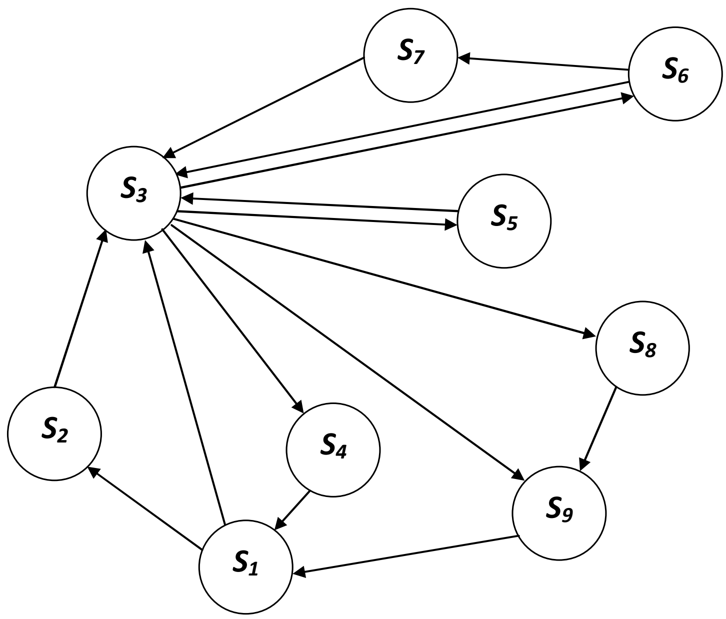

Fig. 1. Directed graph representing the transport means operation process
Rys. 1. Graf skierowany odwzorowania procesu eksploatacji środków transportu

3. Semi-Markov model of the availability of transport means

In general, the availability of technological object to realise the assigned goal is described as a technological object feature that characterises it from the point of view of the possibility of timely obtaining and maintaining the state of efficiency (facilitating the realisation of the goal) at moment t or during time range τ_r of time reserve for supplies and/or object repair.

In this paper, the value of technological object availability is defined for three selected levels:

- First level availability $G_{OT(1)}^{24}$ is determined for objects, which at any given moment t are efficient and supplied; in case of the tested system, it pertains to technological objects which realise the assigned goal or await the beginning of goal realisation at the parking place of the bus depot or following repair done by technical support unit.
- Second level availability $G_{OT(2)}^{24}$ is additionally assigned for efficient technological objects which are to be supplied, e.g. in fuel; however, the supply process will be realised in $T_z \le \tau_z$ shorter than the period of time reserve for supplying technological objects without losing a trip, i.e. during the breaks between trips determined by the schedule of transport goals realisations (bus schedule).
- Third level availability $G_{OT(3)}^{24}$ is additionally determined for inefficient technological objects which were damaged at the parking place of the bus depot while awaiting the realisation of the transport goal or en route while realising the transport assignment but were repaired by bus depot technicians or technical support unit in $T_u \le \tau_u$ time shorter than the period of time reserve without losing a trip, i.e. during the breaks between trips determined by the schedule of transport goals (bus schedule).

In the examined model, it is assumed that both the supplies of the technological object during time reserve τ_z and the repair during time reserve τ_u do not stop the goal from being realised and, at the same time, do not make it necessary to substitute the supplied or repaired object with a different one (reserve object).

3.1. Mathematical model of the operation process

Using the semi-Markov processes in the mathematical modelling of the operation process, the following assumptions were put forward:

- The modelled operation process has a finite number of states S_i, $i = 1,2,\ldots,9$.
- The random process $X(t)$ being the mathematical model of the operation process is a homogenous process.
- At moment $t = 0$, the process finds is in state S_3 (the initial state is state S_3).

The homogenous semi-Markov process is unequivocally defined when initial distribution and its kernel are given. From our assumptions and based on the directed graph shown in Figure 1, the initial distribution $p_i(0)=P\{X(0)=i\},\ \ i=1,2,...,9$ takes the following form:

$$p_i(0)=\begin{cases}1 & when\ \ i=3\\ \\ 0 & when\ \ i\neq 3\end{cases} \tag{1}$$

whereas, the kernel of process $Q(t)$:

$$Q(t)=\begin{bmatrix}0 & Q_{12}(t) & Q_{13}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{23}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) & 0 & Q_{38}(t) & Q_{39}(t)\\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{53}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & Q_{63}(t) & 0 & 0 & 0 & Q_{67}(t) & 0 & 0\\ 0 & 0 & Q_{73}(t) & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t)\\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \tag{2}$$

where:

$$Q_{ij}(t)=P\{X(\tau_{n+1})=j,\tau_{n+1}-\tau_n\leq t|X(\tau_n)=i\},\ \ i,j=1,2,...,9 \tag{3}$$

means that the state of semi-Markovian process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where $\tau_1, \tau_2, \ldots, \tau_n, \ldots$ are arbitrary moments in time, so that $\tau_1<\tau_2<\ldots<\tau_n<\ldots$;
as well as

$$Q_{ij}(t)=p_{ij}\cdot F_{ij}(t) \tag{4}$$

where:

$$p_{ij}=\lim_{t\to\infty}p_{ij}(t) \tag{5}$$

p_{ij} – means that the conditional probability of transfer from state S_i to state S_j,

$$p_{ij}(t) = P\{X(t) = j | X(0) = i\} \tag{6}$$

as well as

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \le t | X(\tau_n) = i, X(\tau_{n+1}) = j\}, \quad i, j = 1,2,...,9 \tag{7}$$

is a distribution function of random variable Θ_{ij} signifying the period of duration of state S_i, under the condition that the next state will be state S_j.

3.2. Availability of transport means

In general, availability of technological objects determined based on the semi-Markov operation process model is defined as the sum of limit probabilities p_i^* of being in states belonging to the set of availability states

$$G = \sum_i p_i^*, \quad dla \quad S_i \in S_G, \quad i = 1,2,...,9 \tag{8}$$

In order to assign the values of limit probabilities p_i^* of staying in the states of semi-Markovian model of transport means operation, based on the directed graph shown in Figure 1, the following matrix P was created of the states change probabilities in process $X(t)$:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 & p_{38} & p_{39} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{63} & 0 & 0 & 0 & p_{67} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Limit probability p_i^* of staying in states of semi-Markov process were assigned on the basis of the limit theorem for semi-Markovian processes [2, 7]:

If hidden Markov chain in semi-Markovian process with finite state S set and continuous type kernel contains one class of positive returning states such that for each state $i \in S, f_{ij} = 1$ and positive expected values $E(\Theta_i), i \in S$ are finite, limit

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)} \tag{10}$$

exist where probabilities $\pi_i, i \in S$ constitute a stationary distribution of a hidden Markov chain, which fulfils the simultaneous linear equations

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \quad \sum_{i \in S} \pi_i = 1. \tag{11}$$

In order to determine the availability of transport system objects (transport means) based on the semi-Markov operation process model, the operational states of the technological object should be divided into availability states S_G and non-availability state S_{NG} of the object to realise the assignment. Technological object availability states are states during which the object, including the operator, remains in the operation system, and is efficient and supplied or will be repaired and/or supplied in a period of time shorter than the time reserve which is to serve the purpose. Non-availability states are states in which the object or the operator remains outside the operation system (efficient or inefficient), as well as when an inefficient and/or unsupplied object remains in the operation system.

In the presented model, for each level of availability the following technological object availability states were defined:

– for the first level:

State S_1 – stopover at bus depot parking space,
State S_3 – carrying out of the transport goal,
State S_7 – awaiting the start of task realisation after technical support repair,

– for the second level:

State S_1 – stopover at bus depot parking space,
State S_3 – carrying out of the transport goal,
State S_7 – awaiting the start of task realisation after technical support repair,
State S_4 – intake between transport peak hours,

– for the third level:

State S_1 – stopover at bus depot parking space,
State S_3 – carrying out of the transport goal,
State S_7 – awaiting the start of task realisation after technical support repair,
State S_4 – intake between transport peak hours,
State S_2 – repair at bus depot parking space,
State S_5 – repair by technical support unit without losing a trip.

Then, with the use of the MATHEMATICA software, the limit probability p_i^* of staying in states of semi-Markov process and the availability of technological objects of the transport system were determined, defined by the following dependencies:

$$G_{OT(1)}^{24} = p_1^* + p_3^* + p_7^* \tag{12}$$

$$G_{OT(1)}^{24} = \frac{(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta_1}+\overline{\Theta_3}+p_{36}\cdot p_{67}\cdot\overline{\Theta_7}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \tag{13}$$

$$G_{OT(2)}^{24} = p_1^* + p_3^* + p_7^* + p_4^* \tag{14}$$

$$G_{OT(2)}^{24} = \frac{(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta_1}+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{36}\cdot p_{67}\cdot\overline{\Theta_7}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \tag{15}$$

$$G_{OT(3)}^{24} = p_1^* + p_3^* + p_7^* + p_4^* + p_2^* + p_5^* \tag{16}$$

$$G_{OT(3)}^{24} = \frac{(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+p_{36}\cdot p_{67}\cdot\overline{\Theta_7}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \tag{17}$$

Then, using the above formulas, values of availability of transport means used in the tested municipal bus transport system were defined (Table 1).

Table 1. Values of availability of transport means used in the tested municipal bus transport system
Tablica 1. Wartości gotowości środków transportu eksploatowanych w systemie autobusowej komunikacji miejskiej

$G_{OT(1)}^{24}$	$G_{OT(2)}^{24}$	$G_{OT(3)}^{24}$
0,8444	0,8509	0,8518

4. Conclusion

Using semi-Markov processes for the modelling of the operational process of transport means facilitates the determination of transport means availability in case of time periods between changes of individual process states having arbitrary probability distribution and transfer to the following state depends only on the current process state.

Availability of technological objects (transport means) used in the municipal bus transport system, determined on the basis of semi-Markovian operational process model, depends directly on the values of limit probabilities

p_i^* of being at the states of the analysed process, and indirectly on the values of probabilities of transfers between process states p_{ij} (values of the elements of matrix P) as well as values of conditional duration periods of process states $\overline{\Theta}_{ij}$ (values of matrix Θ).

Change of the values of probabilities p_{ij} as well as duration periods $\overline{\Theta}_{ij}$ causes the change of the values of transport system technological objects availability G_{OT}. Values of availability of transport system technological objects G_{OT} depend on many factors:

- In case of the first level:
  - The reliability of the technological objects in use,
  - The efficiency of repair processes carried out at logistics subsystem posts and by technical support units,
- In case of the second level:
  - The number of fuel supply posts in the subsystem,
  - The efficiency of the subsystem fuel intake posts,
  - The values of the time reserve for supply of technological objects without losing a trip,
- In case of the third level:
  - The number of technical support units,
  - Equipping the technical support units with tools and devices used in technological object repair ensuring high efficiency of the performed repairs,
  - The serviceability and repair efficiency of technological objects,
  - The values of time reserve for technological object repair without losing a trip.

Values of transport means availability obtained on the basis of operational data presented in Table 1 are not high. However, one should take into consideration the fact that high availability of the utilisation subsystem comprised of N number of transport means may be obtained as a result of using the appropriate structure which integrates technological objects and numbers of reserve objects. The model of defining the availability of the utilisation subsystem comprised of N transport means will be prepared at future stages of the work conducted.

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Semimarkowski model gotowości środków transportu systemu komunikacji miejskiej

S t r e s z c z e n i e

W artykule przedstawiono metodę wyznaczania gotowości obiektów technicznych, użytkowanych w złożonych systemach eksploatacji, z wykorzystaniem teorii dotyczącej procesów semi-Markowa. Całość rozważań przedstawiono na przykładzie wybranego rzeczywistego systemu eksploatacji środków transportu – systemu autobusowej komunikacji miejskiej w wybranej aglomeracji. Bezpośrednią realizacją zadań przewozowych systemu komunikacji miejskiej zajmuje się podsystem wykonawczy złożony z podsystemów elementarnych typu operator–środek transportu (kierowca–autobus), których gotowość w istotny sposób wpływa na możliwość prawidłowej realizacji tych zadań.

W celu zbudowania modelu gotowości środków transportu wyznaczono istotne stany eksploatacyjne procesu eksploatacji oraz dokonano podziału i redukcji liczby stanów ze względu na kryterium gotowości do działania. Na tej podstawie zbudowano zdarzeniowy oraz matematyczny model procesu eksploatacji środków transportu, zakładając, że matematycznym modelem badanego procesu eksploatacji jest jednorodny proces semi-Markowa. Następnie dla danych eksploatacyjnych, uzyskanych z badań rzeczywistego procesu eksploatacji, wyznaczono graniczne wartości współczynnika gotowości dla wyróżnionych poziomów gotowości środków transportu eksploatowanych w systemie komunikacji miejskiej.

W prezentowanej pracy przedstawiono sposób wyznaczania gotowości pojedynczego obiektu technicznego – środka transportu, na podstawie zbudowanego semimarkowskiego modelu procesu eksploatacji, realizowanego w podsystemie wykonawczym badanego systemu transportu miejskiego. Przedstawiony w pracy model wyznaczania gotowości środków transportu jest częścią składową opracowywanego w ramach szerszego projektu badawczego, decyzyjnego modelu sterowania gotowością systemu transportowego.