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The modelling of the reliability of selected devices in an aircraft under conditions of the accumulation of effects of destructive processes

K e y w o r d s

Reliability, limit state, diagnostic parameter, nominal value, deviation from nominal value.

S ł o w a k l u c z o w e

Niezawodność, stan graniczny, parametr diagnostyczny, wartość nominalna, odchyłka od wartości nominalnej.

S u m m a r y

The article presents the way of determining the operational reliability of a device under conditions of destructive processes leading to the change of value of diagnostic parameters. It was assumed that, among diagnostic parameters determining the technical state of a device, there is a dominant parameter. Its values are the highest ones and the limit state is reached in the fastest way. It was assumed that effects of destructive processes accumulate, e.g. increase variance. The model of the symmetric random walk of the deviation value from the nominal value of the dominant parameter was used to determine the reliability.

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1. Introduction

Examining the reliability of aircraft in operation connects with forecasting their technical state. In most cases, this state is described with the use of diagnostic parameters. The change of their values enables the evaluation of the technical state change of aircraft devices. Destructive processes occurring during aircraft operation have a decisive influence on the change of diagnostic parameters. Examples of such processes are as follows:

- Wear processes of construction elements;
- Processes connected with surface wear which lead to the change of element sizes or the increase in clearances connected with the deterioration of the state of cooperating surfaces;
- Corrosion and ageing processes, which cause systems to get out of adjustment.

The physics and analytical description of these processes create lots of difficulties due to their complexity. That is why, different simplifications are used to describe the effects of these processes. Due to a wide range of the effects of these processes, usually only one process is considered. In this article, we will consider and describe the effects of ageing processes leading to the change of diagnostic parameter values.

Parameters describing a technical state of a device change due to ageing processes and internal conditions of a device, which are connected with its operation. Effects of these processes can lead to the following:

- A random increase of parameter values in the function of operation time;
- A random decrease of parameter values in the function of operation;
- A random fluctuation of parameter values in the function of operation time.

In this article, we will consider the case when the diagnostic parameter value changes undergo random fluctuations around the nominal value.

The following establishments are accepted:

1) The number of diagnostic parameters for the device that is being considered is N, i.e. the vector of the technical state has the following form

$$X = \left[x_1 + x_2 + \ldots x_N\right].$$

Diagnostic parameter values are independent, i.e. the change of value of one of them does not result in the change of values of other parameters.

2) The reliability state of a device is determined according to a selected diagnostic parameter (for that the exceedance of the limit value is reached in the fastest way). This parameter is regarded as a dominant one and is marked with x.
3) If current values of a dominant parameter are included in the range

$$X \in \left\lfloor x_d, x_g \right\rfloor$$

where: x_d – the lower limit value of a dominant parameter, and
x_g – the upper limit value of a dominant parameter,

a device is considered to be able to work. Otherwise, the device is considered to be unable to work.

2. The model of the evaluation of the reliability state of a device

It is assumed that

1) The mean value of a dominant diagnostic parameter agrees with the nominal value and is a constant value. Ageing processes lead to the increase of the variance of deviation value from the nominal value
2) Deviation value is determined in the following way:

$$\begin{aligned} z &= x_p - x_n \quad \text{for} \quad x_p > x_n, \\ -z &= x_p - x_n \quad \text{for} \quad x_p < x_n. \end{aligned} \tag{1}$$

where: x_p – the measured value of a dominant parameter,
x_n – the nominal value of a dominant diagnostic parameter.

3) The check-up of a diagnostic parameter is performed at the time interval Δt;
4) The measurement of a diagnostic parameter is performed in the discrete system with step h, where $h = \frac{x_p - x_n}{L}$, and L is selected suitably.
5) Parameter deviations from the nominal value has the following discrete values:

$$z_1 = lh \qquad \text{where} \qquad 1 = ...,-2,-1,0,1,2,...$$

6) The diagram of changes of parameter deviation value is presented in Fig. 1.

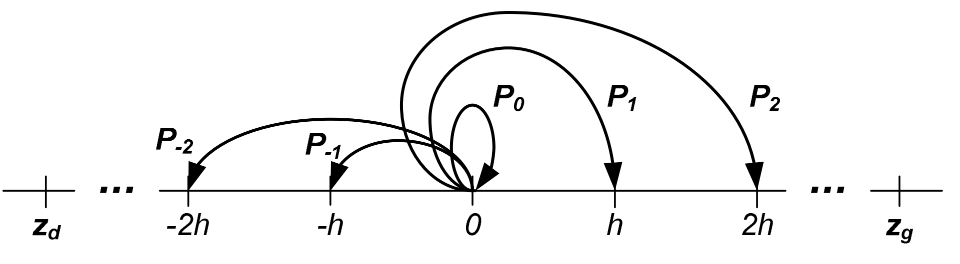

Fig. 1. The diagram of changes of diagnostic parameter deviation
Rys. 1. Schemat zmian odchyłki parametru diagnostycznego

7) At the time interval Δt, changes of diagnostic parameter deviation can equal to 0, h, $2h$, $-h$, $-2h$ with the probability P_0, P_1, P_2, P_{-1}, P_{-2}, where:

$$P_0 + P_1 + P_2 + P_{-1} + P_{-2} = 1 .$$

Thus, other values of parameter deviation at the time interval Δt are unlikely.

Having the above-mentioned assumptions, we can move to the determination of the diagram of the dynamics of changes of diagnostic parameter deviation. Having sufficiently long operation time and large number of measurements of parameter deviations, we can move from the probability of adopting particular values by deviation to the description of changes with the use of the density function.

Let $u(z,t)$ mean the density function of deviation value at the time $t=\Delta tk$ (where k means k-th measurement of the parameter z).

Having the above mentioned determinations and assumptions, we can present the dynamics of deviation changes in a probabilistic point of view with the use of the following difference equation:

$$u(z,t+\Delta t) = P_2 u(z-2h,t) + P_1 u(z-h,t) + P_0 u(z,t) + P_{-2} u(z+2h,t) + P_{-1} u(z+h,t) \tag{2}$$

The difference equation (2) can be converted to the partial differential equation with the use of the following approximation:

$$u(z,t+\Delta t) \cong u(z,t) + \frac{\partial U(z,t)}{\partial t}\Delta t ,$$

$$u(z-2h,t) = u(z,t) - \frac{\partial u(z,t)}{\partial z}2h + \frac{1}{2}\frac{\partial^2 u(z,t)}{\partial z^2}(2h)^2 ,$$

$$u(z-h,t) = u(z,t) - \frac{\partial u(z,t)}{\partial z}h + \frac{1}{2}\frac{\partial^2 u(z,t)}{\partial z^2}h^2 ,$$

$$u(z+2h,t) \cong u(z,t) + \frac{\partial u(z,t)}{\partial y}2h + \frac{1}{2}\frac{\partial^2 u(z,t)}{\partial y^2}(2h)^2 ,$$

$$u(z+h,t) \cong u(z,t) + \frac{\partial u(z,t)}{\partial y}h + \frac{1}{2}\frac{\partial^2 u(z,t)}{\partial y^2}h^2 .$$

The way of converting the difference equation (2) to the partial differential equation is presented in the treatise [1]. Thus, we obtain

$$\frac{\partial u(z,t)}{\partial t}=-b\frac{\partial u(z,t)}{\partial z}+\frac{1}{2}a\frac{\partial^2 u(z,t)}{\partial z^2}. \tag{3}$$

where:

$$b=\frac{\left(2P_2+P_1-P_{-1}-2P_{-2}\right)h}{\Delta t}, \tag{4}$$

$$a=\frac{\left(4P_2+P_1+P_{-1}+2P_{-2}\right)h^2}{\Delta t}. \tag{5}$$

The partial equation (3) for the symmetric walk of parameter deviation, i.e. $P_2=P_{-2}$ i $P_1=P_{-1}$ has the following form:

$$\frac{\partial u(z,t)}{\partial t}=\frac{1}{2}a\frac{\partial^2 u(z,t)}{\partial z^2}. \tag{6}$$

The solution of the partial differential equation (6) is the following density function:

$$u(z,t)=\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}. \tag{7}$$

In order to confirm that the function (7) is the solution of the equation (6), we must calculate derivatives in the equation (6) and check whether the left side is equal to the right side.

$$\frac{\partial u(z,t)}{\partial t}=\frac{-\frac{1}{2}2\pi a\left(2\pi at\right)^{-\frac{1}{2}}}{2\pi at}e^{-\frac{z^2}{2at}}+\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}\left(-\frac{z^2 2a}{4a^2t^2}\right)=$$

$$=\frac{-\pi a}{\sqrt{2\pi at}\,2\pi at}e^{-\frac{z^2}{2at}}+\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}\left(-\frac{z^2}{2at^2}\right)=u(z,t)\left(\frac{z^2-at}{2at^2}\right),$$

$$\frac{\partial u(z,t)}{\partial t}=u(z,t)\left(\frac{z^2-at}{2at^2}\right),$$

$$\frac{\partial^2 u(z,t)}{\partial z^2}=u(z,t)\left(\frac{z^2-at}{a^2t^2}\right).$$

Substituting derivatives obtained into the equation (6), we obtain

$$\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}\left(\frac{z^2-at}{2at^2}\right)=\frac{1}{2}a\left(\frac{z^2-at}{a^2t^2}\right)\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}. \tag{8}$$

The dependence (8) shows that the density function (7) is the solution of equation (6), and we can use the function to determine the reliability of a device.

The reliability of a device, which was determined according to the selected dominant diagnostic parameter, will be as follows:

$$R(t)=\int_{z_d}^{z_g}\frac{1}{\sqrt{2\pi at}}\,e^{-\frac{z^2}{2at}}\,dz\,. \tag{9}$$

where: z_d – the lower limit of parameter deviation;
z_g – the upper limit of parameter deviation.

Considering symmetric changes of deviation, the unreliability of a device can be written in the following form:

$$Q(t)=\int_{-\infty}^{-z_d}u(z,t)\,dz+\int_{z_g}^{\infty}u(z,t)\,dz=2\int_{z_g}^{\infty}u(z,t)\,dz\,, \tag{10}$$

where:

$$u(z,t)=\frac{1}{\sqrt{2\pi at}}\,e^{-\frac{z^2}{2at}}\,dz\,.$$

3. Determining the time distribution of the exceedance of limit values by deviation value

Using the density function (7) and the dependence (10), then the time density function of limit state exceedance, we obtain

$$f(t)=\frac{\partial}{\partial t}Q\left(t,z_g,z_d\right). \tag{11}$$

Thus, we can write

$$f(t)=2\int_{zg}^{\infty}\left\{\frac{\partial}{\partial t}Q\left(t,z_g,z_d\right)\right\}dz\,.$$

Thus:

$$f(t)=2\int_{zg}^{\infty}\left\{u(z,t)\left(\frac{z^2-at}{2at^2}\right)\right\}dz\,. \tag{12}$$

In order to calculate the integral in the dependence (12), we must determine the antiderivative.

We can determine the antiderivative with the use of the antiderivative in the following form:

$$w(z,t)=u(z,t)\Theta(z,t). \tag{13}$$

where: $\Theta(z,t)$ – is the unknown expression.

The derivative of the antiderivative $w(z,t)$ with respect to parameter deviation will be

$$\frac{\partial w(z,t)}{\partial z}=u'(z,t)\Theta(z,t)+u(z,t)\Theta'(z,t). \tag{14}$$

The derivative of the antiderivative with respect to deviation shall be equal to the integrand expression in the dependence (12).

Thus, after determining the derivative $\frac{\partial u(z,t)}{\partial z}$ we obtain:

$$\frac{\partial w(z,t)}{\partial z}=\underbrace{\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}dz\left(-\frac{z}{at}\right)}_{u(z,t)}\underbrace{\left(\;?\;\right)}_{\Theta(z,t)}+\underbrace{\frac{1}{\sqrt{2\pi at}}e^{-\frac{z^2}{2at}}dz}_{u(z,t)}\underbrace{\left(\;?\;\right)}_{\Theta'(z,t)}.$$

Thus:

$$\begin{aligned}\Theta(z,t)&=-\frac{z}{2t},\\ \Theta'(z,t)&=-\frac{1}{2t}.\end{aligned} \tag{15}$$

We check whether we obtain the integrand in the dependence (12)

$$\left[\left(-\frac{z}{at}\right)\left(-\frac{z}{2t}\right)-\frac{1}{2t}\right]=\frac{z^2}{2at^2}-\frac{t}{2t^2}=\frac{z^2-at}{2at^2}.$$

The antiderivative has the following form:

$$w(z,t)=u(z,t)\left(-\frac{z}{2t}\right). \tag{16}$$

Using the dependence (16), we can calculate the integral in the dependence (12), and we obtain the function of the first passage of the limit state. Thus,

$$f(t,z_g)=2w(z,t)\Big|_{z_g}^{\infty}=2u(z_g,t)\left(\frac{z_g}{2t}\right). \tag{17}$$

where:

$$u(z_g,t)=\frac{1}{\sqrt{2\pi at}}e^{-\frac{z_g^2}{2at}}.$$

Then, we check whether the function (17) is the function of the first passage of the limit state, i.e. we must demonstrate that

$$\int_0^\infty 2u\left(z_g,t\right)\left(\frac{z_g}{2t}\right)dt \overset{?}{=} 1. \tag{18}$$

Thus:

$$2\int_0^\infty \left(\frac{z_g}{2t}\right)\frac{1}{\sqrt{2\pi\, at}}\; e^{-\frac{z_g^2}{2at}}dt = \int_0^\infty \frac{z_g}{t\sqrt{2\pi\, at}}\; e^{-\frac{z_g^2}{2at}}dt =$$

$$= \frac{z_g}{\sqrt{2\pi\, a}}\int_0^\infty \frac{1}{t\sqrt{t}}\; e^{-\frac{z_g^2}{2at}}dt = \frac{z_g}{\sqrt{2\pi\, a}}\int_0^\infty \frac{1}{t^{\frac{3}{2}}}\; e^{-\frac{1}{2\frac{a}{z_g^2}t}}dt.$$

The following equality shall occur:

$$\int_0^\infty t^{\frac{3}{2}}\; e^{-\frac{1}{2\frac{a}{z_g^2}t}}dt \overset{?}{=} \frac{\sqrt{2\pi a}}{z_g}. \tag{19}$$

We check the equation (19) by calculating the integral

$$\int_0^\infty t^{\frac{3}{2}}\; e^{-\frac{1}{2\frac{a}{z_g^2}t}}dt = \frac{\sqrt{2\pi a}}{z_g}.$$

For this purpose, we make a substitution

$$u = \frac{1}{2\frac{a}{z_g^2}t}, \quad \text{hence} \quad t = \frac{z_g^2}{2a}\frac{1}{u},$$

$$dt = \frac{z_g^2}{2a}\left(-\frac{1}{u^2}\right)du.$$

Thus,

$$\int_0^{\infty}\left(\frac{z_g^2}{2a}\frac{1}{u}\right)^{-\frac{3}{2}}e^{-u}\frac{z_g^2}{2a}\left(-\frac{1}{u^2}\right)du=\left(\frac{z_g^2}{2a}\right)^{-\frac{3}{2}}\frac{z_g^2}{2a}\int_0^{\infty}\left(\frac{1}{u}\right)^{-\frac{3}{2}}\left(-\frac{1}{u^2}\right)e^{-u}du=$$

$$=\frac{(2a)^{\frac{3}{2}}}{z_g^3}\frac{z_g^2}{2a}\int_0^{\infty}\left(\frac{1}{u}\right)^{-\frac{1}{2}}e^{-u}du=\frac{(2a)^{\frac{1}{2}}}{z_g}\underbrace{\int_0^{\infty}u^{-\frac{1}{2}}e^{-u}du}_{\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}}=\frac{\sqrt{2\pi a}}{z_g}. \tag{20}$$

Thus, the solution of the integral from the formula (19) is $\frac{\sqrt{2\pi a}}{z_g}$, which validates the equality (19).

4. Final remarks

Conditions in which the operation process of aircraft proceeds (changes of temperature and pressure, oscillations, etc.) cause the accumulation of destructive factor influences on devices mounted on board. The method presented enables the determination of the reliability of a device. The reliability of a device with respect to the diagnostic parameter considered can be determined with the use of the dependence (9) or the distribution of the limit state exceedance (17) and the following formula:

$$R(t)=1-\int_0^t f(t,z_g)dz.$$

In a similar way, we can determine the dependence in the case of the unsymmetrical walk of the diagnostic parameter deviation.

References

[1] Będkowski L., Dąbrowski T.: *Podstawy eksploatacji,* cz. 1. *Podstawy diagnostyki technicznej.* WAT, Warszawa 2000.

[2] Gerebach J.B., Kordowski CH.B.: *Modele niezawodnościowe obiektów technicznych.* Wydawnictwo Naukowo-Techniczne. Warszawa 1968.

[3] Szczepanik R., Tomaszek H.: *Zarys metody oceny niezawodności i trwałości urządzeń lotniczych z uwzględnieniem stanów granicznych.* Problemy eksploatacji Nr 3, 2005, s. 247÷256.

[4] Szczepanik R., Tomaszek H.: *Metoda określenia rozkładu czasu do przekroczenia stanu granicznego.* ZEM Zeszyt 4(144), 2005, s. 35÷44.

[5] Szczepanik R., Tomaszek H.: *Niektóre charakterystyki rozkładu czasu przekraczania stanu granicznego*. ZEM Zeszyt 1(145), 2006, s. 99÷105.
[6] Tomaszek H., Żurek J., Jasztal M.: *Prognozowanie uszkodzeń zagrażających bezpieczeństwu lotów statków powietrznych*. Wydawnictwo Naukowe Instytutu Technologii Eksploatacji, Radom 2008.
[7] Tomaszek H., Ważny M.: *Zarys metody oceny wpływu skutków działania procesów destrukcyjnych na stan techniczny systemów celowniczych*. ZEM Zeszyt 3(147), 2006, s. 91÷102.
[8] Tomaszek H., Ważny M.: *Rozkład czasu narastania odchyłek parametrów diagnostycznych lotniczych systemów celowniczych do wartości granicznych*. ZEM Zeszyt 4(148) 2006, s. 81÷90.
[9] Żurek J.: *Modele oceny niezawodności obiektów technicznych*. Materiały Konferencyjne na temat „Problematyka normalizacji, jakości i kodyfikacji w aspekcie integracji z NATO i UE", Warszawa 2009, s. 81÷90.

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Modelowanie niezawodności wybranych urządzeń statku powietrznego w warunkach kumulowania skutków działania procesów destrukcyjnych

S t r e s z c z e n i e

W artykule przedstawiono sposób określenia eksploatacyjnej niezawodności urządzenia w warunkach działania destrukcyjnych procesów powodujących zmiany wartości parametrów diagnostycznych. Przyjęto, że wśród parametrów diagnostycznych, określających stan techniczny urządzenia, istnieje parametr dominujący, którego zmiany wartości są największe i najszybciej osiągany jest stan graniczny. Założono, że skutki działania procesów destrukcyjnych kumulują się np. wzrostem wariancji. Do określenia zależności na niezawodność wykorzystano model symetrycznego błądzenia przypadkowego wartości odchyłki od wartości nominalnej parametru dominującego.