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Accuracy of reliability measures of water supply and sewage facilities

K e y w o r d s

Reliability, reliability measures, method's error

S ł o w a k l u c z o w e

Niezawodność, miary niezawodności, błąd metody.

S u m m a r y

Understanding of reliability measures for different technical facilities plays an important role in a decision-making process. Most methods used for the estimation of reliability measures impose an additional error on the final result. The error, called the method's error, is not related in any way to data accuracy and reliability. It is the method's error. Both assumptions and application conditions formulated for the analysed methods, though not very accurate, seem to be sufficient for theoretical validation of the method. So far, due to a lack of appropriate ways of determination of errors in reliability estimation methods, such errors are not considered in practical applications and the accuracy of the final results is not analysed. Rough estimates without proper error evaluation may become useless. The paper analyses the accuracy of the most popular methods used for the determination of reliability measures for technical facilities. The relationships presented in the paper help to determine the errors of the presented methods. The relationships may be used for different major technical facilities.

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1. Introduction

Water and sewage systems are a part of a strategic municipal infrastructure; therefore, the ability to determine their principal reliability parameters (indicators and measures of reliability) becomes very important. Equally important is the evaluation of the accuracy of these parameters, since measures bearing large errors may lead to wrong decisions. The paper [1] examines ways for the estimation of an impact of inaccuracies or the scarcity of operational input data on the final outcome's error. In the paper, the ways of the evaluation of methods' errors are reviewed. The method's error results directly from using engineering calculations, usually simplified, where elements of low value are ignored. The simplified method gives approximate results.

In the following sections, it is assumed that systems (S) consist of two-state renewable elements, which fail independently; their failures are random and operating and disability times for the elements and the system itself are described with an exponential distribution. Additionally, it is assumed that the entire system is a two-state system. Analysis of these assumptions has been performed in the paper [2].

Particular elements of a water supply system (SZW) or a sewage system (SUŚ) may have different reliability structures. The structure depends on capacity or throughput of system elements and the requirements set for the particular object. It may be a serial (e.g. for a sustainable SZW, for a typical water supply system), parallel or threshold (e.g. for a pump station), mixed (e.g. for a water supply system or a wastewater treatment plant) or a complex structure (e.g. for a water distribution system). To determined reliability measures for these structures, different one or two- parameter methods are used.

2. Errors of a one-parameter method

One- parameter methods [4], [5] help to determine one basic measure of system reliability, i.e. an asymptotic availability K_S, known as reliability and written as:

$$K_S = \frac{Tp_S}{Tp_S + Tn_S} \tag{1}$$

where Tp_S, Tn_S – average system operation and disability times, respectively. K_S is interpreted as the probability that, at any time after the system's start-up, the system remains in operation.

For more complex systems, for which a reliability scheme cannot be constructed review methods (MP) are used. A table of the system elementary states is arranged in this method. Depending on the number of the states, two review methods may be used: a complete review method (MPZ) for all possible elementary states, and a partial review method (MPCz) for only the most probable elementary states, where no more then k_{max} elements failed; usually k_{max}= 2 is assumed. In the following discussion, K_S shows an unknown, accurate value while K_S(MPCz) shows an approximate value obtained with MPCz as [4] and [5]:

$$K_S(\text{MPCz}) = \sum_{i \in E1} P_i \tag{2}$$

where i – number of a system elementary state, Pi – probability of this state occurrence, E1 – set of system operation states. Estimation

$$K_S(\text{MPCz}) \le K_S < K_S(\text{MPCz}) + \varepsilon \tag{3}$$

seems accurate, where an estimation error

$$\varepsilon = P(k > k_{max}) = 1 - P(k \le k_{max}) \tag{4}$$

defines the probability of the occurrence of states not included in MPCz. If the maximum error ε is too high, than states with a higher number of simultaneous failures k_{max} have to be considered. Error estimation is performed by the engineer.

3. Errors in two-parameter methods

An asymptotic availability K_S does not define the system explicitly. To do so, two independent system reliability measures are required – the average operation time Tp_S and the average disability time Tn_S. The measures can be determined with two-parameter methods. To use two-parameter methods, two independent parameters have to be known for each system element (i = 1,...n): the average operation time Tp_i and the average disability time Tn_i. Instead of these parameters, a failure rate $\lambda_S = 1/Tp_S$ or $\lambda_i = \dfrac{1}{Tp_i}$ may also be used.

3.1. Classic method of a failure frequency

A failure frequency can be defined as [4], [5]

$$f_S = \frac{1}{Tp_S + Tn_S} \tag{5}$$

It defines the average number of system failures at a time unit. If Ks and fs are known, then Tp_S and Tn_S are calculated from relationships:

$$Tp_S = \frac{K_S}{f_s} \quad \text{and} \quad Tn_S = \frac{1 - K_s}{f_S} \tag{6}$$

A classic method of failure frequency may be used for systems in which K_S and f_S may be precisely determined with analytical equations.

For a n-element serial structure:

$$K_s = \prod_{i=1}^{n} K_i \quad \text{and} \quad f_S = \sum_{i=1}^{n} f_i \cdot \prod_{j \neq i} K_j \tag{7}$$

Then, after necessary transformations

$$\lambda_S = \sum_{i=1}^{n} \lambda_i \quad \text{and} \quad Tn_S \approx \frac{\sum_{i=1}^{n} \lambda_i \cdot Tn_i}{\lambda_S} \tag{8}$$

An approximation sign for Tn_S indicates that a number of terms in the total value of

$$\Delta(Tn) = \frac{\sum_{i<j} \lambda_i \lambda_j Tn_i Tn_j + \sum_{i<j<k} \lambda_i \lambda_j \lambda_k Tn_i Tn_j Tn_k + \ldots + \lambda_1 \lambda_2 \ldots \lambda_n Tn_1 Tn_2 \ldots Tn_n}{\lambda_S} \tag{9}$$

have been ignored.

For engineering applications $Tp_i >> Tn_i$. In theory, it means that the ignored values are very small. The Δ(Tn) value is an absolute error for Tn_S.

For a n-element parallel structure:

$$K_s = 1 - \prod_{i=1}^{n} (1 - K_i) \quad \text{and} \quad f_S = \sum_{i=1}^{n} f_i \cdot \prod_{j \neq i} (1 - K_j) \tag{10}$$

Then, after necessary transformation the equation is in the form:

$$\lambda_S \approx \prod_{i=1}^{n} \lambda_i \left(\sum_{i=1}^{n} \prod_{j \neq i} Tn_j \right) \quad \text{and} \quad Tn_S = \frac{\prod_{i=1}^{n} Tn_i}{\sum_{i=1}^{n} \prod_{j \neq i} Tn_j} \tag{11}$$

As before, the approximation sign for λ_S means that the formula has been simplified and terms of low value have been ignored. A relative error may be calculated as:

$$\delta(\lambda_S)=(\lambda_S-\lambda_S(f_S))/\lambda_S=(1-\lambda_S(f_S)/\lambda_S)\cdot 100\% \tag{12}$$

where λ_S, $\lambda_S(f_S)$ – accurate failure rate and approximate failure rate, respectively; a term $\lambda_S(f_S)/\lambda_S$ is calculated from the relationship:

$$\frac{\lambda_S}{\lambda_S(f_S)}=1+\sum_{i=1}^{n}\lambda_i Tn_i+\sum_{i<j}\lambda_i\lambda_j Tn_i Tn_j+\sum_{i<j<k}\lambda_i\lambda_j\lambda_k Tn_i Tn_j Tn_k+\ldots+\sum_{i=1}^{n}\prod_{j\neq i}\lambda_j Tn_j \tag{13}$$

For threshold structures of a “n from M” type, values of K_S and f_S are determined from the review method table. In such case, Tp_S and Tn_S obtained from (6) should be accurate. However, in some cases, i.e. when the entire system is highly reliable (highly reliable elements or a substantial redundancy of elements), the results become less accurate due to numerical errors; they originate from calculations carried out on infinitesimal values ($1-K_S$, f_S) Analysis of such problems has been performed in the work [3].

3.2. General method of failure frequency

In complex systems, where many different types of reserves exist or where elements are not uniform, a failure frequency function may be difficult or even impossible to develop as a mathematical formula. In such case, a general method of failure frequency may be used [2]. A failure rate for any system is equal to [6]:

$$\Phi_S=\sum_{z\in E0}\sum_{i\in E1}P_i\lambda_{iz} \tag{14}$$

where i, z – number of system elementary states, E1,E0 – sets of system operation and disability states, respectively, P_i – probability of occurrence of i-state, λ_{iz} – transfer rate from i-state to z-state in a system. Moreover [6]:

$$\Phi_S=\frac{1}{Tp_S+Tn_S}\qquad Tp_S=\frac{1}{\Phi_S}\sum_{i\in E1}P_i\qquad Tn_S=\frac{1}{\Phi_S}\sum_{i\in E0}P_i \tag{15}$$

Application of MPCz means that calculations have not been performed on accurate K_S and Φ_S values but only on their approximations given here as K_S(MPCz) and Φ_S(MPCz), respectively.

The average system operating and disability times may be determined using the following relationships:

$$\frac{K_S(MPCz)}{\Phi_S(MPCz)+\Delta(\Phi_S)} \le Tp_S \le \frac{K_S(MPCz)+\varepsilon}{\Phi_S(MPCz)} \tag{16}$$

$$\frac{U_S(MPCz)}{\Phi_S(MPCz)+\Delta(\Phi_S)} \le Tn_S \le \frac{U_S(MPCz)+\varepsilon}{\Phi_S(MPCz)} \tag{17}$$

where the estimation errors are equal: ε from formula (4) and

$$\Delta(\Phi_S) = \frac{\varepsilon \cdot \sum\limits_{k>k_{max}} (n-k)}{\min\limits_{i=1,..n} Tp_i} \tag{18}$$

If widths of intervals including Tp_S and Tn_S are small enough (from the engineering perspective) not to confuse a decision making process, then the calculation may be finished. Otherwise, the range of MPCz should be increased (i.e. k_{max} should be increased).

3.3. Method of minimum disability cross-sections

System reliability parameters such as Tp_S and Tn_S are calculated once the minimum disability cross-sections have been determined. A system disability cross-section is defined as a set of elements: if all the elements are disabled than the entire system is disabled too. The minimum disability cross-section does not include any other cross-section. In practice usually one, two or three-element cross-sections are determined and the following formulas are used [4], [5]:

- To determine measures for the minimum disability cross-sections
  - one-element: $\lambda_{[i]} = \lambda_i$, $Tn_{[i]} = Tn_i$ (19)
  - two-elements: $\lambda_{[i,j]} \approx \lambda_i \lambda_j (Tn_i + Tn_j)$, $Tn_{[i,j]} = \dfrac{Tn_i Tn_j}{Tn_i + Tn_j}$ (20)
  - three- elements: $\lambda_{[i,j,k]} \approx \lambda_i \lambda_j \lambda_k (Tn_i Tn_j + Tn_i Tn_k + Tn_j Tn_k)$ (21)

$$Tn_{[i,j,k]} = \frac{Tn_i Tn_j Tn_k}{Tn_i Tn_j + Tn_i Tn_k + Tn_j Tn_k} \tag{22}$$

- To determine the measures for the entire system

$$\lambda_S = \sum_{[i]} \lambda_{[i]} + \sum_{[i,j]} \lambda_{[i,j]} + \sum_{[i,j,k]} \lambda_{[i,j,k]} \qquad (23)$$

$$Tn_S \approx \frac{\sum_{[i]} \lambda_{[i]} Tn_{[i]} + \sum_{[i,j]} \lambda_{[i,j]} Tn_{[i,j]} + \sum_{[i,j,k]} \lambda_{[i,j,k]} Tn_{[i,j,k]}}{\lambda_S} \qquad (24)$$

The method of minimum disability cross-sections is a simplified method, since it is based on a simplified classic method of a failure frequency. Analytical estimation of the final Tp_S and Tn_S errors becomes impossible for a general case. From the analysis of the results obtained during numerous example calculations, it may be concluded that parameters Tp_S and Tn_S determined with this method may be inaccurate due to low reliability of system elements, system's complexity or both.

4. Example application

A town is supplied by n = 5 independent water supply systems. Their operation capacities are 10%Qn, 50%Qn, 5%Qn, 31%Qn and 46%Qn, where Qn – a nominal (designed) system capacity. The average operation times (Tpi, i = 1...5) are 720, 1200, 680, 1000 and 1000 hours, respectively. The average failure times (Tni, i = 1...5) are 24, 48, 18, 24 and 36 hours, respectively. The goal is to determine the reliability measures for the entire water supply system.

If evaluation and the decision making process concerning a possible system modernisation are based only on K_S then the MPCz method should be used. For $k_{max} = 1$ $0.926127 \le K_S < 0.935057$. Since the method's error ε = 0.008929 has been rather high, then the number of states is increased. For $k_{max} = 2$; $0.928108 \le K_S < 0.928384$; in that case the method's error ε = 0.000276 is acceptable.

If a decision maker needs to know the Tp_S and Tn_S values, then the general method of failure frequency (MPCz+Φs) or the method of minimum disability cross-sections (MMPN) may be used. Using (MPCz+Φs) for $k_{max} = 1$, the estimates $521.3 \le Tps \le 550.7$hours and $36.6 \le Tns \le 43.5$ hours were obtained. The error for Tns has been found as too high. For $k_{max} = 2$ the estimates $545.1 \le Tps \le 545.6$hours and $42.1 \le Tns \le 42.3$ hours were obtained. For the other method (MMNP) the minimum disability cross-sections were determined [2] and [6]. Values of Tps = 540.7h and Tns = 41.2h were obtained using formulas (19), (23) and (24). However, MMPN does not allow for error evaluation.

5. Summary

Water and sewage works, together with central heating and gas facilities, comprise a municipal strategic infrastructure. Therefore, apart from technical and economic criteria, reliability criteria have to be developed for them. The criteria are based on evaluation of basic reliability measures for these units. Determination of these measures is equally important as ability to evaluate the possible error. Estimated reliability measures show some input data errors; additionally, they also may carry on errors assigned to the methods used for the estimation of reliability measures. Methods' errors result from ignoring less probable states of a system, using simplified formulas or calculating with infinitesimal values. The ability to evaluate the methods errors as well as the errors associated with the input data helps to acquire results with accuracy satisfactory for practical applications. In a popular engineering practice, higher estimation accuracy of unknown operation parameters of the unit reduces risk during a decision making process.

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Dokładność metod wyznaczania miar niezawodności obiektów zaopatrzenia w wodę i usuwania ścieków

S t r e s z c z e n i e

Podstawą podejmowania decyzji o ewentualnej modernizacji obiektów technicznych jest znajomość ich miar niezawodności. Większość metod szacowania miar niezawodności wprowadza do wyniku dodatkowy, niezwiązany z niedokładnością i niepewnością danych błąd. Jest to błąd metody. Formułowane dla analizowanych metod założenia i warunki stosowalności, choć są nieprecyzyjne, wystarczają do teoretycznego uzasadnienia metody. Dotąd, wobec braku odpowiednich sposobów wyznaczania błędów metod szacowania niezawodności, w praktycznych zastosowaniach nie uwzględniano tych błędów i nie analizowano dokładności uzyskanych wyników. Wyniki przybliżone bez oceny popełnianych błędów mogą być nieprzydatne. W artykule przeanalizowano dokładność najczęściej stosowanych metod wyznaczania miar niezawodności obiektów technicznych. Przedstawione w artykule zależności pozwalają na określenie błędów analizowanych metod. Zależności mogą być stosowane dla różnych ważnych obiektów technicznych.