SCIENTIFIC PROBLEMS
OF MACHINES
OPERATION
AND MAINTENANCE
4 (156) 2008

HENRYK TOMASZEK*, MICHAŁ JASZTAL**

Comments on the improvement of the military aircraft operation by limiting the possibilities of damage formation

Key words

Reliability, safety, diagnostic parameter, limit state, preventive measures.

Słowa kluczowe

Niezawodność, bezpieczeństwo, parametr diagnostyczny, stan graniczny, profilaktyka.

Summary

Aircraft safety is one of the most important issues concerning their operation. Aircraft safety has a direct influence on economic indicators. In turn, aircraft safety depends on the reliability of an aircraft. Taking care of this reliability is equivalent to concern for safety. The reliability of an aircraft depends on an appropriate technical maintenance aimed at preventive measures, which include the development of technical projects that eliminate damage causes.

This article presents an attempt at describing the reliability of an aircraft with appropriately developed preventive measures that enable an assessment of an aircraft's reliability with the use of the exponential distribution.

1. Introduction

As far as safety aspect is concerned, the operation of aircraft mainly consist in the way of servicing (which is being developed), which is aimed at the elimi-

* Instytut Techniczny Wojsk Lotniczych, ul. Księcia Bolesława 6, 01-494 Warszawa, skr. poczt. 96, tel. (0-22) 6851956; faks (0-22) 836 44 71.

** Wojskowa Akademia Techniczna, Wydział Mechatroniki, ul. Kaliskiego 2, 00-908 Warszawa 49, tel. (0-22) 683 73 74, faks (0-22) 685 75 81.

nation of damage causes during flight, especially damages which may cause failures or crashes.

Despite all efforts, it is still impossible to make a full assessment of an aircraft's technical condition under the influence of destructive processes (fatigue and other types). Non-detection (overlooking) of signalling damages results in catastrophic (sudden) damages that usually causes serious failures and plane crashes.

This way of servicing aircraft engineering and assessing the reliability results in a suggestion concerning the classification of all types of damages of an aircraft. The classification is as follows:

- Signalling damages that can be removed without consequences;
- Signalling damages which are detectable; when such damages are not detected in due time they result in serious consequences;
- Catastrophic damages (non-signalling) which usually result in serious consequences, for example, aircraft equipment failures and air crashes.

So, it can be assumed that all damages concerning the period of an aircraft operation can be divided into two separate groups, i.e. signalling damages and catastrophic damages. It is accepted that an aircraft maintains its fitness state till the first out of two groups of damages.

Unfitness of an aircraft can be determined in the following way:

$$R(t) = R_1(t) \cdot R_2(t) \tag{1}$$

where: $R_1(t)$ – probability that until t there will be no sudden damage in an aircraft;

$R_2(t)$ – probability that until t there will be no signalling damage in an aircraft.

Above all, concern for safety includes taking care of high level of an aircraft's reliability during flight. In the next part of the study the possibility of assessment of the reliability and ways of its improvement are discussed.

2. The outline of the method of reliability assessment of an aircraft taking into consideration signalling and catastrophic damages

2.1. Description of the operation conditions and acceptance of assumptions

It is accepted that the operation of an aircraft includes the following assumptions:

1. n diagnostic parameters are used to assess technical condition. Thus, technical condition vector has the following form:

$$x = (x_1, x_2, ..., x_n).$$

2. As far as the reliability assessment is concerned, instead of diagnostic parameter values, the following deviations are used:

$$z_i = x_i - x_{i\,nom} \quad (i = 1,2,...,n),$$

where: x_i – i- diagnostic parameter;
$x_{i\,nom}$ – nominal value of i parameter.

3. Values of deviations z_i (i=1, 2, ... , n) are positive numbers.
4. Limit values of deviations are z_i^d. If $0 \le z_i < z_i^d$ $(i = 1,2,...,n)$ then an aircraft is faultless. If at least one deviation exceeds the admissible value, then an aircraft is non-operational.
5. It is assumed that deviations z_i $(i = 1,2,...,n)$ are independent random variables, i.e. value change of one deviation does not result in value change of another deviation.
6. Value change of deviations, z_I, results from an aircraft work, i.e. during an aircraft flight.
7. Deviation value change rate can be described by the following dependency:

$$\frac{dz_i}{dN} = g(z_i, c_i), \tag{2}$$

where: z_i – deviation of diagnostic parameter;
c_i – indicators of local operation conditions of elements which affect the growth in deviation of a diagnostic parameter;
N – the number of flights of an aircraft.

Using dependency (2), we can determine the value of deviation during one flight:

$$\Delta z_i = g(z_i, c_i)\Delta N, \qquad \text{for } \Delta N = 1. \tag{3}$$

8. Intensity of aircraft flights λ is determined by the following dependency:

$$\lambda = \frac{P}{\Delta t}, \tag{4}$$

where: Δt – time interval in which there will be flight of an aircraft with probability P. Time interval Δt shall be matched in such way (depends on functioning of an aircraft operational system) that $\lambda \Delta t \le 1$.

Using intensity of aircraft flights λ we can determine the number of flights performed up to t in accordance with the following dependency:

$$N = \lambda t. \tag{5}$$

9. It is assumed that an aircraft is operated. Technical service is supposed to prevent signalling damages and reduce the possibility of catastrophic damages which cause failures and air crashes.

2.2. Determining aircraft reliability taking into consideration signalling and catastrophic damages

Probabilistic description of the growth in the deviation of diagnostic parameters in the function of time of operation can be considered separately for each diagnostic parameter. Thus, we accept that we consider changes of deviation of *i*- a diagnostic parameter.

$U_{z_i,t}$ means the probability that at the moment *t* deviation of *i*- parameter is z_i.

Dynamics of changes (growth) of *i*- deviation can be determined by the following difference equation:

$$U_{z_i,t+\Delta t} = (1-\lambda\Delta t)U_{z_i,t} + \lambda\Delta t\, U_{z_i-\Delta z_i,t}\,, \tag{6}$$

where: $(1-\lambda\Delta t)$ – the probability that within time interval Δt there will be no flight of an aircraft;

$\lambda\Delta t$ – the probability of an aircraft flight within time interval Δt.

Hence:

$$(1-\lambda\Delta t)+\lambda\Delta t = 1.$$

Equation (6) determines the possibility that at the moment $t+\Delta t$, the value of deviation of *i*- diagnostic parameter will be z_i if, at the moment *t* deviation had this value and it did not increase because there was no flight of an aircraft, or at the moment *t* deviation had the following value: $z_i - \Delta z_i$ and during time interval Δt it increased Δz_i because the flight happened.

Difference equation (6) in the functional notation has the following form:

$$u(z_i,\, t+\Delta t) = (1-\lambda\Delta t)u(z_i,t) + \lambda\Delta t u(z_i-\Delta z_i,t)\,, \tag{7}$$

where: $u(z_i,t)$ – the function of the density of deviation of *i*- diagnostic parameter from the nominal value.

We will transform difference equation (7) into partial differential equation using the following approximations:

$$\begin{aligned} u(z_i,\, t+\Delta t) &= u(z_i,t) + \frac{\partial u(z_i,t)}{\partial t}\Delta t, \\ u(z_i-\Delta z_i,t) &= u(z_i,t) - \frac{\partial u(z_i,t)}{\partial z_i}\Delta z_i + \frac{1}{2}\frac{\partial^2 u(z_i,t)}{\partial z_i^2}(\Delta z_i)^2. \end{aligned} \tag{8}$$

Substituting (8) into equation (7) we obtain:

$$u(z_i, t) + \frac{\partial u(z_i, t)}{\partial z} \Delta t = (1 - \lambda \Delta t)\ u(z_i, t) + \lambda \Delta t$$
$$\left[u(z_1, t) - \frac{\partial u(z_i, t)}{\partial z_i} \Delta z_i + \frac{1}{2} \frac{\partial^2 u(z_i, t)}{\partial z_i^2} (\Delta z_i)^2 \right].$$

Hence, after reduction we obtain:

$$\frac{\partial u(z_i, t)}{\partial t} \Delta t = -\lambda \Delta z_i \Delta t \frac{\partial u(z_i, t)}{\partial z_i} + \frac{1}{2} \lambda \Delta t (\Delta z_i)^2 \frac{\partial^2 u(z_i, t)}{\partial z_i^2} . \qquad (9)$$

Dividing both sides of equation (9) by Δt, we obtain:

$$\frac{\partial u(z_i, t)}{\partial t} = -b_i(t) \frac{\partial u(z_i, t)}{\partial z_i} + \frac{1}{2} a_i(t) \frac{\partial^2 u(z_i, t)}{\partial z_i^2} , \qquad (10)$$

where: $b_i(t) = \lambda \Delta z_i$ – average growth in deviation of *i*- diagnostic parameter from the nominal value per unit of time;

$a_i(t) = \lambda (\Delta z_i)^2$ – average square of growth in deviation of *i*-diagnostic parameter from the nominal value per unit of time;

Δz_i – value of diagnostic parameter deviation during one flight (determined by dependency (3)).

We are looking for a special solution of equation (10) which meets the following conditions: if $t \to 0$, the solution is convergent with the Dirac function, i.e. $u(z_i,t) \to 0$, *for* $z \neq 0$ $u(0,t) \to \infty$, but in such way that the integral of function u is equal one for $t > 0$.

According to the accepted condition, the solution of equation (10) has the following form:

$$u(z_i, t) = \frac{1}{\sqrt{2\pi A_i(t)}} \quad e^{-\frac{(z_i - B_i(t))^2}{2A_i(t)}} , \qquad (11)$$

where:

$$B_i(t) = \int_0^t b_i(t) dt , \qquad (12)$$

$$A_i(t) = \int_0^t a_i(t)dt \,. \tag{13}$$

Dependency (12) determines the average value of deviation, and dependency (13) determines the variance of deviation.

Using dependency (11), we can write down the reliability in the aspect of signalling damage for *i*- diagnostic parameter in the following form:

$$R_i(t) \cong \int_{-\infty}^{z_i^d} u(z_i, t)\, d\, z_i \,. \tag{14}$$

Taking into consideration all diagnostic parameters and the above assumptions, we can write down formula determining the reliability in respect of signalling damage. The formula has the following form:

$$R_2(t) = \prod_{i=1}^{n} R_i(t) \,. \tag{15}$$

In order to determine the reliability defined by dependency (1), we must determine the dependency for the second component of an aircraft reliability $R_1(t)$ in respect of a catastrophic damage.

Catastrophic (sudden) damages result from an incomplete control and recognition of a technical condition of an aircraft.

The monitoring of aircraft operation indicates that this group of damages results from sudden changes of measurable and immeasurable parameters, which is caused by the impossibility to monitor the changes of parameter values. Moreover, the crossing of existing restrictions also increases the level of damage risk.

In a probabilistic description of generation of catastrophic damages, damage intensity is the main source of this type of damages. Damage intensity is determined by the following dependency:

$$\chi(t) = \lim_{\Delta t \to 0^+} \frac{P(t < T < t + \Delta t \mid T > t)}{\Delta t} \,, \tag{16}$$

where: T – random variable of time to catastrophic damage formation;
t – operation time of an aircraft;
$P(\,)$ – conditional probability of an event.

$$\chi(t) = \lim_{\Delta t \to 0^+} \frac{P\{t < T \le t + \Delta t\}}{\Delta t \; P\{t < T\}}$$

$$\chi(t) = \frac{-R_1^{'}(t)}{R_2(t)}$$

Transforming dependency (16), we can obtain the following differential equation:

$$R_1^{'}(t) + \chi(t) R_2(t) = 0 \,. \tag{17}$$

Equation (17) for an initial condition $R_1(t = 0) = 1$ has the following solution:

$$R_1(t) = e^{-\int_0^t \chi(t) dt} \,. \tag{18}$$

If: $\chi(t) = \chi = const$, then

$$R_1(t) = e^{-\chi t} \,. \tag{19}$$

To use dependency (19), we must assess parameter χ. From the monitoring of operation of aircraft, we can obtain the times of damage formation t_k, where $k = 1, 2, ..., \omega$.

Time t_k is the time of formation of the first damage in k- an aircraft. The time is calculated from the beginning of operation. To assess parameter χ, we will use the method of moments. We will compare an expected value of operation time (determined from a theoretical dependency) with an average value (determined from observations).

Theoretical average time up to catastrophic damage formation is

$$E[T] = \int_0^\infty R_1(t) dt = \int_0^\infty e^{-\chi t} dt = \frac{1}{\chi} \,.$$

An average value of operation time of an aircraft (up to catastrophic damage formation) is

$$\hat{E}[T] = \frac{\sum_{k=1}^{\omega} t_k}{\omega} \,.$$

Hence,

$$\frac{1}{\chi^*} = \frac{\sum_{k=1}^{\omega} t_k}{\omega},$$

$$\chi^* = \frac{\omega}{\sum_{k=1}^{\omega} t_k}. \tag{20}$$

If we know the probability of sudden damage formation during one flight, we can assess damage intensity using the following dependency:

$$\chi^* = \frac{Q}{\Delta t}.$$

Dependency for the assessment of reliability $R_1(t)$ has the following form:

$$R_1(t) = e^{-\chi^* t}. \tag{21}$$

Taking into consideration dependency (15) and (21), we obtain formula determining the reliability of an aircraft. The formula has the following form:

$$R(t) = e^{-\chi t} \prod_{i=1}^{n} R_i(t). \tag{22}$$

3. Comments on the way of forming the reliability

The above-presented outline of the method for determining the dependency for aircraft reliability is conditioned by assumptions that formalise reality. The above-presented method can be modified by accepting particular assumptions. The more real assumptions, the more precise predicted reliability of an aircraft.

The above-presented method concerns events in which the results of destructive processes are accumulated and correlated with the operation time of an aircraft. The process is disrupted by the possibility of the formation of sudden damages which are caused by, for example, overload pulses, crash landings, etc.

After some development, the method can enable the assessment of fatigue life in respect of particular diagnostic parameters. The obtained data can be used to improve technical servicing. The course of diagnostic controls that are ade-

quately determined and distributed in operation time prevents the results of signalling damages.

Thus, it can be assumed that

$$R_2(t) = \prod_{i=1}^{n} \int_{-\infty}^{z_d} u_1(z_i, t) dz_i \approx 1. \tag{23}$$

Thus, the reliability of an aircraft, taking into consideration technical servicing, can be assessed by the following dependency:

$$R(t) = e^{-\chi^* t}. \tag{24}$$

It seems that the above-presented outline of the method for predicting the reliability of an aircraft can be used as an auxiliary material for considering specific problems concerning the assessment of the reliability and fatigue life of elements, units, and devices.

All methods that were presented in the study are used to assess the reliability and fatigue life of construction elements in the aspect of fatigue.

The above-presented models can be adapted for specified cases of determining the probability of catastrophic damage taking into consideration phenomena physics and conditions of operation.

This conventional division of damages will help determine the reliability of an aircraft and present an algorithm of the reliability of an aircraft in the process of operation (Fig. 1).

The algorithm enables among other things:

- The analysis of the physics of damage formation in the form of increasing deviations of diagnostic parameters.
- The assessment of risk concerning the crossing of admissible values on the basis of the assessment of the growth in deviations.
- The assessment of reliability for particular diagnostic parameters and the determination of the range of technical interventions.
- The investigation of relationship concerning reliability requirements.
- The development of adjustment in respect of aircraft diagnostics in the case of negative results connected with the assessment of reliability (not meeting requirements).
- The introduction of additional diagnostic procedures into the process of operation in order to obtain required values of reliability by the following:
  - Reducing the results of signalling damages and increasing their delectability in order to prevent their transformation into catastrophic damages;
  - Reducing the number of catastrophic damages and the risk of their formation.

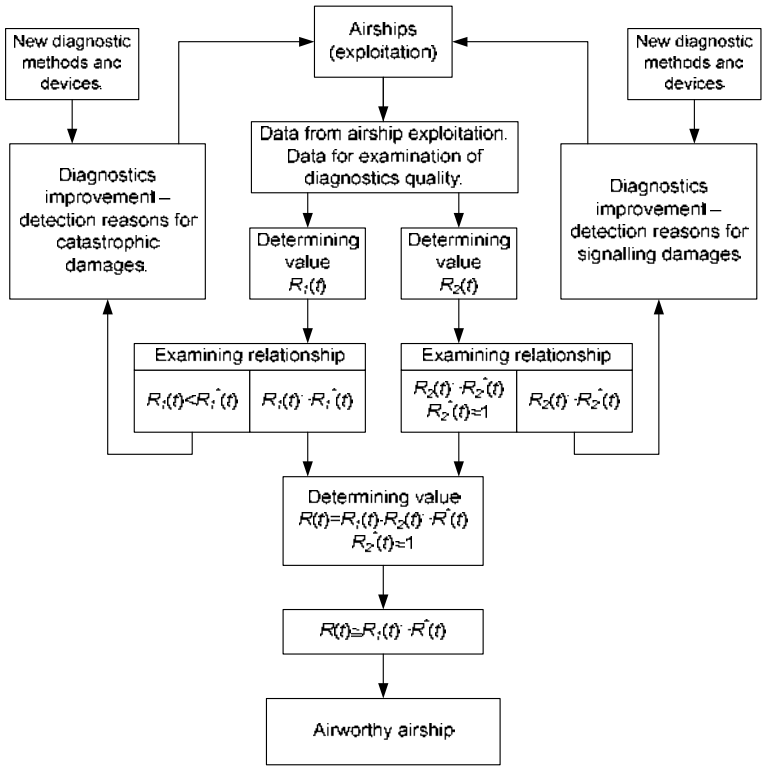

Fig. 1. Diagram of forming reliability and improving technical service (diagnostics) of aircraft
Rys. 1. Schemat kształtowania niezawodności i doskonalenia obsługi technicznej (diagnostyki) statków powietrznych

4. Reliability in stretch of time of the length τ

Service works and preventive measures, which are being continuously improved, indicate that the exponential distribution can be used to assess the reliability of an aircraft during flight. Moreover, the assessment of reliability can be reduced to the diagram of operation with replacement when the time for replacement is negligible. Thus, we can determine the probability of a non-failure operation of an aircraft in a finite time period $(t, t + \tau)$.

This probability is marked with $R_t(\tau)$. For a stationed aircraft, the formula has the following form:

$$R(\tau) = \lim_{t\to\infty} R_t(\tau) = \frac{1}{\theta}\int_0^\infty [1 - F(x+\tau)]dx ,$$

where: τ – time of flight of an aircraft;

$F(x)$ – distribution function for time of operation up to the occurrence of damage.

Because, for exponential distribution $\theta = \frac{1}{\lambda}$, the reliability in stretch of time of the length τ will be determined by the following dependency:

$$R(\tau) = \frac{1}{\theta}\int_{\tau}^{\infty} [1 - F(t)]dt = \frac{1}{\theta}\int_{\tau}^{\infty} \left[1 - \left(1 - e^{-\chi t}\right)\right]dt = \frac{1}{\theta}\int_{\tau}^{\infty} e^{-\chi t}dt =$$

$$= \frac{1}{\theta}\left[-\frac{1}{\chi}e^{-\chi t}\right]_{\tau}^{\infty} = e^{-\chi t}$$

5. Conclusions

The above-presented outline of the method for determining the dependency for aircraft reliability concerns events in which the results of destructive processes are accumulated and correlated with the operation time of an aircraft. The process is disrupted by the possibility of formation of sudden damages which are caused by, for example, overload pulses, crash landings, etc.

After some developments, the method can enable the assessment of fatigue life in respect of particular diagnostic parameters. The obtained data can be used to improve technical servicing. The course of diagnostic controls that are adequately determined and distributed in operation time prevents the results of signalling damages.

The presented division of damages helped to determine the reliability of an aircraft and create an algorithm of the reliability of an aircraft in the process of operation (Fig. 1).

References

[1] Tomaszek H., Wróblewski H.: *„Podstawy oceny efektywności eksploatacji systemów uzbrojenia lotniczego*”. Dom Wydawniczy „Bellona” , Warszawa 2001.

[2] Tomaszek H., Żurek J., Jasztal M.: *„Prognozowanie uszkodzeń zagrażających bezpieczeństwu lotów statków powietrznych”*. Wydawnictwo naukowe JTE, Radom 2008.

[3] Loroch L., Tomaszek H., Żurek J.: *„Zarys metody oceny niezawodności i trwałości elementów konstrukcji lotniczych na podstawie opisu procesów destrukcyjnych”*. Materiały XXXII Zimowej Szkoły Niezawodności 2004.

[4] Tomaszek H., Żurek J.: *„Zarys metody oceny niezawodności statku powietrznego z uwzględnieniem uszkodzeń sygnalizowanych i katastroficznych”*. XXXIII Zimowa Szkoła Niezawodności. Materiały str. 537–550, 2005.

[5] Żurek J.: *„Wykorzystanie wyników diagnozowania do oceny i sterowania eksploatacją techniki lotniczej”*. Materiały konferencyjne „DIAG-2003”.
[6] Tomaszek H., Klimaszewski S.: *„Zarys metody oceny resztkowej trwałości wybranych elementów statku powietrznego”*. X Jubileuszowy Kongres Eksploatacji Urządzeń Technicznych. Materiały str. 209–218.
[7] Gerebach J.B., Kordoński Ch.B.: „*Modele niezawodności obiektów technicznych*”. WNT, Warszawa.
[8] Stępień S., Tomaszek H.: *„Zarys probabilistycznego opisu rozwoju pęknięcia zmęczeniowego oraz określanie ryzyka katastroficznego zniszczenia elementu podczas eksploatacji”*. ZEM, Zeszyt 1 (133), 2003r., s. 23–31.
[9] Piasecki S.: „Elementy teorii niezawodności i eksploatacji obiektów o elementach wielostanowych”, Wydawnictwo Instytutu Badań Systemów PAN, Warszawa 1995.

This scientific work was financed by the Ministry of Science and Higher Education in the years 2006-2008 as a research project.

Manuscript received by Editorial Board, November 25th, 2008.

Uwagi na temat doskonalenia eksploatacji wojskowych statków powietrznych przez ograniczenie możliwości powstawania uszkodzeń

S t r e s z c z e n i e

Bezpieczeństwo statku powietrznego jest jedną z najważniejszych charakterystyk eksploatacji mających bezpośredni wpływ na wskaźniki ekonomiczne. Z kolei duży wpływ na bezpieczeństwo ma niezawodność statku powietrznego. Troska o tę niezawodność jest jednocześnie troską o bezpieczeństwo. Ostateczna postać zależności opisujących niezawodność statku uzależniona jest w dużej mierze od odpowiednio opracowanej obsługi technicznej nakierowanej na profilaktykę. Natomiast profilaktyka obejmuje opracowanie takich przedsięwzięć technicznych, które usuwają przesłanki do powstania uszkodzeń.

W artykule podjęto próbę opisu niezawodności statku z odpowiednio opracowaną profilaktyką, która umożliwia ocenę niezawodności statku z wykorzystaniem rozkładu wykładniczego.