SCIENTIFIC PROBLEMS
OF MACHINES OPERATION
AND MAINTENANCE
3 (155) 2008

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Selected issues concerning preparation for the monitoring and detection of fatigue fractures in elements of aircraft construction

K e y w o r d s

Reliability, durability, fatigue, fracture (crack), monitoring interval, risk of damage, detection of a crack.

S ł o w a k l u c z o w e

Niezawodność, trwałość, zmęczenie, pęknięcie, przedział kontroli, ryzyko uszkodzenia, wykrycie pęknięcia

S u m m a r y

All accepted and applied strategies concerning the operation of aircraft oblige engineering services to monitor fatigue fractures of aircraft construction.
Fatigue fractures belong to a dangerous type of damages. Engineering services are to detect fractures before reaching the admissible length, which is determined by taking flight safety into consideration. Detection of a fracture results in the necessity of an engineering interference preventing a catastrophic failure.

The article presents selected issues concerning preparation for the monitoring and detection of fatigue fractures in elements of aircraft construction. The article stresses the prognosis of the development of a fracture.

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1. Introduction

All accepted and applied strategies concerning the operation of aircraft oblige engineering services to assess the risk of damages, especially catastrophic failures with far-reaching consequences. The assessment concerning the risk of aircraft damages is based on predicting their evolution.

With regard to engineering issues, fatigue fractures are dangerous types of damages because they cause fractures in elements.

The task of engineering services is to perform the following:

- Work out the prognosis concerning the development of fractures in elements that are significant in respect of flight safety;
- Develop a way of monitoring the development of fractures;
- Carry out the repair before the admissible state is reached (the state is determined by the length of a fracture).

There is already a lot of literature concerning the methods for predicting the development of fatigue fractures, for example, studies [1-5].

It should be pointed out that no published study includes all aspects of fatigue fractures because of their complexity, especially in case of fractures of complex shapes. Thus, the monitoring of fatigue structures in construction elements is of crucial importance during aircraft operation.

Developing a system monitoring the fatigue fracture growth requires the consideration of many issues concerning the theory of fatigue crack growth. Figure 1 presents the course of a crack growth in relation to the function of operation time, including significant values.

The main aim of monitoring fatigue fracture growth is to guarantee that an engineering interference will be carried out at the appropriate time, which will prevent a catastrophic failure of an element.

For this purpose, the following issues shall be determined:

- The time interval at which a fracture is possible to detect;
- The number of check-ups and their distribution in a particular time interval, enabling the detection of a fracture with the required possibility.

The diagram in Fig. 1 indicates that basic tasks will include the following:

- Determining the admissible length of a fracture;
- Determining the course of fracture growth (at random); and,
- Determining flying time (t_d) of an aircraft up to the moment when the risk of exceeding permitted length by a fracture will be lower than or equal to Q_d.

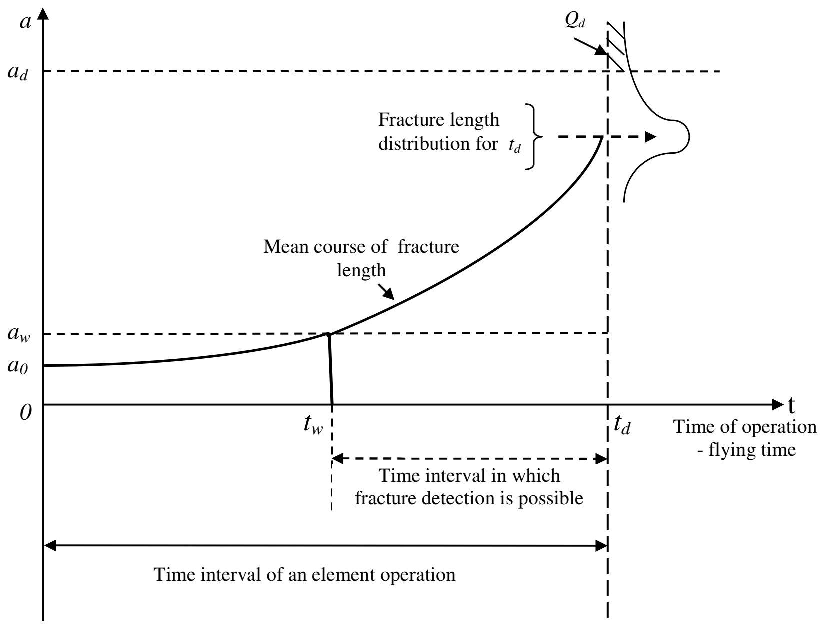

Fig. 1. Diagram of elements concerning preparation for monitoring a fracture growth: a – length of a fracture; a_d – admissible length of a fracture for the accepted level of safety; a_w – length of a fracture possible to detect with conventional diagnostic methods; a_0 – initial length of a fracture; Q_d – accepted level of risk of exceeding permitted length of a fracture; t_d – flying time (probability of exceeding permitted length of a fracture will not exceed Q_d)

Rys. 1. Schemat elementów z zakresu przygotowań do kontroli wzrostu pęknięcia: a – długość pęknięcia; a_d – dopuszczalna długość pęknięcia dla przyjętego poziomu bezpieczeństwa; a_w – długość pęknięcia możliwa do wykrycia przez stosowane metody diagnostyczne; a_0 – początkowa długość pęknięcia; Q_d – przyjęty poziom ryzyka przekroczenia dopuszczalnej długości pęknięcia; t_d – nalot statku, dla którego prawdopodobieństwo przekroczenia dopuszczalnej długości pęknięcia nie przekroczy Q_d

Having determined the above-mentioned issues, it is possible to determine the time interval of monitoring fatigue fracture growth that corresponds to the flying time of an aircraft, which is equal to t_d. Having determined the time interval for monitoring fracture growth, it is possible to distribute a particular number of check-ups.

2. Determining admissible length of a fracture

To determine admissible length of a fracture of a construction element, we will use the stress intensity factor in the following form:

$$K = M_k \sigma \sqrt{\pi a} \tag{1}$$

where: a – length of the fracture of a construction element;
M_k – the correction factor which includes the geometric characteristics of the finite dimensions of an element and the shape of a fracture;
σ – load (stress) of a construction element.

In case of the critical length of a fracture a_{kr} and the critical stress σ_{kr} , the stress intensity factor determined in dependence (1) becomes the critical value K_c called *material resistance to fracture* (fatigue strength)

$$K_c = M_k \sigma_{kr} \sqrt{\pi a_{kr}} \tag{2}$$

Using (2), we can determine the value of the critical length of a fracture. The formula has the following form:

$$a_{kr} = \frac{K_c^2}{M_k^2 \sigma_{kr}^2 \pi} \tag{3}$$

Probabilistically, we can assume that the above-determined critical length of a fracture is the mean value. To obtain the admissible length of a fracture in respect to safety, we must insert the safety factor into the formula (3). Thus,

$$a_d = \frac{K_c^2}{k M_k^2 \sigma_{kr}^2 \pi} \tag{4}$$

where: k – the safety factor.

3. Determining the density function of a fracture of an element during the process of the operation of an aircraft

The following conditions are assumed:

- The technical condition of a device element is determined by the value of fracture length "*a*".
- The fracture length changes during the operation of a device.
- The Paris formula has the following form:

$$\frac{da}{dN_z} = C M_K^m \left(\sigma^{\max}\right)^m \pi^{\frac{m}{2}} a^{\frac{m}{2}} \tag{5}$$

where: M_K – factor of finite dimensions of an element and a fracture location;

$\sigma^{\max}$ – max. stress;
C, m – material constants;

- Dependence (5) can be represented in the form of the time function, or more precisely, the function of flying time. For this purpose, we assume:

$$N_z = \lambda t \tag{6}$$

where: λ – the rate of load cycles (in our case – the rate of determining loading cycles);
t – flying time of an aircraft.

In our case $\lambda = \frac{1}{\Delta t}$, where Δt is the duration of the loading cycle. We can assume the following working formula for Δt:

$$\Delta t = \frac{T}{N_c},$$

where: T – the duration of a standard flight;
N_c – the number of loading cycles during a standard flight.

Dependence (5) in the function of flying time has the following form:

$$\frac{da}{d\lambda t} = CM_K^m \left(\sigma^{\max}\right)^m \pi^{\frac{m}{2}} a^{\frac{m}{2}},$$

Thus:

$$\frac{da}{dt} = \lambda C M_K^m \left(\sigma^{\max}\right)^m \pi^{\frac{m}{2}} a^{\frac{m}{2}} \tag{7}$$

Using the above-mentioned findings and symbols, we can go on to the description of dynamics of fracture growth in a probabilistic aspect. For this purpose, we will use the following difference equation:

$$U_{a,t+\Delta t} = P_1 U_{a-\Delta a_1,t} + P_2 U_{a-\Delta a_2,t} + \ldots + P_L U_{a-\Delta a_L,t} \tag{8}$$

where: $U_{a,t}$ – probability that in the case of flying time equal to *t*, the length of fracture will be "*a*";

Δa_i – the crack growth at interval Δt for the stress value equal to $\sigma_i^{\max}\,(i=1,2,...,L)$;

P_i – probability of stress element appearance equal to $\sigma_i^{\max}$ in loading cycle.

After transforming the difference equation (8), we can obtain the Fokker-Planck differential equation. The transformation is presented in the thesis [3]. The equation has the following form:

$$\frac{\partial u(a,t)}{\partial t} = -\alpha(a)\frac{\partial u(a,t)}{\partial a} + \frac{1}{2}\beta(a)\frac{\partial^2 u(a,t)}{\partial a^2} \tag{9}$$

where: $u(a,t)$ – the density function of a fracture length in the function of flying time of an aircraft.

$$\alpha(a) = \lambda\sum_{i=1}^{L} P_i \Delta a_i \tag{10}$$

$$\beta(a) = \lambda\sum_{i=1}^{L} P_i (\Delta a_i)^2 \tag{11}$$

$$\Delta a_i = C_m \left(\sigma_i^{\max}\right)^m a^{\frac{m}{2}} \tag{12}$$

$$C_m = C\, M_K^m\, \pi^{\frac{m}{2}} \tag{13}$$

The article presents the general solution for coefficient m ≠ 2 (solution for m = 2 was presented in previous publications).

For m ≠ 2, factor $\alpha(a)$ in the equation (9) can be presented in the following form:

$$\begin{aligned} \alpha(a) &= \lambda C_m\, a^{\frac{m}{2}} \underbrace{\left[P_1\left(\sigma_1^{max}\right)^m + P_2\left(\sigma_2^{max}\right)^m + ... + P_L\left(\sigma_L^{max}\right)^m\right]}_{E\left[\left(\sigma^{max}\right)^m\right]} = \\ &= \lambda C_m\, E\left[\left(\sigma^{max}\right)^m\right] a^{\frac{m}{2}} = \lambda C M_K^m \pi^{\frac{m}{2}} E\left[\left(\sigma^{max}\right)^m\right] \cdot a^{\frac{m}{2}} \end{aligned} \tag{14}$$

where:

$$E\left[\left(\sigma^{\max}\right)^m\right] = P_1\left(\sigma_1^{\max}\right)^m + P_2\left(\sigma_2^{\max}\right)^m + \ldots + P_L\left(\sigma_L^{\max}\right)^m \tag{15}$$

Factor $\beta(a)$ in the equation (9) can be presented in the following form:

$$\beta(a) = \lambda \sum_{i=1}^{L} P_i\left(\Delta a_i\right)^2,$$

$$\beta(a) = \lambda\left[P_1\left(C_m\left(\sigma_1^{max}\right)^m a^{\frac{m}{2}}\right)^2 + P_2\left(C_m\left(\sigma_2^{max}\right)^m a^{\frac{m}{2}}\right)^2 + \ldots + P_L\left(C_m\left(\sigma_L^{max}\right)^m a^{\frac{m}{2}}\right)^2\right] =$$

$$= \underbrace{\left[P_1\left(\sigma_1^{max}\right)^{2m} + P_2\left(\sigma_2^{max}\right)^{2m} + \ldots + P_L\left(\sigma_L^{max}\right)^{2m}\right]}_{E\left[\left(\sigma^{max}\right)^{2m}\right]}\lambda C_m^2 \cdot a^m = \tag{16}$$

$$= \lambda C_m^2 \; E\left[\left(\sigma^{max}\right)^{2m}\right] a^m$$

After substituting for C_m, we obtain the following dependence:

$$\beta(a) = \lambda\, C^2 M_K^{2m} \pi^m E\left[\left(\sigma^{\max}\right)^{2m}\right] a^m \tag{17}$$

Fracture length "a" which is present in formulas (15) and (17) is determined from the dependence (7) when $m \neq 2$

$$\frac{da}{dt} = \lambda\, C\, M_K^m E\left[\left(\sigma^{\max}\right)^m\right] \pi^{\frac{m}{2}} a^{\frac{m}{2}} \tag{18}$$

$$a = \left(a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda\, C\, M_K^m E\left[\left(\sigma^{\max}\right)^m\right] \pi^{\frac{m}{2}} t\right)^{\frac{2}{2-m}} \tag{19}$$

We substitute the above-obtained dependence (19) into (15)

$$\alpha(t) = \lambda\, C\, M_K^m \pi^{\frac{m}{2}} E\left[\left(\sigma^{\max}\right)^m\right] \cdot \left[a_o^{\frac{2-m}{2}} + \frac{2-m}{2}\lambda\, C\, M_K^m E\left[\left(\sigma^{\max}\right)^m\right] \pi^{\frac{m}{2}} t\right]^{\frac{m}{2-m}} \tag{20}$$

After substituting (19) into the dependence (17), we obtain factor $\beta(a)$, which is dependent on flying time:

$$\beta(t)=\lambda C^2 M_K^{2m}\pi^m E\left[\left(\sigma^{\max}\right)^{2m}\right]\cdot\left[\left[a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda C M_K^m E\left[\left(\sigma^{\max}\right)^m\right]\pi^{\frac{m}{2}}t\right]^{\frac{2}{2-m}}\right]^m \quad (21)$$

Dependencies (20) and (21) and the equation (9) can be written in the following form:

$$\frac{\partial u(a,t)}{\partial t}=-\alpha(t)\frac{\partial u(a,t)}{\partial a}+\frac{1}{2}\beta(t)\frac{\partial^2 u(a,t)}{\partial a^2} \quad (22)$$

The solution of equation (22) for the accepted symbols has the following form:

$$\partial u(a,t)=\frac{1}{\sqrt{2\pi A(t)}}\, e^{-\frac{(a-B(t))^2}{2A(t)}} \quad (23)$$

where: $B(t)$ – mean value of fracture length for flying time assuming the value of t;
$A(t)$ – variance of fracture length for flying time assuming the value of t.

The mean value B(t) is the solution of the following integral

$$B(t)=\int_0^t \alpha(z)dz \quad (24)$$

The variance $A(t)$ is the solution of the following integral:

$$A(t)=\int_0^t \beta(z)dz \quad (25)$$

Flying time of an aircraft is determined according to the following dependence:

$$t=t_N=\sum_{i=1}^{N} t_i\,,$$

where: t_i – duration i-of this flight.

The solution of integral (24) has the following form:

$$B(t)=\int_0^t \alpha(z)\,dz=\left[a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda C\,M_K^m\pi^{\frac{m}{2}}E\left[\left(\sigma^{max}\right)^m\right]t\right]^{\frac{2}{2-m}}-a_o \qquad (26)$$

The dependence (26) determines the average growth of fracture length at flying time (flying time length - t). The solution of integral (25) has the following form:

$$A(t)=\frac{2}{2+m}C\,M_K^2\,\pi^{\frac{m}{2}}\cdot\frac{E\left[\left(\sigma^{max}\right)^{2m}\right]}{E\left[\left(\sigma^{max}\right)^m\right]}\cdot$$
$$\cdot\left[\left(a_o^{\frac{2-m}{2}}+\frac{2-m}{2}\lambda C\,M_K^m\,\pi^{\frac{m}{2}}E\left[\left(\sigma^{max}\right)^m\right]t\right)^{\frac{2+m}{2-m}}-a_o^{\frac{2+m}{2}}\right] \qquad (27)$$

Dependence (23) is an unknown function of the density of fracture length as a function of flying time. In the case of this function of density, dependence (26) represents the mean value of fracture length as a function of flying time, and dependence (27) represents the variance.

4. Determining the range of monitoring fatigue crack growth

The function of the density of crack length which is dependent on flying time (23) and formula (3) can be used to determine the dependency concerning the assessment of the risk of a catastrophic crack for flying time equal to t.

$$\overline{Q}(t)=\int_{a_{kr}}^{\infty}u(a,t)\,da \qquad (28)$$

The risk of construction element damage, taking the safety factor (4) under consideration, will be determined by the following dependency:

$$Q(t)=\int_{a_d}^{\infty}u(a,t)\,da \qquad (29)$$

Using dependency (29), we can assess a fatigue life of an element for the accepted risk level of damage. In this case, it is assumed that a fatigue life of an element is the upper limit of the range of a crack detection. The calculation formula has the following form:

$$Q(t)_{dop} = \int_{a_d}^{\infty} \frac{1}{\sqrt{2\pi\, A(t)}} e^{-\frac{(a-B(t))^2}{2A(t)}} da \tag{30}$$

To transform the density function of crack length, we use the following dependency:

$$Z_t = \frac{a - B(t)}{\sqrt{A(t)}} \tag{31}$$

After transformation, dependency (29) has the following form:

$$Q(t)_{dop} = \frac{1}{\sqrt{2\pi}} \int_{\frac{a_d - B(t)}{\sqrt{A(t)}}}^{\infty} e^{-\frac{1}{2}Z_t^2} dz \tag{32}$$

We replace the lower limit of the integral (32) with value γ_t. Thus, we obtain the following dependency:

$$\gamma_t = \frac{a_d - B(t)}{\sqrt{A(t)}} \tag{33}$$

Using dependency (33), we can rewrite formula (32) in the following form:

$$Q(t)_{dop} = \frac{1}{\sqrt{2\pi}} \int_{\gamma_t}^{\infty} e^{-\frac{1}{2}Z_t} dz \tag{34}$$

Using tables of normal distribution, we can determine the right side of dependency (33).

Assuming the required value $Q^*(t)_{dop}$, we find such value γ_t^*, for which the left side of dependency (34) will be equal to the right side. Then, we settle dependency (35):

$$\gamma_t^* = \frac{a_d - B(t)}{\sqrt{A(t)}} \tag{35}$$

and find value "t_d" - flying time of an aircraft, which is assumed to be upper limit of the range concerning the detection of a construction element crack.

Thus, the range of monitoring crack growth is (0, t_d). The possibility of detecting a crack is limited within the range (t_w, t_d), which is presented in Fig.1.

5. Distribution of crack monitoring and determining the probability of crack detection

Developing a schedule of monitoring a construction element, we assume the following:

1) Initially, the length of a crack is small, i.e. $a_0 < a_w$, where a_0 – crack length for $t=0$, and a_w - crack length possible to detect with a diagnostic method.
2) The diagnostic method is sensitive and detects cracks of $a \geq a_w$ length in a single check-up with probability P_w.
3) The admissible length of a crack is relatively long, and the speed of crack growth is low, i.e. the range of monitoring is wide (0, t_d).
4) The range of monitoring includes the possibility of performing several check-ups confirming the existence of a crack.
5) Initially, principles concerning an element operation will be fulfilled.

Using the above-mentioned assumptions, we will determine the probability of detection as a function of the number of control.

The probability of crack detection at the first attempt is P_w, at second attempt - (1- P_w) P_w, at third one - (1- P_w)^2 P_w, etc.

The number of check-ups of an element to detect a crack is a random variable. Its values are 1, 2, 3, ... and its distribution is as follows:

$$P_n = (1 - P_w)^{n-1} P_w, \text{ n = 1,2,3,....} \tag{36}$$

where: P_n – probability of crack detection at n-attempt.

If $(1-P_w)=q$, thus $P_n = q^{n-1}P_w$.

The total of probabilities P_n(n=1,2,3,.....) is

$$\hat{P}_w = \sum_{n=1}^{\infty} q^{n-1}P_w = P_w\sum_{n=1}^{\infty} q^{n-1} = P_w\left(q^0+q^1+q^2+....\right) = P_w\frac{1}{1-q} = \frac{P_w}{1-\left(1-P_w\right)} = 1.$$

Dependence (36) determines the geometrical distribution. Assuming the level of probability of crack detection at several attempts, we can determine the number of check-ups.

$$\hat{P}_w = \sum_{n=1}^{n*} \left(1-P_w\right)^{n-1}P_w = P_w\sum_{n=1}^{n*} \left(1-P_w\right)^{n-1} \tag{37}$$

Value $n*$ in dependency (37) where there is equality between the left and right side determines the number of check-ups ensuring crack detection with probability $\hat{P}_w$.

The distribution of check-ups shall ensure crack detection in the most rational way. The first check-up shall be performed, for example, after 1/3 of length concerning the planned operation of an element. Next check-ups shall be performed more and more often.

6. Final notes

Planned check-ups shall be included in the system of engineering service of an aircraft in order to detect a crack. Within the above-considered range of monitoring, the probability that crack length will be smaller than admissible crack length is:

$$R(t_d) = 1 - Q^*(t_d).$$

If we assume that there are three attempts of crack detection in the range of monitoring, the probability of detection after three attempts will be:

$$\hat{P}_w = 0{,}9 + (1-0{,}9)0{,}9 + (1-0{,}9)^2 \cdot 0{,}9 = 0{,}999\,.$$

Modern diagnostic methods enable the detection of a crack at one attempt with probability 0.9, if in a construction element there is a crack and its length is longer than a_w.

References

[1] Kocańda S.: „*Zmęczeniowe pękanie metali*”. WNT, Warszawa, 1985.

[2] Kocańda S., Tomaszek H.: „*Probabilistyczna ocena trwałości zmęczeniowej elementów konstrukcyjnych w warunkach rozwoju pęknięć*”. Zeszyty Naukowe Politechniki Świętokrzyskiej, Mechanika nr 50, Kielce 1993, s. 259 – 272.

[3] Tomaszek H., Szczepanik R., Jasztal M.: „*Zarys metody określania rozkładu czasu narastania pęknięcia elementu do wartości granicznej w warunkach zmęczenia w procesie eksploatacji statku powietrznego*”. ZEM Zeszyt 3(147), 2006, s. 81 – 89.

[4] Tomaszek H., Szczepanik R.: „*Metoda określania rozkładu czasu do przekroczenia stanu granicznego*”, ZEM Zeszyt 4(144), 2005, s. 35 – 44.

[5] Klimaszewski S., Tomaszek H.: „*Zarys metody oceny resztkowej trwałości wybranych elementów statku powietrznego*”. X Jubileuszowy Kongres Eksploatacji Urządzeń Technicznych, Materiały 2005, s. 209 – 218.

[6] Fahr, A., Forsyth, D.S., and Bullock, M.: „*A comparison of probability of detection (POD) data determined using different statistical methods*”. NRC/IAR report LTR-ST-1947, December 1993.

[7] Alten F. Grandt Jr, „*Fundamentals of Structural Integrity. Damage Tolerant Design and Nondestructive Evaluation*”. Wiley, 2004.

[8] Dragan K.: „*Sprawozdanie Nr 41/31/2006. Opis i modelowanie kształtowania się uszkodzeń w poszyciu statku powietrznego i możliwości ich wykrywania*”. ITWL, 2006

[9] *„Aging of U.S. Air Force Aircraft, Final Report”,* Publication NMAB-488-2, National Academy Press, USA, Washington D.C. 1997.

[10] *„NDC of Cracks in Aircraft”,* NTIAC-SR-98-04, USA, Texas 1998.

Manuscript received by Editorial Board, November 03, 2008

Wybrane problemy z zakresu przygotowań do kontroli i wykrywania pęknięć zmęczeniowych elementów konstrukcji statku powietrznego

S t r e s z c z e n i e

Wszystkie przyjęte i stosowane strategie eksploatacji statków powietrznych zobowiązują służby techniczne do kontroli konstrukcji statku pod kątem istnienia pęknięć zmęczeniowych.

Pęknięcia zmęczeniowe są niebezpiecznym rodzajem uszkodzeń i zadaniem służb technicznych jest ich wykrycie przed osiągnięciem dopuszczalnej długości określonej z uwzględnieniem bezpieczeństwa lotów. Wykrycie pęknięcia pociąga za sobą konieczność interwencji technicznej uniemożliwiającej powstanie uszkodzenia katastroficznego.

W artykule przedstawiono wybrane problemy z zakresu przygotowania do wykrywania pęknięć zmęczeniowych elementów konstrukcji statku powietrznego, zwracając głównie uwagę na prognozę rozwoju pęknięcia.

This research was made with the financial support of the Ministry of Science and Higher Education as a working project in 2006–2008 years.