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Technical stability – a new modelling perspective for building solutions of monitoring systems for machinery state[1]

K e y w o r d s

Phase trajectory, technical diagnostics, monitoring, technical stability

S ł o w a k l u c z o w e

Trajektorie fazowe, diagnostyka techniczna, monitoring, stateczność techniczna.

S u m m a r y

Starting with the synthetic overview of the machinery state monitoring systems the article presents a need to search for new methodological guidelines for the process of developing such systems without their common limitations. The paper shows theory of technical stability as a useful tool for machinery state monitoring algorithms building, related to dynamics analysis of the monitored object, choice of the diagnostic symptoms and levels of their quantification thus allowing diagnostic decision making. It also shows the development methods. It shows purposefulness in controlling the phase images of the tested vibration signals, giving them the value of an useful tool for fault development process identification in the monitored object.

1. Introduction

When reviewing practically functioning monitoring systems for machinery state, and the results of currently held research it can be briefly concluded, in several general thoughts, that:

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- Watching the machine state is made by monitoring systems where several quantities are observed: particular values of numeric estimates (*e.g. rms, peak and mean values or their combination*), or particular functional images of monitored diagnostic signals (*movement trajectory of shaft neck in a bearing, spectral density function, correlation, coherence, cepstrum, envelope etc*.)
- Criteria values for monitored diagnostic symptoms (defining particular states of the object) are described with relevant standards, regulations and agreements emerging out of exploitation experience or assuming acceptable scenarios of faults of the object or from statistical processing of controlled diagnostic signals.
- There are only few monitoring system solutions which functions are based on partial relation-binding monitored state of the objects with the changes of the signal observed.
- In construction rules of monitoring system there is no consistent theory enforcing logical relations of conditions of lack of safe functioning capabilities in monitored object to the choice rules for diagnostic symptoms for state change observation of the monitored objects and the conditions of their undisturbed estimation.
- Often the construction and exploitation features of the object are not well enough taken into consideration in the process of monitoring system building.

The above assessment does not tend to name all the problematic questions that appear during the development of the monitoring system or describe known research and experimental results. Yet it might be an inspiration to look for new methodological guidelines for construction process of monitoring system without the limitations presented above.

The purpose of the paper is to show some possibilities. It seems that a good tool to perform such a task might be a theory of technical stability [7], which allows to meet and solve a number of tasks in the process of monitoring system development. That theory will be the basis for the framework of research actions and their algorithms management dedicated to search for new, related to the dynamics of the monitored object, diagnostic symptoms and the choice of the quantification levels allowing diagnostic decision making.

2. Methodological relationships of technical stability to monitoring process of the state change of the controlled object

The methodological base for **building the monitoring system** for machine state change, according to the authors proposal, might be **solutions based on the stability theory** and its research procedures, related to the definitional understanding of the stability (e.g. technical stability, orbital stability, stochastic stability, stability in the terms of some coordinates etc) [5, 6, 7, 11]; related to the diagnostic task in consideration. Its results allow testing of the dynamic behavior conditions of the considered object when its stationary state (balance conditions or trajectory movement) is disturbed.

In translation to the language of technical diagnostics theory, the tasks of stability theory, its mathematical formalism, have tight connections to the performance of the building process of diagnostic machine state change recognition conditions, including the matter of **diagnostic symptoms and criteria values choice** describing process of qualification of the changes happening in the monitored object.

State changes of the monitored object are tightly connected with the changes of construction parameters of its parts, kinematic pairs or the conditions of their cooperation. They are related to relevant ratios changes: mass, elasticity, damping, which generate disturbances in characteristic movements of the initial states, and the changes of which might be the subject of control assessment in the monitoring system.

As an example, mass wear of particular part influences its elasticity and damping parameters, which reflects in deviation of the object's dynamic behavior, considered to be safe. The theoretical analysis of particular disturbances is a recognition task of stability theory. The observed results of small disturbances in monitored object's stationary state (its movement along fixed trajectories) – referenced to programmed work conditions and expected exploitation flow – are widely considered to be recognition criteria of stability.

From the technical diagnostics point of view, the criteria of technical stability [7] might be suitable research criteria to perform tasks appearing in the machine state change monitoring process, deciding about the resistance of the controlled object to disturbances characteristic for its normal exploitation in its stationary state.

For such assessment it is necessary to make some assumptions:

- acceptable deviation of movement trajectory from its stationary state (from the point of view of safe exploitation of the analyzed object);
- acceptable range of changes for initial conditions;
- predicted level of external and internal disturbances constantly influencing the controlled object during its exploitation.

The question of stability of the analyzed technical object with forces $f\left(\dot{x}, x, t\right)$ and disturbances $R\left(\dot{x}, x, t\right)$ acting on it, and its movement described by the equation

$$\ddot{x} = f\left(\dot{x}, x, t\right) + R\left(\dot{x}, x, t\right) \tag{1}$$

demands analysis of its solution and answering the questions:

- Does the system have zero-solution and what is the course of the solution in the neighborhood of the zero-solution?

- What are the areas of initial conditions, where solutions coming out of them have the same qualitative course?
- What is the influence of perturbations $R(\dot{x}, x, t)$ of right side of the equation (1) on qualitative course of the solution?

Taking into consideration that requirements, the definition of technical stability for mechanical system described with the differential equations:

$$\dot{x} = f(x,t) + R(x,t) \tag{2}$$

where ***x, f, R*** are vectors in the $\Re^n$ space:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} \quad \mathbf{f} = \begin{bmatrix} f_1 \\ f_2 \\ \ldots \\ f_n \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} R_1 \\ R_2 \\ \ldots \\ R_n \end{bmatrix} \tag{3}$$

where functions ***f(t, x)*** ***R (t ,x)*** are defined in a range included in n+1-dimensional space:

$$t \geq 0, \qquad (x_1, x_2, \ldots, x_n) \in \mathbf{G} \subset \mathbf{E}_n \tag{4}$$

where: $\mathbf{E}_n$ denotes linear normed n-dimensional space and functions $R_i(t, x_1, x_2, \ldots, x_n)$ are disturbances acting constantly, with assumption:

$$\| R(t, x_1, \ldots x_n) \| \leq \delta \tag{5}$$

might be phrased as follows.

Let there be two ranges Ω and ω included in G such that Ω is closed, limited and includes origin of coordinates and ω is open and included in Ω.

Let us assume that the solution of the analyzed system (2) is x(t) with the initial condition $x(t_0) = x_0$.

If for each x_0 belonging to ω, x(t) stays in the range Ω for $t \geq t_0$ with disturbance function satysfying inequality (5), then the system (2) is technically stable in terms of range ω, Ω and limited constantly acting disturbances (5). According to this definition of technical stability, each movement trajectory derived from the range ω is supposed to stay in the range Ω for $t \geq t_0$.

For monitoring systems, allowing monitored signals to momentarily exceed the accepted levels, the term of technical stability might be weakened to the condition where each trajectory exceeding the range ω is supposed to stay in the range Ω for $t_0 \leq t < T_0$, where $T - t_0$ is the duration of the movement. With such a condition we deal with a technical stability in limited time.

3. Phase trajectory as a tool of monitored machine state change assessment

From the point of view of technical diagnostics, including the need of application of the technical stability theory solutions for the monitoring system development, there are interesting questions of algorithm building for condition recognition for technical stability loss of the overviewed object. It could be realized by solving the system of differential equations (2) describing dynamic behavior of the monitored object or analyzing the course of their solution with qualitative methods.

The latter method might be tightly connected to the machine state changes monitoring process. It is related to the examination of the phase portraits of the solutions, that is curves $x(t)$, $\overset{\circ}{x}(t) = y$ on the plane $x, \overset{\circ}{x}$ called phase plane, which might be the subject of the monitoring. Such methods are usually included to the set of topological methods of solutions testing of differential equations (2). They allow dynamic behavior analysis of the monitored object, with the disturbances constantly acting and non-linear, which are significant for the fault appearing process [8, 10, 15], including their early phases.

Their testing procedures, based on some topological facts, related to the existence of some constants of homomorphic transformations formed as theorems, allow qualitative assessment of dynamic behavior of the analyzed object and related to it conditions of technical stability loss.

The most often used method is Lapunov method [12], where properties are used of properly chosen, for the dynamics description of the controlled object, scalar function ***V(x,t)***. Its derivative testing along the solutions (behaviors) of the equation set (2) determines the decision leading to its stability.

The theorem on which it is based states that if there exist a scalar function ***V(x,t)*** of the class $\mathbf{C}^1$, defined for each $\boldsymbol{x}$ and $t \geq 0$ fulfilling requirements:

$$\boldsymbol{V}(\boldsymbol{x}, t) > 0 \qquad \text{for } \boldsymbol{x} \neq \mathbf{0}$$

$$\dot{V}(x,t) \leq 0 \ \text{ along solutions of (2)} \qquad \text{for } \boldsymbol{x} \notin \boldsymbol{G} - \boldsymbol{\omega} \tag{6}$$

$$\boldsymbol{V}(\boldsymbol{x}_1, t_1) < \boldsymbol{V}(\boldsymbol{x}_2, t_2) \ \text{ for } \boldsymbol{x}_1 \notin \boldsymbol{\omega} \text{ and } \boldsymbol{x}_2 \notin \boldsymbol{G} - \boldsymbol{\Omega}; \ \ \boldsymbol{t}_1 < \boldsymbol{t}_2$$

then the object described by (2) is technically stable.

Referring results of that theorem to the issue of creating foundation for machinery state monitoring system, the Lapunov function ***V(x,t)*** should be built and should be checked by means of measured trajectories x, y of the tested object conditions (6). In building the Lapunov function ***V(x,t)*** directions from [2,9] might be useful, or an effort made to define its form as total energy of the tested object.

Another way of testing the properties of the monitored trajectories from the point of view of the stability assessment of the monitored dynamic system is its testing by means of two functions [4]:

$$\Phi\,(x, y\,) = x\,y + \dot{x}\,\dot{y} \quad ; \quad \Psi(x, y\,) = x\,\dot{y} - \dot{x}\,y \tag{7}$$

of which the positive or negative definition allow to assess the character of the monitored movement. Their dependent values allow assigning to the points of trajectories a direction characteristic for the point of entry, exit or slip related to the analyzed curve, which helps in determining the ranges ***G*** and ***Ω*** in the range of the monitored phase space.

The construction foundation for quantifier of the monitored trajectories properties from their stability point of view might also be searched based on the topological retract method – Ważewski method [4]. In that method ranges are built, limited with curves of the points of entry and exit of the equation set (2) solutions, from the ranges accepted as allowable.

As it emerges from the synthetic review of the technical stability testing methods their usage for the monitoring system development is related to two tasks:

1. Creation of the measurement tools providing observation of the phase portraits changes for the dynamic behavior of the monitored node of the system, defined by measurement:

$$x\,(\,t\,), \quad \mathring{x}\,(\,t\,) = y$$

2. Building of the quantifier for the monitored courses by implementation of the technical stability testing algorithms, based on the Lapunov function method or two functions Φ(x, y) and ψ(x, y) method or the retract method.

From the practical realization of the monitoring system point of view, the solution for the task one is not problematic. There are more inconveniences with determining positively or negatively defined Lapunov function for the monitored construction and the differential equation set describing its dynamics. The significant simplification of the task appears when the monitoring system with defined location of the measurement sensors is dedicated to modal parameters change of the controlled object. In case of using the two functions Φ(x, y) and ψ(x, y) method there may some difficulties appear in solving their functional equations, necessary to draw their zero-adjustment curves. A significant advantage of the method is fact that both functions for a non-linear system have identical form, which makes the method more universal for different applications. The usage of the retract method in turn forces the necessity of building some curve-limited range on which there are only points of entry or points of exit which might appear as a significant problem.

4. The meaning of phase portraits – an example

In this chapter the attention is brought to the usefulness of the vibration phase portraits for fault recognition in monitored elements of steel constructions. Its mean of information is a comparison of the phase beam vibration trajectory image (Fig. 1) responding to variable input drive (fig. 2, 3, 4), with a crack of a certain depth. The base for that were simulation experiments made on analytic model of Bernoulli-Euler beam with constantly open gap (that is without taking the closing of the gap effect into consideration), which are part of the wider research and analysis [3] and [14]. The modeling of the gap was described in [13], where also the results of the research are found for sensitivity of different vibration symptoms for the size of the gap and its location. The article [13] is a valuable supplement to the results presented below. They show possibility of easy recognition of phase trajectories changes for beam vibration with the gap as compared to the beam without the gap.

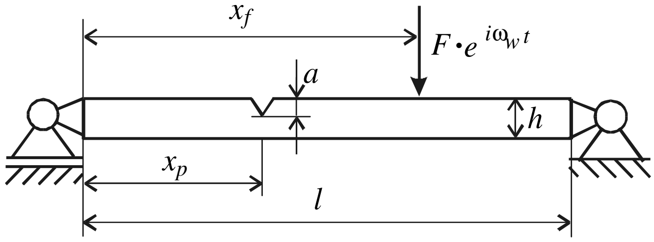

Fig. 1. Tested structure with the gap and its parameters: l = 1.2 m, x_f = 0.6 m, x_p = 0.4 m, a = 0.3*h
Rys. 1. Schemat badanego elementu z pęknięciem i określające go parametry:
l = 1,2 m; xf = 0,6 m; xp = 0,4 m; a = 0,3*h

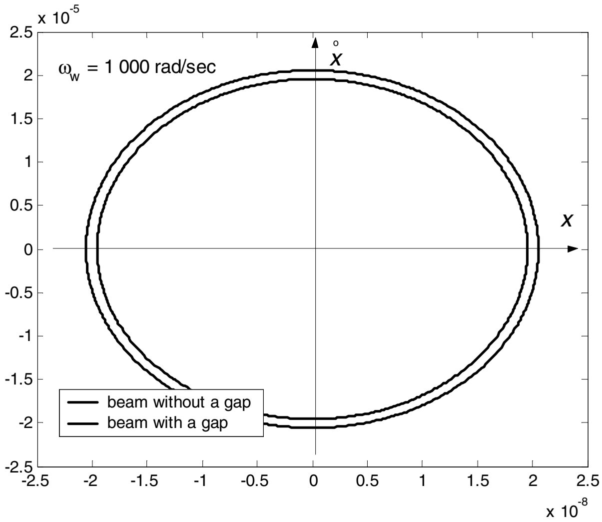

Fig. 2. Phase image change course for beam vibration with the driving frequency ω_w =1000 rad/s
Rys. 2. Przebieg zmian portretów fazowych drgań belki dla częstotliwości wymuszenia ww = 1000 rad/s

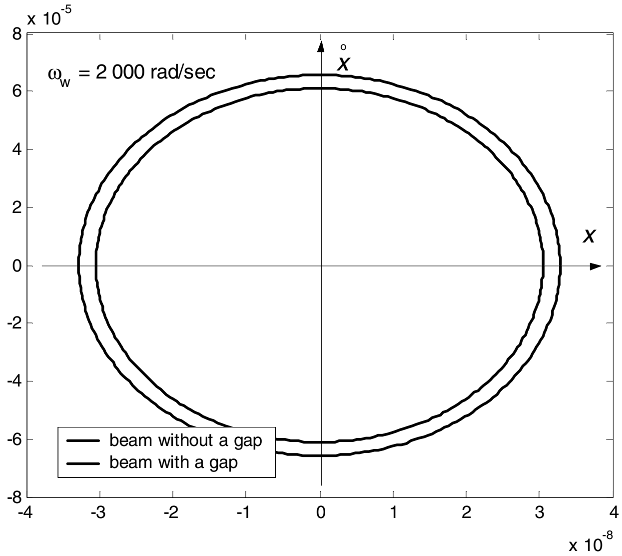

Fig. 3. Phase image change course for beam vibration with the driving frequency ω_w = 2000 rad/s
Rys. 3. Przebieg zmian portretów fazowych drgań belki dla częstotliwości wymuszenia ww = 2000 rad/s

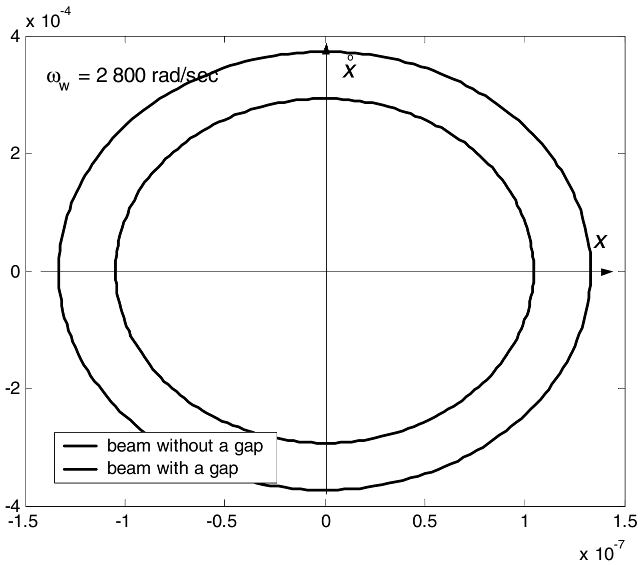

Fig. 4. Phase image change course for beam vibration with the driving frequency ω_w = 2800 rad/s
Rys. 4. Przebieg zmian portretów fazowych drgań belki dla częstotliwości wymuszenia ww = 2800 rad/s

The conclusion from the figures is that the gap caused significant growing of the phase trajectory in its size for all analyzed frequencies of the input drive of vibration. Such a variability is more convenient for beam faults detection than described in [13] conjunctions between transverse and longitudinal vibration or the variability of natural frequency with the gap. It might be a good diagnostic mean for beam crack monitoring system building.

5. Final notes

Presented mathematical formalization, directed towards search of new research tools dedicated to better recognition of the monitored object state change, seems to be promising. Its methodological approach, based on the technical stability analysis of the controlled object gives a good definition of the process of object's transition into functional disability. It relates fully to the non-linear physics of the phenomenons describing the process. It ties realized recognition with the dynamic state of the monitored object and with its constructional and exploitational parameter changes, which makes it universal.

The practical application of the phase trajectory change control method seems to be very useful tool of fault emerging and development process identification. It might be its main quality factor and is easily adoptable to practical application. It is not filtering non-linear effects and phenomenons of frequency structure change of the monitored diagnostic signals related to the fault development, which might be its very unique advantage.

The number of problems related to test algorithm building for technical stability of the monitored dynamics of the controlled object remains still unrecognized and makes an interesting research field. It will be concentrated around developing useful algorithms and guidelines for the choice of the Lapunov function for the monitored object, providing the process of the monitored state quantification.

Even though many mentioned problems were not thoroughly explained the author hopes that the research idea will be developed and its results would generate more objective rules for machine state change monitoring.

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Stateczność techniczna – nowa perspektywa modelowa dla budowy rozwiązań systemów monitorujących zmiany stanu maszyn

S t r e s z c z e n i e

Wychodząc z syntetycznego przeglądu systemów monitorujących zamiany stanu maszyn, autor nakreślił potrzebę poszukiwań nowych wskazań metodologicznych dla procesu ich konstruowania, pozbawionych zasygnalizowanych w nim ograniczeń. Zwrócił uwagę na rozwiązania teorii stateczności technicznej jako użytecznych narzędzi budowy algorytmów rozpoznawania zmian stanu monitorowanej maszyny, powiązanych z: analizą dynamiki monitorowanego obiektu, wyborem symptomów diagnostycznych oraz doborem poziomów ich kwantyfikacji, umożliwiającej podejmowanie decyzji diagnostycznych. Omówił możliwe metody dla ich realizacji. Wskazał na celowość kontroli zmian obrazów fazowych kontrolowanych sygnałów drganiowych, przypisując im walor użytecznego narzędzia identyfikacji procesu powstawania i rozwoju uszkodzeń monitorowanego obiektu.