The influence of dynamic viscosity changes caused by pressure on the capacity of conical slide bearings

Summary

Oil dynamic viscosity is one of the essential properties in hydrodynamic lubrication of slide bearings. The one increases with the pressure. The pressure-viscosity effect is especially significant when the pressure is much larger than the atmospheric pressure. In this paper, the analysis of influence of this phenomenon on the basic slide bearing parameters (especially on bearing capacity) is studied. Here, the results of numerical calculations of influence of pressure-viscosity effect on parameters of conical slide bearings as well as journal slide bearings are presented.

1. Introduction

Proper design process of slide bearings should take into account all the factors which may have essential influence on their operating parameters. Among those parameters, the most important are: temperature and pressure. The ones have influence on dynamic viscosity of the lubricant. It is commonly know that viscosity of lubricant decreases with temperature and increases with pressure. Moreover, for most of lubricants the pressure-viscosity effect is much larger than the one of temperature or shear rate when the pressure is significantly above the atmospheric pressure.
atmospheric pressure [6]. In conical slide bearings in which ones according to
conical shape of the elements (bush and journal) the lubricant viscosity changes
may very easy lead to their steel contact or in some extreme cases even to their
jam. Therefore, it is seem to be very useful to performance exact analysis of
influence of the mentioned phenomenon on the working condition occurring in
conical slide bearings. In this paper the dynamic viscosity changes caused by
pressure changes are studied. This problem, to the best of author knowledge, has
not been sufficiently developed yet.

In order to simulation of performance conditions of conical slide bearings
as similar as possible to real the considering numerical model of conical slide
bearing gap predicates all possible cases of their work. Classical model mainly
includes the terms describing the gap height in circumferential direction and the
ones arising from eccentricity phenomenon. This model, in addition, takes into
account the misaligning phenomena of bush and journal axes as well as different
conical angles. Due to, the study of lubrication of conical slide bearings espe-
cially the analysis of pressure distribution and their capacity is more precisely
and as near as possible to their real values.

Among other things, the problems of lubrication of conical slide bearings
were studied in the following papers: [2], [3], [4], [7], [8], [9]. Authors of those
papers assumed in their analysis the axial-symmetric oil flow through the coni-
cal slide bearings gap. Moreover, they have considered the geometric model of
bearings with permanent height of the gap along generating line of conical jour-
nal. Those assumptions considerably deviate from real working conditions of
conical slide bearings and make such models less accurate. Therefore, such
analysis didn’t give real values of working parameters of conical slide bearings
i.e. values of pressure distribution in the gap, capacity, friction coefficients etc.
In the work [1] its authors have conducted the study on aerostatic-aerodynamic
as well as hydrostatic-hydrodynamic conical slide bearings. The authors’ of
paper [10] have focused on the analysis of thermal and pressure-viscosity effect
on the misaligned conical-cylindrical slide bearing.

The similar analysis of the influence of the viscosity changes caused by
pressure and temperature was made in the work [5]. Anyway, the last deals to
hydrodynamic lubrication problems of journal slide bearings.

2. Geometrical model

In this work the following geometrical model of conical slide bearing is
considered (fig. 1).

The misaligning phenomenon of bush and journal bearing may be created
as result of imposed external load as showed in the fig. 1 or in consequence of
improper bearing mounting.
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3. Mathematical model

To determine the influence of pressure on oil viscosity the Barus equation was employed and the one is expressed as:

\[ \eta = \exp(\zeta p) \]  

(2)

where: \( \zeta \) – dimensionless piezocoefficient of viscosity, \( p \) – dimensionless pressure value.

A steady three-dimensional mathematical model expressing a pressure-viscosity interaction on performance parameters of conical slide bearings is consisted of continues equation, momentum equation and heat equations. Those equations were written in dimensionless form and have the following form:

\[ U_1 \frac{\partial v_{21}^{(1)}}{\partial y_1} + \frac{\partial v_{11}^{(1)}}{\partial \varphi} + U_1 \frac{\partial v_{31}^{(1)}}{\partial x_1} + \frac{1}{L_4} v_{31}^{(1)} \cos \gamma = 0 \]

\[ p_1^{(0)} U_1^2 \eta_1 \frac{\partial v_{11}^{(0)}}{\partial y_1^2} + U_1^2 \eta_1 \frac{\partial v_{11}^{(1)}}{\partial y_1^2} - \frac{\partial p_1^{(1)}}{\partial \varphi} = 0 \]

\[ \frac{\partial p_1^{(1)}}{\partial y_1^2} = 0 \]

\[ p_1^{(0)} U_1 \eta_1 \frac{1}{L_4} \frac{\partial^2 v_{31}^{(0)}}{\partial y_1^2} + U_1 \eta_1 \frac{1}{L_4} \frac{\partial^2 v_{31}^{(1)}}{\partial y_1^2} - \frac{1}{L_4} \frac{\partial p_1^{(1)}}{\partial x_1} = 0 \]  

(3)

where: \( v_{11}^{(0)}, v_{21}^{(0)}, v_{31}^{(0)} \) denote the corrections of the individual oil velocity components for classical case of lubrication slide bearings, \( v_{11}^{(1)}, v_{21}^{(1)}, v_{31}^{(1)} \) – mean the corrections of the individual oil velocity components for the case in which one the oil dynamic viscosity depend on pressure, \( p_1^{(0)}, p_1^{(1)} \) – denote the corrections of pressure for classical case and the one taking into account the pressure-viscosity effect.

Eq. (3) were derived from the basic set of partial differential equations [8] (i.e. continues equation, momentum equation and heat equations) by means of the classical small parameter method [5]. In that model, the stress and strain relation was expressed by Rivlin-Erickson formula.

To assign the corrections of oil vector velocity components the following boundaries conditions should be taken into account:
\[ \nu_{11}^{(1)} = 0, \nu_{21}^{(1)} = 0, \nu_{31}^{(1)} = 0 \quad \text{for} \quad y_1 = h_1, \]
\[ \nu_{11}^{(1)} = 0, \nu_{21}^{(1)} = 0, \nu_{31}^{(1)} = 0 \quad \text{for} \quad y_1 = 0 \] (4)

hence, the corrections of oil vector velocity components have the following form:

\[ \nu_{11}^{(0)} = \frac{1}{2\eta U_1^2} (h_1 y_1 - y_1^2) \left( p_1^{(0)} \frac{\partial p_1^{(0)}}{\partial \varphi} - \frac{\partial p_{11}^{(1)}}{\partial \varphi} \right) \] (5)

\[ \nu_{21}^{(0)} = \frac{h_1^2}{4L_1^2 \eta U_1} \left[ \frac{\partial h_1}{\partial x_1} \frac{\partial p_{11}^{(1)}}{\partial x_1} \right] - \left[ p_1^{(0)} \frac{\partial h_1}{\partial x_1} \frac{\partial p_1^{(0)}}{\partial \varphi} + \frac{L_1^2}{U_1^2} \frac{\partial h_1}{\partial \varphi} \frac{\partial p_1^{(0)}}{\partial \varphi} \right] + \frac{L_1^2}{U_1^2} \frac{\partial h_1}{\partial \varphi} \frac{\partial p_{11}^{(1)}}{\partial \varphi} \right] + 
\]
\[ + \frac{h_1^2}{12\eta U_1^2} \left[ - \frac{1}{U_1^2} \left( \frac{\partial p_{11}^{(1)}}{\partial y_1} \right)^2 - p_1^{(0)} \frac{\partial^2 p_1^{(0)}}{\partial y_1^2} + \frac{\partial^2 p_{11}^{(1)}}{\partial y_1^2} + \frac{1}{L_1^2} \left[ \frac{\partial^2 p_{11}^{(1)}}{\partial x_1^2} - p_1^{(0)} \frac{\partial^2 p_1^{(0)}}{\partial x_1^2} \right] \right] + 
\]
\[ - \left( \frac{\partial p_{11}^{(1)}}{\partial x_1} \right)^2 \right] \] + \frac{h_1^3 \cos \gamma}{12L_1^2 \eta U_1^2} \left[ p_1^{(0)} \frac{\partial h_1}{\partial x_1} \frac{\partial p_1^{(0)}}{\partial \varphi} - \frac{\partial p_{11}^{(1)}}{\partial x_1} + \left( \frac{\partial p_{11}^{(1)}}{\partial x_1} - p_1^{(0)} \frac{\partial p_1^{(0)}}{\partial x_1} \right) \right] \] (6)

\[ \nu_{31}^{(0)} = \frac{1}{2\eta U_1^2} (h_1 y_1 - y_1^2) \left( p_1^{(0)} \frac{\partial p_1^{(0)}}{\partial x_1} - \frac{\partial p_{11}^{(1)}}{\partial x_1} \right) \] (7)

where: \( U_1 = 1 + L_1 \left( x_1 + 1 \right) \cos \gamma \).

The differential equation describing the oil pressure distribution in the gap of conical slide bearing has the following form:

\[ \frac{\partial}{\partial \varphi} \left( \frac{h_1^3}{\eta} \frac{\partial p_{11}^{(1)}}{\partial \varphi} \right) + U_1^2 \frac{\partial}{\partial x_1} \left( \frac{h_1^3}{\eta} \frac{\partial p_{11}^{(1)}}{\partial x_1} \right) = p_1^{(0)} \frac{\partial}{\partial \varphi} \left( \frac{h_1^3}{\eta} \frac{\partial p_1^{(0)}}{\partial \varphi} \right) + 
\]
\[ + p_1^{(0)} \frac{U_1^2}{L_1^2} \frac{\partial}{\partial x_1} \left( \frac{h_1^3}{\eta} \frac{\partial p_1^{(0)}}{\partial x_1} \right) + \frac{h_1^3}{\eta} \left( \frac{\partial p_{11}^{(1)}}{\partial \varphi} \right)^2 + U_1^2 \left( \frac{\partial p_{11}^{(1)}}{\partial x_1} \right)^2 \] (8)

where: \( U_1 = 1 + L_1 \left( x_1 + 1 \right) \cos \gamma \).

Eq. (8) was obtained from the eq. (6) at assumption that the oil vector velocity component \( v_{21}^{(0)} \) is equal to 0.

It easy to see that for \( \gamma = 90^\circ \) the eq. (8) is equivalent to the one describing the pressure distribution in journal slide bearing gaps.
4. Numerical calculations

Eq. (8) was written in the terms of the finite difference form (for the first derivation the forward scheme was applied, for the second derivation the central scheme was used) to yield the pressure field in the conical as well as journal slide bearings gap. Then, the corrections of pressure distribution and bearing capacity were found using Matlab 7.2. The numerical calculations were carried out for Gümbl’s conditions. The one were performed for some given values of $L_1 = 1$, $\lambda = 0,2,0,4$, tilt angle $\nu = 0,001^\circ$ and conical angles $\gamma = 90^\circ, 89^\circ$ and $\gamma_1 = 88,99^\circ$. The obtained results of pressure distribution and bearing capacities were presented in the graphical form (fig. 2 and fig. 3).

![Graphs showing corrected pressure distributions](image)

Fig. 2. Dimensionless corrections of pressure distribution $p_1^{(0)}$ with constant viscosity and corrections of pressure distributions $p_1^{(1)}$ taking into account the pressure-viscosity effect

Rys. 2. Rozkład bezwymiarowych korekt ciśnienia hydrodynamicznego $p_1^{(0)}$ w szczelinie stożkowego łożyska ślizgowego oraz bezwymiarowych korekt ciśnienia hydrodynamicznego $p_1^{(1)}$, wynikających z uwzględnienia przyrostu lepkości dynamicznej ze wzrostem ciśnienia
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Fig. 3. Values of the slide bearings capacity, 1 – journal slide bearing, 2 – conical slide bearing, $C_{i}^{(0)}$ – correction of the bearing capacity for the case with constant oil dynamic viscosity, $C_{i}^{(1)}$ – dimensionless correction of the bearing capacity taking into consideration the increase of oil dynamic viscosity caused by pressure.

Rys. 3. Bezwymiarowe wartości nośności łożyska ślizgowego, 1 – ślizgowe łożysko walcowe, 2 – ślizgowe łożysko stożkowe, $C_{1}^{(0)}$ – bezwymiarowe wartości korekt nośności łożyska ślizgowego dla przypadku ze stałą lepkością dynamiczną oleju, $C_{11}^{(1)}$ – bezwymiarowe wartości nośności łożyska dla przypadku uwzględniającego przyrost lepkości dynamicznej oleju ze wzrostem ciśnienia.

4. Conclusions

From the fig. 2 and 3 arise that the influence of the pressure-viscosity effect on hydrodynamic pressure distribution and bearing capacity is significant. The one strongly increases with the increases of the eccentricity $\lambda$. It is important to know that the numerical calculations were carried out for permanent values of tilt angle $\nu$ and conical angles $\gamma$ and $\gamma_1$. The one have very strong influence on pressure distribution and with this on bearing capacity. In order to estimate influence of pressure-viscosity effect on the total value of dimensionless bearing capacity $C_1$ the values of $C_{11}^{(1)}$ should be multiplied by coefficient $\varsigma$ ($\varsigma \approx 0.094$ [5]) and add to value of $C_1^{(0)}$. For example, for $\lambda = 0.4$ the increases in the $C_1$ value caused by the pressure-viscosity effect is by 14.5% for conical slide bearings and about by 55% for journal slide bearings.

Presented in this paper the mathematical model of height gap (eq. 1) and model describing the pressure distribution in the slide bearing gap (eq. 8) are equivalent to conical slide bearings as well as to journal slide bearings. Moreover, due to dimensionless form of this model the last is more general and available for large number of type-series of journal and conical slide bearings.

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Wpływ zmian lepkości dynamicznej oleju od ciśnienia na nośność stożkowego łożyska ślizgowego

Streszczenie

Podczas prawidłowego procesu projektowania stożkowych łożysk ślizgowych powinny być uwzględnione wszystkie czynniki mogące mieć istotny wpływ na jego parametry eksploatacyjne. Nośność łożysk ślizgowych zależy od wartości rozkładu ciśnienia hydrodynamicznego powstającego w jego szczelinie. Natomiast ciśnienie hydrodynamiczne zależy głównie od lepkości dynamicznej czynnika smarującego. W pracy, przedstawione zostały wyniki analizy numerycznej, której celem było określenie wpływu ciśnienia na lepkość dynamiczna oleju a przez to na ciśnienie hydrodynamiczne łożyska. Wyznaczone zostały odpowiednie korekty ciśnienia a następnie po ich uwzględnieniu wyznaczono nośność rozważanego w pracy łożyska.

W celu symulacji jak najbardziej dokładnych warunków pracy stożkowych łożysk ślizgowych przyjęto do opisu zmian szczeliny model analityczny uwzględniający wszystkie możliwe przypadki pracy łożyska stożkowego. W odróżnieniu od modelu klasycznego, który zawierał głównie człony określające zmiany wysokości szczeliny po kącie opasania i od mimośrodowości względnej zastosowany w obliczeniach numerycznych model uwzględnia dodatkowo możliwość wystąpienia przekoszenia czopa względem panewki, jak też dopuszcza różne wartości kątów rozwarcia stożków czopa i panewki. Dzięki takiemu modelowi analiza jest znacznie dokładniejsza i bliższa rzeczywistym warunkom pracy stożkowych łożysk ślizgowych.